GENETIC ALGORITHM BASED DESIGN
OF
POWER SYSTEM STABILIZERS

by

ADRIAN ANDREOIU

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Department of Electric Power Engineering
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ADRIAN ANDREIOIU

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Department of Electric Power Engineering
Chalmers University of Technology
SE-412 96 Göteborg
Sweden

Telephone: +46 (0)31 772 1632
Fax: +46 (0)31 772 1633
http://www.elteknik.chalmers.se

Cover: Distribution of solutions during genetic process for SMIB GA based PSS

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For large scale power systems comprising many interconnected machines, the problem of power system stabilizer (PSS) parameter tuning is a complex exercise due to the presence of several poorly damped modes of oscillation. The problem is further complicated by continuous variation in power system operating conditions. In the simultaneous tuning approach, exhaustive computational tools are required to obtain optimal parameter settings for the PSS, while in case of sequential tuning, although the computational burden is lesser, evaluating the tuning sequence is an additional requirement. There is a further problem of eigenvalue drift.

Genetic Algorithms (GA) are global search techniques providing a powerful tool for optimization problems by miming the mechanisms of natural selection and genetics. These operate on a population of potential solutions (individuals) applying the principle of survival of the fittest to produce better and better approximations to a solution. In each generation, a new set of approximations is created by selecting the individuals according to their level of fitness in the problem domain, and breeding them together using operators borrowed from natural genetics. Thus, the population of solutions is successively improved with respect to the search objective by replacing least fit individuals with new ones (offspring of individuals from the previous generation), better suited to the environment, just as in natural evolution.

This thesis proposes a novel approach using the powerful properties of GA, to simultaneously optimize the PSS parameter settings. The classical Lyapunov’s parameter optimization method using an Integral of Squared Error Criterion has been integrated with the GA search objective function. The method has been used for tuning of lead-lag-, derivative- and PID-PSS, and has been found to perform very satisfactorily.

The thesis also examines classical approaches to tuning of lead-lag and derivative PSS that consider one nominal operating condition. Investigations reveal that the classical approach does provide satisfactory performances for operating conditions up to the nominal but deteriorated responses when the load increases. However, the
classically tuned PSS fails to stabilize the system at certain operating conditions. The proposed GA based method on the other hand, provides the option of including any operating point within its tuning domain, thus ensuring system stability over a large domain, and in particular, the tuning domain.

*Keywords*: genetic algorithm, power systems, small-signal stability, power system stabilizers, design
PREFACE

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Göteborg
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<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$A$</td>
<td>state (plant) matrix of the system</td>
</tr>
<tr>
<td>$B$</td>
<td>control matrix</td>
</tr>
<tr>
<td>$D$</td>
<td>damping coefficient</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>perturbation matrix</td>
</tr>
<tr>
<td>$P$</td>
<td>active power</td>
</tr>
<tr>
<td>$Q$</td>
<td>reactive power</td>
</tr>
<tr>
<td>$Q$</td>
<td>weighing matrix</td>
</tr>
<tr>
<td>$H$</td>
<td>inertia constant</td>
</tr>
<tr>
<td>$I_d, I_q$</td>
<td>direct and quadrature components of armature current</td>
</tr>
<tr>
<td>$M$</td>
<td>inertia coefficient</td>
</tr>
<tr>
<td>$\delta$</td>
<td>rotor angle</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>denotes small perturbation or deviation of a variable from its steady-state value</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular speed</td>
</tr>
<tr>
<td>$x_{d}, x_q$</td>
<td>synchronous reactances in $d$ and $q$ axes, respectively</td>
</tr>
<tr>
<td>$x_{d}'$</td>
<td>direct axis transient reactance</td>
</tr>
<tr>
<td>$x_{q}'$</td>
<td>quadrature axis reactance</td>
</tr>
<tr>
<td>$E_{q,d}$</td>
<td>equivalent excitation voltage (field circuit voltage)</td>
</tr>
<tr>
<td>$E_{q}'$</td>
<td>internal voltage behind transient reactance $x_{d}'$</td>
</tr>
<tr>
<td>$f$</td>
<td>frequency</td>
</tr>
<tr>
<td>$J$</td>
<td>performance index</td>
</tr>
<tr>
<td>$K_{d}, T_{d}$</td>
<td>AVR gain and time constant, respectively</td>
</tr>
<tr>
<td>$K_C$</td>
<td>lead-lag PSS gain</td>
</tr>
<tr>
<td>$K_d$</td>
<td>derivative PSS gain</td>
</tr>
<tr>
<td>$K_p, K_{ip}, K_i$</td>
<td>proportional, derivative and integral components of PID-PSS</td>
</tr>
<tr>
<td>$T_W$</td>
<td>Washout time constant</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------</td>
</tr>
<tr>
<td>$T_m$, $T_e$</td>
<td>mechanical and electrical torque, respectively</td>
</tr>
<tr>
<td>$T_{do}'$</td>
<td>time constant of excitation circuit</td>
</tr>
<tr>
<td>$T_q$</td>
<td>sampling time</td>
</tr>
<tr>
<td>$V_d$, $V_q$</td>
<td>direct and quadrature components of terminal voltage</td>
</tr>
<tr>
<td>$Y$</td>
<td>admittance matrix</td>
</tr>
<tr>
<td>$T_i$, ..., $T_d$</td>
<td>lead-lag PSS time constants</td>
</tr>
<tr>
<td>$\mathbf{x}$</td>
<td>state vector</td>
</tr>
<tr>
<td>$\mathbf{p}$</td>
<td>perturbation vector</td>
</tr>
<tr>
<td>$s$</td>
<td>Laplace operator</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$u$</td>
<td>stabilizing signal</td>
</tr>
<tr>
<td>$^t$</td>
<td>transpose</td>
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</tbody>
</table>

ac alternative current
ANN Artificial Neural Networks
AVR Automatic Voltage Regulator
dc direct current
GA Genetic Algorithm
GEP Generator Exciter Power system
ISE Integral of Squared Error
MIMO Multi Input Multi Output
OLS Orthogonal Least Squares
PSS Power System Stabilizer(s)
PID Proportional Integral Derivative
RBF Radial Basis Function
RLS Recursive Least Squares
SMIB Single Machine to Infinite Bus
SISO Single Input Single Output
TGR Transient Gain Reduction
VSC Variable Structure Control
VSCPSS Variable Structure PSS
1 INTRODUCTION

1.1 Background

Power system stability problem has received a great deal of attention over the years. From the beginning, for convenience in analysis, gaining a better understanding of the nature of stability problems and developing solutions to the problems, it has been the usual practice to classify power system stability problems into two broad categories (Figure 1.1):

- **Angle Stability** - representing the ability of the system to maintain synchronism;
- **Voltage Stability** - representing the ability of the system to maintain steady acceptable voltage.

![Power System Stability Classification Diagram](image)

**Figure 1.1** Power System stability classification [1]
This work focuses on the angle stability category, with particular reference to the small-signal stability problem.

Amongst the different types of angular stability problems, the transient stability problem is related to the short term or transient period, which is usually limited to the first few seconds following the disturbance. It is concerned with the system response to a severe disturbance, such as transmission system fault. Much of the electric utility effort and interest related to system stability have been concentrated on the short-term response, and therefore the system is designed and operated so as to meet a set of reliability criteria concerning transient stability. Well-established analytical techniques and computer programs exist for the analysis of transient stability.

Small signal stability on the other hand is concerned with the system response to small changes and is a fundamental requirement for the satisfactory operation of power systems. Usually, the problem is one of ensuring sufficient damping of system oscillations. Small signal stability can be analyzed by linearizing the system about an equilibrium point represented by a steady state operating condition. This allows the use of powerful analytical tools of linear systems to determine the stability characteristics, which aid in the design of corrective controls.

In the past, many utilities took small signal stability for granted and carried out no studies at all to reveal problems related to small signal performance. This was primarily because in the past, a system that remained stable for the first few seconds following a severe disturbance was sure to be stable for small perturbations about the post fault system condition. This is not true for present day systems. As power systems have developed, the need for small signal studies and measures to ensure sufficient stability margins has being recognized.

Attention has been focused on the effect of excitation control on the damping of oscillations, which characterizes the phenomena of stability. In particular it has been found useful and practical to incorporate transient stabilizing signals derived from speed, terminal frequency or accelerating power superimposed on the normal voltage error signal to provide for additional damping to these oscillations. Such devices are known as Power System Stabilizers (PSS).

Power system stabilizers have been extensively used as supplementary excitation controllers to damp out the low frequency oscillations and enhance the overall system stability. The PSS extends the system stability limits by modulating generator excitation to provide damping to the oscillations of synchronous machine rotors relative to one another. They produce a component of torque in phase with rotor speed deviations, in order to enhance the system damping.
1.2 Brief Review of Literature on PSS Design

Over the last four decades, a large number of research papers have appeared in the area of PSS. Research has been directed towards obtaining such a PSS that can provide an optimal performance for a wide range of machine and system parameters. However, as noted in [2], a universally applicable stabilizing function is not practically feasible. Various control strategies and optimization techniques have found their applications in this area as also various degrees of system modeling have been attempted. While it is difficult to bring out a detailed discussion of the historical development of PSS and its applications, a modest attempt has been made in this section to discuss the most significant works in the area.

Heffron and Phillips [3] analyzed the effect of modern amplidyne voltage regulators on under-excited operation of large turbine generators. They were the first to present the small perturbation model in terms of $K_r-K_e$ constants of a machine-infinite bus system. Their investigations revealed that the use of modern continuously acting regulators greatly increased the steady-state stability limit of turbine generators in the under-excited region. And that the trend towards lowering the short circuit ratio of large turbine generators was sound from steady state stability standpoint, provided a modern continuously acting voltage regulator was used.

DeMello and Concordia [2] examined the case of a single machine connected to an infinite bus through external reactance. Their analysis developed insights into the effects of thyristor-type excitation systems and established understanding of the stabilizing requirements for such systems. These stabilizing requirements included the voltage regulator gain parameters as well as the PSS parameters. They explored the effect of a variety of machine loading, inertia and system external impedance (length of the transmission line) on damping characteristics of voltage or speed following a small perturbation in mechanical torque. They developed some unifying concepts that explained the stability phenomena of concern and predicted desirable phase and magnitude characteristics of stabilizing functions.

Larsen and Swann [4] presented application of PSS utilizing either speed, frequency or power input signals. Guidelines were presented for tuning PSS that enable the user to achieve desired dynamic performance with limited effort. The need for torsional filters in the PSS path for speed input PSS was also discussed.

Kundur et al. [5] described the details of a Delta-$P$-Omega PSS design for generating units in Ontario Hydro. Two alternate excitation schemes were considered, one with and the other without Transient Gain Reduction (TGR). It was shown that with appropriate selection of PSS parameters, both schemes provided
satisfactory performance. Appropriate choice of washout time-constant, PSS output limits and phase-lead compensation circuit parameters was demonstrated.

Yu and Siggers [6] presented the application of state-feedback optimal PSS while Moussa and Yu [7] proposed an eigenvalue shifting technique for determining the weighing matrix in the performance index. The technique involved shifting of the dominant eigenvalue to the left, on the $s$-plane until a satisfactory shift is made or the controller’s practical limit is reached. The optimal state-feedback controllers were also applied to a multi-machine system. However, in spite of the powerfullness of optimal control theory, the controllers so achieved, failed to appeal to utilities because their realization was difficult, cumbersome and costly.

A lot of work has also been reported on coordinated tuning of PSS for multi-machine systems. DeMello et al. [8] presented an eigenvalue-eigenvector analysis to identify the most effective generating units to be equipped with PSS in multi-machine systems that exhibit dynamic instability and poor damping of several inter-area modes of oscillations.

Fleming et al. [9] proposed a sequential eigenvalue assignment algorithm for selecting the parameters of stabilizers in a multi-machine power system. In sequential tuning, the stabilizer parameters are computed using repeated application of single-input/single-output (SISO) analysis. The suggested approach enables the selection of parameters of stabilizers such that a specified improvement in the damping ratio of each poorly damped mode can be realized approximately. The stabilizers are applied sequentially at different locations as ascertained by modal analysis outlined by DeMello et al. [8]. However, it should be noted that the sequential addition of stabilizers disturbs the previously placed eigenvalues to some extent.

Abdalla et al. [10] also presented a procedure for the selection of the most effective machines for stabilization. They suggested the addition of a damping term to each machine’s equation of motion, one at a time. An eigenvalue-based measure of relative improvement in the damping of oscillatory modes is implemented and used as a criterion to find the best candidate machine for stabilizer application.

The sequential tuning methods discussed in [8]-[10] are computationally simple compared to the simultaneous tuning methods, but they incur eigenvalue drift within the sequence. The eigenvalue drift problem does not arise with simultaneous tuning methods, and thus they provide the true optimal solution, but, on the other hand, these methods are computationally expensive.

Doi and Abe [11] proposed the coordinated design/tuning of PSS in multi-machine system by combining eigenvalue sensitivity analysis and linear programming. The PSS parameters are determined by minimizing a performance index, which is the
sum of all PSS gains. This method is simultaneously able to select generators where PSS can be effectively applied and to synthesize the adequate transfer function of the PSS for these generators.

Lim and Elangovan [12],[13] presented a method for designing decentralized stabilizers in a multi-machine system using complex frequency domain approach. Using this approach, the PSS parameters are obtained so that some or all of the system mechanical mode eigenvalues may be placed at the prescribed locations in the s-plane. The problem of exact eigenvalue assignment is transformed to that of solving iteratively a set of equivalent characteristic equations, the solution of which are the desired stabilizer parameters.

In all the above works we discussed, the PSS structure was considered to be fixed and were tuned considering a set of nominal operating conditions and system parameters.

However, such a fixed structure optimum PSS would provide sub-optimum performance under variations in system parameters and operating conditions. Several modern control strategies such as self-tuning control, variable-structure control (VSC), fuzzy-logic systems (FLS), artificial neural networks (ANN), genetic algorithms (GA), etc. have been reported in the recent literature, aiming to develop robust PSS configurations.

The applicability, advantages and disadvantages of minimum variance, pole assigned, linear quadratic and pole shifting adaptive controllers for power systems were examined in detail by Ghosh et al. [15]. They presented a comparison of system dynamic performances obtained using three alternate PSS, i.e., adaptive pole-shifting, adaptive linear quadratic and a conventional PSS. Their studies show that the adaptive pole-shifting PSS provides the best performance.

Cheng et al. [16] presented an adaptive PSS using a self-searching pole-shifting control algorithm. The adaptive PSS so designed is effective in damping system oscillations under both small as well as large perturbations. Cheng et al [17] further proposed a dual-rate adaptive PSS based on self-searching pole-shifting algorithm for damping multi-mode oscillations. In this algorithm the system parameters are identified every 80 msec while the control signal is updated every 20 msec.

Lim [18] proposed a method for designing a self-tuning PSS based on the minimization of a quadratic performance index. The effectiveness of the self-tuning PSS for either excitation or governor control under different disturbances and over a wide range of operating conditions has been demonstrated.

As an alternative to self-tuning PSS, Variable Structure PSS (VSPSS) has been proposed in the literature in order to counteract the problem of variation of system
parameters and operating condition. The VSC are insensitive to system parameter variations and can easily be realized using microcomputers. A systematic procedure for the selection of the proper switching vector is though very important for their design.

Chan and Hsu [23] proposed an optimal VSPSS for a machine-infinite bus system as well as for a multi-machine system. The proposed VSPSS is optimal in the sense that the switching hyperplane is obtained by minimizing a quadratic performance index, in the sliding mode operation. The resulting switching vector and hence the switching hyperplane depends on the weighing matrices associated with the performance index.

Kothari et al. [24] have presented the design of a VSPSS with desired eigenvalues in the sliding mode, where the switching hyperplane is obtained using a pole placement method. This has been further extended in [25] to apply a radial pole-shifting technique for design of VSPSS in the discrete-mode.

A fuzzy set theory based PSS was reported by Hsu and Cheng in [26]. The proposed stabilizer adopted a decentralized output feedback control law that required only local measurements within each generating unit, thus providing scope for further implementation.

In [27], Hoang and Tomsovic proposed a systematic approach to fuzzy logic control design, where the controller parameters are either calculated off-line or computed in real time in response to system changes. In this design approach it was shown that the controller is insensitive to the precise dynamics of the system.

Artificial neural network is based on the concept of parallel processing and has great ability in realizing complicated non-linear mappings from the input space to the output space, thus providing an extremely fast processing facility for complicated non-linear problems.

Zhang et al. [28] presented a PSS design approach that employs the multi-layer perceptron with error back-propagation training method. The ANN was trained with the training data group generated by an adaptive power system stabilizer.

In [29], Segal et al. presented a new approach for real-time tuning of conventional PSS using a radial basis function network, which is trained using an orthogonal least squares (OLS) learning algorithm.

Abido and Abdel-Magid presented in [30] a PSS design that combines numerical (ANN) and linguistic (FLS) information in a uniform fashion, thus providing a model-free description of the control system and overcoming the ANN and FLS weaknesses and facilitates on-line implementation.
1.3 Genetic Algorithms in PSS Tuning

Genetic Algorithms are global search techniques providing a powerful tool for optimization problems by miming the mechanisms of natural selection and genetics. These operate on a population of potential solutions applying the principle of survival of the fittest to produce better and better approximations to a solution. In each generation, a new set of approximations is created by selecting the individuals according to their level of fitness in the problem domain and breeding them together using operators borrowed from natural genetics [31]. Thus, the population of solutions is successively improved with respect to the search objective by replacing least fit individuals with new ones (offspring of individuals from the previous generation), better suited to the environment, just as in natural evolution.

According to Goldberg [31], GA are different from other optimization and search procedures in four ways:

1. GA work with a coding of the parameter set, not the parameters themselves.
2. GA search from a population of points, not a single point.
3. GA use payoff information, not derivatives or other auxiliary knowledge
4. GA use probabilistic transition rules, not deterministic rules.

Figure 1.2 shows a schematic diagram of a genetic algorithm. The process commences with random generation of a pool of possible solutions, i.e. the population and the individuals that form it. Each individual in the population, also called chromosome is represented by a string, which is formed by a number of sub-strings equal to the number of the problem's variables. Each variable is coded in a suitable coding system (binary, integer, real-valued, etc). The population size and the chromosome size are kept constant during the whole search process.

The performance of each individual in the population is evaluated through an objective function, which models the dynamic problem and has as output a fitness value. The fitness value is a measure of how good the respective individual is with respect to the problem objective.

Individuals will be selected in accordance with their fitness value to take part in the genetic process. The purpose of selection is to keep the best well-fit individuals and increase the number of their offspring in the next generation, on the account of the least fit individuals.

The recombination process consists in the grouping of the selected individuals in pairs (i.e. parents) in which they exchange genetic information forming two new individuals (i.e. children, or offspring). This process helps the optimization search to escape from possible local optima and search different zones of the search space.
A *mutation* genetic operator that replaces allele of genes is implemented to increase the probability of complete search, by allowing the investigations in vicinity of local optima.

![Genetic Algorithm flowchart](image)

**Figure 1.2** Genetic Algorithm flowchart

*Reinsertion* is the process in which children will populate the next generation by replacing parents. Reinsertion can be made partially or completely, uniformly (offspring replace parents uniformly at random) or fitness-based. All genetic operators are implemented with a certain predefined probability.

Power System Stabilizers designed using the GA based search and optimization are more likely to converge to a global optima than a conventional optimization based PSS, since they search from a population of possible solutions, and are based on probabilistic transition rules.

In the recent literature, applications of genetic algorithm to tune the parameters of PSS have been reported [32]-[35]. A GA based optimization method has been used in [33] to tune the parameters of a rule-based PSS. This way, the advantages of the
rule-based PSS such as its robustness, less computational burden and ease of realization are maintained. Introduction of GA helps obtain an optimal tuning for all PSS parameters simultaneously, which thereby takes care of interactions between different PSS.

In [34], simultaneous tuning for all the PSS in the system using a GA based approach has been developed. The GA seeks to shift all eigenvalues of the system within a region in the stable domain.

Zhang and Coonick [35] proposed a GA based computational procedure to select PSS parameters simultaneously in multi-machine power systems, by solving a set of inequalities that represent the objectives of optimization problem.

1.4 Outline of the Thesis

This thesis examines the application of Genetic Algorithms to PSS tuning in order to determine a globally optimum PSS parameter set that will ensure a stable and robust operation of a multi-machine power system, for each operating point within a wide range.

Chapter 1 introduces the problem of small-signal stability in power systems, with emphasis on the low frequency oscillation phenomena occurring due to small disturbances and its mitigation by means of PSS. A review of literature discusses the relevant work in this area of tuning of PSS and lays down the motivations and objectives of the work. The field of Genetic Algorithms is also introduced, as the method chosen to perform the simultaneous tuning of PSS parameters.

Chapter 2 presents the small-signal stability models of single machine connected to infinite bus (SMIB) and multi-machine power systems. The mathematical formulations have been detailed in Appendices for interested readers. The models have been formulated in state-space form and their open-loop (uncontrolled mode) characteristics have been examined.

Chapter 3 presents the classical optimization method based on Lyapunov’s parameter optimization to tune the parameters of the lead-lag PSS. Phase compensation characteristics of the lead-lag PSS have been examined and a method of PSS tuning using the exact phase compensation approach has been developed. Further, the Lyapunov’s parameter optimization method has been extended to PSS tuning in multi-machine systems.

Chapter 4 presents the application of a GA search on the classical Integral of Squared Error (ISE) based method for tuning of PSS. The method ensures that for any operating condition within a pre-defined domain, the system remains stable.
when subjected to small perturbations. The optimization criterion employs a quadratic performance index that measures the quality of system dynamic response within the tuning process. The solution thus obtained is globally optimal and robust. The proposed method has been tested on different PSS structures— the conventional lead-lag and the derivative type. System dynamic performances with PSS tuned using the proposed technique are satisfactory for different load conditions and system configurations.

Chapter 5 considers the optimum tuning of fixed structure proportional plus integral plus derivative (PID) PSS for the single-machine infinite bus and multi-machine power systems. A GA based tuning technique is developed and tested for SMIB and multi-machine power systems. The tuning scheme proposed in this chapter uses a genetic algorithm (GA) based search that integrates a classical parameter optimization criterion based on Integral of Squared Error (ISE). This method succeeds in achieving a robust, simultaneously tuned and globally optimal PID-PSS parameter set, while maintaining the simplicity of the classical optimization method. The tuning method implicitly builds-in an increased robustness through an objective function, that depends on the operating domain. The system is represented in a discrete state-space form and the influence of the sampling time on the PSS parameter tuning is investigated.

Chapter 6 highlights the significant contributions of the present work and draws the scope for future work in this area.
2 Dynamic Models for Small Signal Stability Analysis

2.1 Background
Small-signal stability is the ability of the power system to maintain synchronous operation when subjected to small disturbances. Since the disturbance is considered to be small, the equations that describe the resulting dynamics of the system may be linearized. Instability that may result can be of two types:
   a) steady increase in generator rotor angle due to lack of synchronizing torque;
   b) rotor oscillations of increasing amplitude due to lack of sufficient damping torque.

In today’s practical power systems, the small-signal stability problem is usually one of insufficient damping of system oscillations.

For the analysis of small-signal stability, linearized models are generally considered to be adequate for representation of the power system and its various components.

2.2 General approach
The state-space representation is concerned not only with input and output properties, but also with its complete internal behavior. In contrast, the transfer function representation specifies only the input/output behavior. If state-space representation of a system is known, the transfer function is uniquely defined. In this sense, the state-space representation is a more complete description of the system, and it is ideally suited for the analysis of multi-variable MIMO systems.
In the development of a dynamic model for a multi-machine power system (classical stability model), the following assumptions are usually made [36]:

a) Mechanical power input is constant.
b) Damping or asynchronous power is negligible.
c) Constant-voltage-behind-transient-reactance model for the synchronous machines is valid.
d) The mechanical rotor angle of a machine coincides with the angle of the voltage behind the transient reactance.
e) Loads are represented by passive impedances.

To study the dynamic behavior of a system, the following data are needed:
- System data (lines, buses, transformers, machines);
- Normal load-flow data ($\overline{S}_i$ and $\overline{V}_i$ - the complex power and voltage at generator nodes, respectively).

Each machine model is first expressed in its own $d$-$q$ frame, which rotates with its rotor. For the solution of interconnecting network equations, all voltages and currents must be expressed in a common reference frame. The real-axis of one machine, rotating at synchronous speed, is used as the common reference. Axis transformation equations are used to transform between the individual machine ($d$-$q$) reference frames and the common ($D$-$Q$) reference frame. The real-axis of the common reference frame is used as the reference for measuring the machine rotor angle. For a machine represented in detail, including dynamics of rotor circuit(s), the rotor angle is defined as the angle by which the machine $q$-axis leads/lags the real axis. Under dynamic conditions, the angle $\delta$ changes with rotor speed [37].

The following calculations are essential in order to prepare the system for a stability study:

1. All system data are converted to a common base.

2. The loads are converted to equivalent admittances. The needed data is taken from a load-flow study. The equivalent shunt admittance at the bus is given by:

$$
\overline{Y}_l = \frac{T_L}{\overline{V}_L} = \frac{\overline{S}_L}{\overline{V}_L^2} = \frac{P_L}{V_L^2} - j \frac{Q_L}{V_L^2} \tag{2.1}
$$

where $\overline{V}_L$, $P_L$, $Q_L$, and $\overline{I}_L$ are the voltage, active power, reactive power and current, respectively, corresponding to a load admittance $\overline{Y}_L = G_L + jB_L$. 

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3. The internal voltages of a generator \( E \angle \delta \) are calculated from a load-flow run. The internal angle of the generator during transients (\( \delta \)) is computed from the pre-transient terminal voltages \( V \angle \alpha \).

4. The admittance matrix \( \mathbf{Y} \) of the network is calculated. \( \mathbf{Y} \) is a \( n \times n \) matrix, where \( n \) is the total number of buses.

5. Obtain the admittance matrix for the reduced network (\( \mathbf{Y}_m \)) by eliminating all the nodes that are not internal generator nodes. All nodes except for the internal generator nodes should have zero injection currents, and this property is used to obtain the network reduction [36].

Note that \( \mathbf{Y}_m \) is a \( n_g \times n_g \) dimension matrix, where \( n_g \) is the number of generators. In Appendix I, Section 8.1.1 provides details of the above.

2.3 Small Perturbation Dynamic Model of the System

The transfer function block diagram (Figure 2.1) describes the dynamics of the \( i^{th} \) machine in a multi-machine system [38]. This is a generalization of the extensively used single machine connected to infinite bus transfer function block diagram [2] and takes into account the interaction between machines via \( \mathbf{K} \) matrices, which are square matrices of order \( n_g \). The diagonal elements of the \( \mathbf{K}_1, \ldots, \mathbf{K}_6 \) matrices determine the machine’s dynamics, while the off-diagonal elements model the dynamic interactions between machines. Observe that in this block diagram the PSS is not represented, for convenience. The number of state variables is \( n \times n_g \), where \( n \) is the number of state variables used to model a machine and its excitation system.

For the calculation of \( \mathbf{K}_1, \mathbf{K}_2, \ldots, \mathbf{K}_6 \) matrices, the armature current components \( I_d \) and \( I_q \), and the terminal voltage components \( V_d \) and \( V_q \) of each machine are expressed with respect to the common frame of reference.

During low-frequency oscillations, the current induced in a damper winding is negligibly small; hence the damper windings are completely ignored in the system model. As for the \( d \) and \( q \)-axis armature windings of the synchronous machine, their natural oscillating frequency being extremely high, their eigenmodes will not affect the low-frequency oscillations, and hence can be described simply by algebraic equations [38]. What is left is the field winding circuit of the machine, which is described by a differential equation, not only because of its low eigenmode frequency, but also because it is connected directly to the excitation system to which the supplementary excitation control is applied. The excitation
system itself must be described by differential equations. Finally, the torque
differential equation of the synchronous machine is included in the model.

\[
\begin{align*}
\frac{d}{dt} (\Delta \omega_i) &= -\frac{D_i}{M_i} \cdot \Delta \omega_i - \frac{K_{1j}}{M_i} \cdot \Delta \delta_j - \frac{K_{2j}}{M_i} \cdot \Delta E'_{q_j} + \frac{1}{M_i} \cdot \Delta T_{M_i} \\
\frac{d}{dt} (\Delta \delta_i) &= 2 \cdot \pi \cdot f \cdot \Delta \omega_i \\
\frac{d}{dt} (\Delta E'_{q_i}) &= -\frac{K_{4j}}{T_{do_i}} \cdot \Delta \delta_j - \frac{1}{T_{do_i} \cdot K_{3j}} + \frac{1}{T_{do_i}} \cdot \Delta E_{fj_i} \\
\frac{d}{dt} (\Delta E_{fj_i}) &= \frac{K_{4j} \cdot K_{5j}}{T_{A_i}} \cdot \Delta \delta - \frac{K_{4j} \cdot K_{6i}}{T_{A_i}} \cdot \Delta E'_{q_i} - \frac{1}{T_{A_i}} \cdot \Delta E_{fj_i} + \frac{K_{4j}}{T_{A_i}} \cdot u_{E_i} \\
&\quad \forall i = 1, \ldots, n_e
\end{align*}
\]

Figure 2.1  Transfer function block diagram representation of a multi-machine system
for small-signal stability analysis
It is to be noted here that when \( n_g = 1 \), the above set of equations reduce to the well-known SMIB system representation.

Using vector-matrix notation, the set of equations (2.2) can be represented in state-space form as follows:

\[
\frac{d}{dt} x(t) = A \cdot x(t) + B \cdot U(t) + \Gamma \cdot p(t)
\]

(2.3)

In equation (2.3), \( A, B \) and \( \Gamma \) are the state, control and perturbation matrices respectively, and \( x(t), U(t) \) and \( p(t) \) are state, control and perturbation vectors, respectively.

State matrix \( A \) is a function of the system parameters and operating conditions, while the perturbation matrix \( \Gamma \) and control matrix \( B \) depend on system parameters only.

For the system operating conditions and parameters considered (see Appendix II), the system eigenvalues are obtained by solving the characteristic equation of the system.

The stability characteristic of the system is dependent on the eigenvalues of the state matrix as follows:

a) A real eigenvalue corresponds to a non-oscillatory mode. A negative real eigenvalue represents a decaying mode, while a positive real eigenvalue represents aperiodic instability.

b) A pair of complex eigenvalues represents an oscillatory mode. The real component of the eigenvalue gives the damping, and the imaginary component gives the frequency of oscillation. A negative real part represents a damped oscillation whereas a positive real part represents oscillation of increasing amplitude.

2.4 Systems Investigated

The systems considered for analysis in this thesis are the single machine connected to infinite bus through a double circuit transmission line (Figure 1.2), and the well-known nine-bus power system [36], which has three generators and three loads (Figure 2.3).
In all generators, IEEE Type ST-1 excitation systems have been considered. System parameters and operating data for both systems investigated have been provided in Appendix II.

\[
 Y = G + jB
\]

**Figure 2.2**  Single machine connected to an infinite bus system

**Figure 2.3**  3-machine system [36]
2.5 Single Machine Infinite Bus (SMIB) System

The small-perturbation transfer-function block diagram of a SMIB system is shown in Figure 2.4.

The state-space model can be expressed as follows:

\[
\frac{d}{dt} \mathbf{x}(t) = A \cdot \mathbf{x}(t) + b \cdot u(t) + \Gamma \cdot \rho(t) \quad (2.4)
\]

where

\[
A = \begin{bmatrix}
\frac{D}{M} & -\frac{K_1}{M} & -\frac{K_2}{M} & 0 \\
\frac{2\pi f}{M} & 0 & 0 & 0 \\
0 & \frac{K_4}{T_{dv}} & -1 & 1 \\
0 & -\frac{K_4}{T_A} \cdot K_5 & \frac{K_4}{T_A} \cdot K_6 & -1 \\
\end{bmatrix}
\]

\[
\mathbf{x} = \begin{bmatrix}
\Delta \omega \\
\Delta \delta \\
\Delta E_q \\
\Delta E_{fd}
\end{bmatrix}^T
\]
Note that the control signal $u$, which is actually the PSS output, would act on the summing junction of the terminal voltage reference of the AVR-excitation system Figure 2.4. Also note that $u$ is now a scalar, since we deal with a SISO system. The perturbation considered consists of a step increase of 1% in the mechanical torque $T_m$ of the synchronous generator.

Table 2.1 shows the open-loop eigenvalues of the SMIB system for the nominal operating point and system parameters considered. Evidently, the system is unstable under small perturbations and require a stabilizing signal from the PSS. The system dynamics without PSS are shown in Figure 2.5 and Figure 2.6.

<table>
<thead>
<tr>
<th>Operating point [p.u.]</th>
<th>Eigenvalues</th>
<th>Damping factor</th>
<th>Natural frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>P = 0.8</td>
<td>0.1028 ± j5.5022</td>
<td>-0.0187</td>
<td>0.875</td>
</tr>
<tr>
<td>Q = 0.6</td>
<td>-6.3710</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>-14.2975</td>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 2.5  Rotor angle deviation for SMIB system under small perturbation
As discussed in Section 2.4, the three-generator, nine-bus system [36] has been considered for our analysis of multi-machine systems. The same set of state variables as that used for SMIB system has been used to describe each machine of the multi-machine system behavior through state-space modeling approach. The state-space model for the three-machine system is expressed as follows:

\[
\frac{d}{dt} \dot{x}(t) = A \cdot \dot{x}(t) + B \cdot u(t) + \Gamma \cdot \bar{p}(t) \tag{2.5}
\]

where \( \dot{x}(t) \), \( U(t) \) and \( p(t) \) are state, control and perturbation vectors, respectively and they are expressed as follows:

\[
\dot{x}(t) = [\Delta \omega_1(t) \Delta \delta_1(t) \Delta E_{q1}(t) \Delta E_{f1}(t) \Delta \omega_2(t) \Delta \delta_2(t) \Delta E_{q2}(t) \Delta E_{f2}(t) \Delta \omega_3(t) \Delta \delta_3(t) \Delta E_{q3}(t) \Delta E_{f3}(t) \ldots \Delta \omega_3(t) \Delta \delta_3(t) \Delta E_{q3}(t) \Delta E_{f3}(t)]^T
\]

\[
U(t) = [\Delta u_1(t) \Delta u_2(t) \Delta u_3(t)]^T
\]

\[
p(t) = [\Delta T_{m1} \Delta T_{m2} \Delta T_{m3}]^T
\]
A, B and Γ are the state, control and perturbation matrices, respectively. They constructed by extending the SMIB model to the 3-machine model and are given in Appendix III Section 8.3.1.

This system is analyzed in a similar manner as the SMIB system. Figure 2.7 and Figure 2.8 show the time response of angular speed- and rotor angle deviations of all the machines of the system without PSS, when a step perturbation in mechanical input of 1% occurs at the shaft of Generator 1.

![Graph](image)

**Figure 2.7** Angular speed deviations of all machines when Generator 1 is perturbed
Table 2.2 presents the eigenvalues and corresponding damping factors and natural frequencies of oscillations. As also suggested by the figures is now evident that the system is unstable under small perturbations on generator 1.

<table>
<thead>
<tr>
<th>Operating point</th>
<th>Eigenvalues</th>
<th>Damping factor</th>
<th>Natural frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₁ = 0.7160</td>
<td>0.0524 ± j8.0334</td>
<td>-0.0065</td>
<td>1.2786</td>
</tr>
<tr>
<td>P₂ = 1.6300</td>
<td>0.0479 ± j6.3195</td>
<td>-0.0076</td>
<td>1.0058</td>
</tr>
<tr>
<td>P₃ = 0.8500</td>
<td>-0.0447 ± j15.3935</td>
<td>-0.0029</td>
<td>2.4500</td>
</tr>
<tr>
<td>Q₁ = 0.2700</td>
<td>-10.3506 ± j2.9481</td>
<td>0.9618</td>
<td>1.7129</td>
</tr>
<tr>
<td>Q₂ = 0.0670</td>
<td>-10.3957 ± j6.6547</td>
<td>0.8156</td>
<td>2.0716</td>
</tr>
<tr>
<td>Q₃ = -0.1090</td>
<td>-4.8010</td>
<td>1.0000</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>-15.5527</td>
<td>1.0000</td>
<td>-</td>
</tr>
</tbody>
</table>
2.7 Concluding Remarks

This chapter presents the details of mathematical models required for the analysis of small signal stability for both single machine connected to infinite bus and multi-machine systems. The mathematical models are in the state-space form, thereby making the application of linear analysis possible. As has been demonstrated, both systems are unstable under small perturbations and require additional stabilizing control from the power system stabilizer, the design of which will be treated in the following chapters.
3 CONVENTIONAL APPROACHES TO DESIGN OF LEAD-LAG PSS

Two distinct types of system oscillations are usually recognized in interconnected power systems [5]. One type is associated with units at a generating station swinging with respect to rest of the power system. Such oscillations are referred to as "local mode" oscillations and have a frequency in the range of 0.8 to 2.0 Hz. The term local is used because the oscillations are localized at one power plant. The second is associated with swinging of many machines in one part of the system against machines in another part. These are "inter-area mode" oscillations, and have frequencies in the range of 0.2 to 0.7 Hz.

The basic function of a power system stabilizer (PSS) is to add damping to the generator rotor oscillations by controlling its excitation using auxiliary stabilizing signal(s). To provide damping, the PSS must produce a component of electrical torque in phase with the rotor speed deviations.

3.1 General Characteristics of Lead-lag Power System Stabilizers

3.1.1 Performance Objectives of PSS
The overall excitation control system (including PSS) is designed to [5]:

a) Maximize the damping of the local plant mode as well as inter-area mode oscillations without compromising the stability of other modes;
b) Enhance system transient stability;
c) Not adversely affect system performance during major system upsets which cause large frequency excursions;
d) Minimize the consequences of excitation system malfunction due to component failures.

Since the purpose of a PSS is to introduce a damping torque component, the speed deviation represents an appropriate signal to be used as input for the PSS. In practice, both generator and its exciter exhibit frequency dependent gain and phase characteristics, $GEP(s)$. Hence, the PSS transfer function should have appropriate phase-lead circuits to compensate for the phase lag between the exciter input and the electrical torque [37].

For large values of $K_A$ and the usual range of constants, the composite transfer-function for $GEP(s)$ and the corresponding phase-lag characteristic can be written as follows [2]:

$$GEP(s) = \frac{K_2}{K_6 \cdot \left(1 + s \cdot \frac{T_{do}'}{K_6 \cdot K_A}\right) \cdot \left(1 + s \cdot T_A\right)}$$

(3.1)

$$\angle GEP = \tan^{-1}\left(\frac{\omega \cdot T_{do}'}{K_6 \cdot K_A}\right) + \tan^{-1}\left(\omega \cdot T_A\right)$$

![Figure 3.1 Phase-lag characteristic of GEP(s) for a SMIB system](image)
Figure 3.1 shows a plot of the phase-lag introduced by $GEP(s)$, as a function of the frequency. It can be seen that for the frequency range of interest (0.2 to 2.0 Hz), the phase lag due to $GEP(s)$ is between $25^\circ$ to $105^\circ$.

The transfer function of the lead-lag PSS on the $i^{th}$ machine is shown in equation (3.2) and the corresponding block diagram is shown in Figure 3.2.

$$G_{PSS}(s) = \frac{\Delta u_i(s)}{\Delta \omega_i(s)} = K_{Cl} \cdot \left[ \frac{s \cdot T_{W_i}}{1 + s \cdot T_{W_i}} \right] \cdot \left[ \frac{1 + s \cdot T_{1_i}}{1 + s \cdot T_{2_i}} \right] \cdot \left[ \frac{1 + s \cdot T_{3_i}}{1 + s \cdot T_{4_i}} \right]$$

(3.2)

![Figure 3.2 Transfer function block diagram for Lead-lag PSS](image)

The corresponding phase-lead characteristic of the lead-lag PSS is given by:

$$\angle G_{PSS} = \frac{\pi}{2} - \tan^{-1}(\omega \cdot T_w) + 2 \cdot \left[ \tan^{-1}(\omega \cdot T_1) - \tan^{-1}(\omega \cdot T_2) \right]$$

(3.3)

Note that the phase angle of PSS signal was expressed in (3.3) under the assumption that $T_1 = T_3$ and $T_2 = T_4$ for the purpose of simplification.

![Figure 3.3 PSS phase lead characteristics for different time constants, $T_1 = T_3$ and $T_2 = T_4 = 0.05$ sec](image)
Figure 3.3 shows the phase-lead characteristics of the PSS for several values of $T_f$. In the figure, we note that for the frequency range under consideration, the lead-lag PSS can provide up to about 90° phase compensation, with $T_f = 0.3$ seconds.

The parameters of the lead-lag PSS are required to be tuned optimally, in order to obtain the best performance of the system.

### 3.1.2 Primary Considerations for the Selection of Lead-lag PSS Parameters

In principle, the lead-lag PSS (also regarded as the conventional\(^1\) PSS) consists of three blocks: a phase compensation block, a signal washout block and a gain block (refer Figure 3.2).

The phase compensation block provides the appropriate phase-lead characteristic to compensate for the phase lag between the exciter input and the generator electrical torque $GEP(s)$. The phase characteristic to be compensated changes with the system conditions, therefore a characteristic acceptable for a range of frequencies (normally 0.1 to 2.0 Hz) is sought. This may result in less than optimum damping at any one frequency. The required phase lead can be obtained by choosing the appropriate values of time constants $T_i, ..., T_s$.

The signal washout block functions as a high-pass filter, which allows the dc signals to pass unchanged, thus avoiding terminal voltage variation due to steady changes in speed. The washout time constant $T_w$ should be long enough to pass stabilizing signals at the frequencies of interest unchanged, but not so long that it leads to undesirable generator voltage excursions during system-island conditions [37].

The stabilizing gain $K_c$ determines the amount of damping introduced by the PSS, and, ideally, it should be set to a value corresponding to maximum damping. However, in practice the gain is set to a value that results in satisfactory damping of the critical system modes without compromising the stability of other modes, or transient stability, and that does not cause excessive amplification of PSS input signal noise.

In order to restrict the level of generator terminal voltage fluctuation during transient conditions, limits are imposed on PSS outputs.

The PSS parameters to be optimized are the time constants, $T_{i1}, T_{i2}, T_{i3}, T_s$ and gain $K_c, T_w = 10$ seconds is chosen at all machines in order to ensure that the phase-lead and gain contributed by the washout block for the range of oscillation

---

\(^1\) In this thesis, the terms "lead-lag PSS" and "conventional PSS" are used interchangeably.
frequencies normally encountered is negligible [5]. The number of PSS parameters to be optimized is reduced by considering the PSS to be comprising two identical cascaded lead-lag networks. Therefore, \( T_{12} = T_{22} \) and \( T_{32} = T_{43} \). Also, \( T_{32} = T_{43} = 0.05 \) seconds is assumed fixed from physical realization considerations [2]. Thus, the optimization problem reduces to determining \( T_{12} \) and \( K_{c1} (i = 1, \ldots, n) \) only.

### 3.2 Analysis of a Single Machine Infinite Bus System with a Lead-lag PSS

#### 3.2.1 Composite Model

Figure 3.4 shows the composite transfer-function block-diagram of the SMIB system equipped with a lead-lag PSS.

![Small perturbation block diagram of a single machine to infinite bus system equipped with a lead-lag PSS](image)

The linear dynamic model of the open-loop power system (i.e. without PSS) was discussed in detail in Chapter 2. Following the same convention, the representation of the above composite system inclusive of the PSS can be described by:

\[
\frac{d}{dt} x(t) = A \cdot x(t) + \Gamma \cdot p(t) \tag{3.4}
\]
\( A \), and \( \Gamma \) are the state and perturbation matrices, respectively, \( \dot{x}(t) \) the state vector, and \( p(t) \) is the perturbation. The state vector \( \dot{x}(t) \) is given as follows:

\[
\dot{x}(t) = \begin{bmatrix}
\Delta \omega \\
\Delta \delta \\
\Delta E_q^r \\
\Delta E_{fd} \\
\Delta N_1 \\
\Delta N_2 \\
\Delta u
\end{bmatrix}
\]  

(3.5)

Note that the two additional intermediate variables of the PSS, namely \( \Delta N_1 \) and \( \Delta N_2 \) appear in (3.4) as state variables, as does the PSS output signal \( \Delta u \).

The state matrix \( A \) of this system is given by:

\[
A = \begin{bmatrix}
0 & -\frac{K_1}{M} & -\frac{K_2}{M} & 0 & 0 & 0 & 0 \\
2 \cdot \pi \cdot f & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{K_4}{T_{do}} & -\frac{1}{T_{do}} & \frac{1}{T_{do}} & 0 & 0 & 0 \\
0 & -\frac{K_A \cdot K_4}{T_A} & -\frac{K_A \cdot K_6}{T_A} & \frac{1}{T_A} & 0 & 0 & K_A \\
-K \cdot T_1 & -K \cdot T_2 & -K \cdot T_3 & 0 & -\frac{1}{T_2} & 0 & 0 \\
-T \cdot T_1 & -T \cdot T_2 & -T \cdot T_3 & 0 & \frac{1}{T_2} & \frac{1}{T_2} & \frac{1}{T_2} \\
-K \cdot T_1 & -K \cdot T_2 & -K \cdot T_3 & 0 & \frac{1}{T_2} & \frac{1}{T_2} & \frac{1}{T_2} \\
-T \cdot T_1 & -T \cdot T_2 & -T \cdot T_3 & 0 & \frac{1}{T_2} & \frac{1}{T_2} & \frac{1}{T_2}
\end{bmatrix}
\]

3.2.2 Optimization of PSS Parameters Using Phase Compensation Technique

The phase compensation technique is based on the objective of tuning the PSS parameters to fully compensate for the phase lag introduced through the exciter and generator characteristics \( GEP(s) \), such that the torque changes provided by the PSS are in phase with the rotor speed deviations. The following step by step approach is used:

a) Find the natural frequency of oscillation \( \omega_n \) of the electromechanical mode. Neglecting the damping, the characteristic equation of the mechanical loop may be written as:

\[
M \cdot s^2 + 2 \cdot \pi \cdot f \cdot K_1 = 0
\]  

(3.6)

and the solutions are:
\[ s = \pm j \omega_n, \quad \omega_n = \sqrt{2 \cdot \pi \cdot f \cdot K_i / M} \tag{3.7} \]

where \( M \) is the inertia constant in seconds
\( f \) is the system frequency in Hz
\( K_i \) is the synchronizing torque coefficient

For the present system investigated and the operating conditions described in Section 8.2, \( K_i = 0.9538 \), and the natural frequency of oscillation of the electromechanical mode is \( \omega_n = 5.474 \) rad/sec.

b) Find the phase lag between \( \Delta u \) and \( \Delta E_q' \) of the electrical loop using the relationship established in (3.1). For \( \omega_n = 5.474 \) rad/sec we find that the phase-lag introduced by \( GEP(s) \) equals 69.87°.

c) Obtain using (3.3) the PSS phase-lead time constants that exactly compensate for the system phase-lag. In the given system, to exactly compensate for the phase-lag of 69.87°, it was found that the phase-lead time constant required is \( T_1 = T_2 = 0.216 \) sec. Figure 3.5 shows the phase-lag characteristic of \( GEP(s) \) vis-à-vis the phase-lead characteristic of the lead-lag PSS for \( T_1 = T_3 = 0.216 \) sec. It can be seen that the so tuned PSS closely compensates for the phase-lag in a frequency range up to 1 Hz and compensates accurately for the electromechanical mode.

![Graph showing phase-lag characteristic and PSS phase characteristics](image)

**Figure 3.5** Open-loop SMIB system and PSS phase characteristics
3.2.3 Lyapunov's Method Based Optimization of PSS Parameters

3.2.3.1 The Performance Index

The choice of a suitable performance index is extremely important for the design of PSS. In this thesis, a performance index as given in (3.8), where \( \mathbf{x} \) is the state vector and \( \mathbf{Q} \) is a weighing matrix has been used:

\[
J = \int_{0}^{\infty} (\mathbf{x}^T \cdot \mathbf{Q} \cdot \mathbf{x}) \, dt
\]  

(3.8)

The performance index \( J \) can be evaluated using the relation:

\[
J = \mathbf{x}^T (0) \cdot \mathbf{P} \cdot \mathbf{x}(0)
\]  

(3.9)

where \( \mathbf{x}(0) \) is the initial state of the state vector, and \( \mathbf{P} \) is a positive definite symmetric matrix obtained by solving the Lyapunov's equation:

\[
\mathbf{A}^T \cdot \mathbf{P} + \mathbf{P} \cdot \mathbf{A} = -\mathbf{Q}
\]  

(3.10)

where \( \mathbf{A} \) is the state matrix of the system.

By appropriate choice of \( \mathbf{Q} \) matrix elements, various penalization weights can be assigned to the state variables (which in this case are deviations from steady-state conditions) and a desirable dynamic performance for the system can be achieved.

3.2.3.2 Determining the PSS Parameters

As described in the previous section, by an appropriate choice of \( \mathbf{Q} \), the performance criterion, and hence the optimal PSS parameters can be manipulated. In this work, we choose an Integral of Squared Error (ISE) criterion that seeks to minimize the squared of the deviation of power angle deviation (\( \Delta \delta \)) from its steady-state value. Thus the state variable \( \Delta \delta \) is assigned a high weight and penalized for deviations and the PSS parameters are obtained accordingly.

Mathematically, this can be written as,
\[ J = \int_0^\infty \sum (\Delta \delta - \Delta \delta_{ss})^2 dt \]  

(3.11)

It can be seen that in this case, \( Q = \text{diag}[0 \ 1 \ 0 \ 0 \ 0 \ 0] \).

In order to obtain the optimal values of \( K_c \) and \( T_i \), the following procedure has been used:

1. Choose a set of PSS parameters for which the state matrix of the composite system is stable.
2. Fix the value of \( T_i \) and vary \( K_c \) over a wide range of values and determine the performance index (using (3.9)). It is seen that for a fixed \( T_i \), when \( K_c \) is increased, the performance index \( J \) decreases continuously, attains a minimum (say \( J_{\text{min}} \)) and then increases with further increase in \( K_c \).
3. Carry out Step-2 for various values of \( T_i \) and determine the minimum of \( J_{\text{min}} \) (say, \( J_{\text{min}*} \))

Figure 3.6 shows the plot of variation of \( J \) as a function of \( K_c \) for different values of \( T_i \). From the figure, we note that \( J \) attains the overall minimum for \( K_c = 38.65 \) and \( T_i = 0.11 \) seconds, which are the optimal settings of the lead-lag PSS.

![Figure 3.6](image)

**Figure 3.6** Performance index as a function of PSS gain \( K_c \), for different \( T_i \)

Figure 3.7 shows a comparison of the system dynamic performances with the optimal lead-lag PSS, but designed using two different techniques: the phase compensation approach and the Lyapunov’s method. It can be seen that both the design approaches provide satisfactory performances, though the phase
compensation approach requires somewhat more settling time, which is evident from a comparison of the values of performance indices $J$ also (Table 3.1).

<table>
<thead>
<tr>
<th>Table 3.1 Performance indices corresponding to different PSS settings</th>
</tr>
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<tr>
<td>PSS settings, $T_L, K_C$</td>
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<td></td>
</tr>
<tr>
<td>Performance Index, $J$</td>
</tr>
</tbody>
</table>

Figure 3.7 Performance of SMIB system with optimal lead-lag PSS

Further, we examine the phase compensation characteristics (Figure 3.8) drawn from the two optimal PSS settings previously obtained. While the phase compensation technique closely compensates for the $GEP(s)$ phase lag up to a frequency of 1 Hz, the system phase-lag is considerably under-compensated when the optimal lead-lag PSS is used.
3.3 Analysis of a Multi-Machine Power System with Lead-lag PSS

3.3.1 Composite Model

The representation of the multi-machine system without PSS has been discussed in Chapter 2, and a state-space model was developed. In this section, the development of the state-space model for the same system is presented in the line of Chapter 2, considering that all the generators are equipped with lead-lag PSS.

For the sake of continuity in understanding, the state-space model of the multi-machine system without PSS is re-stated below:

\[
\frac{d}{dt} \mathbf{x}(t) = \mathbf{A} \cdot \mathbf{x}(t) + \mathbf{B} \cdot \mathbf{u}(t) + \mathbf{\Gamma} \cdot \mathbf{p}(t)
\]  

\( (3.12) \)

\( \mathbf{A}, \mathbf{B} \) and \( \mathbf{\Gamma} \) are the state, control and perturbation matrices and have been described in Section 8.3.1. The associated state, control and perturbation vectors are given below:

\[
\mathbf{x}(t) = [\Delta \omega_1(t) \Delta \delta_1(t) \Delta E_{q_1}(t) \Delta E_{\delta_1}(t) \Delta \omega_2(t) \Delta \delta_2(t) \Delta E_{q_2}(t) \Delta E_{\delta_2}(t) \ldots \Delta E_{\delta_2}(t) \Delta \omega_3(t) \Delta \delta_3(t) \Delta E_{q_3}(t) \Delta E_{\delta_3}(t)]^T
\]
\[
\begin{align*}
    \mathbf{u}(t) &= \begin{bmatrix} \Delta u_1(t) & \Delta u_2(t) & \Delta u_3(t) \end{bmatrix}^T \\
    \mathbf{p}(t) &= \begin{bmatrix} \Delta T_{m_1} & \Delta T_{m_2} & \Delta T_{m_3} \end{bmatrix}^T
\end{align*}
\]

The control vector \( \mathbf{u}(t) \) is a vector of stabilizing signals that represents the PSS output at different machines.

The dynamic equations of the PSS in state-space form as obtained from the transfer function block-diagram is given below:

\[
\begin{align*}
    \frac{d}{dt} \left( \Delta N_{i_1}(t) \right) &= K_C \cdot \frac{d}{dt} \left( \Delta \omega_i \right) - \frac{\Delta N_{i_1}(t)}{T_{w_i}} \\
    \frac{d}{dt} \left( \Delta N_{i_2}(t) \right) &= \left[ \Delta N_{i_1}(t) - \Delta N_{i_2}(t) + T_1 \cdot \frac{d}{dt} \left( \Delta N_{i_1}(t) \right) \right] \cdot \frac{1}{T_{2i}} \\
    \frac{d}{dt} (\Delta u_i(t)) &= \left[ \Delta N_{i_2}(t) + T_3_i \cdot \frac{d}{dt} (\Delta N_{i_2}(t)) \right] \cdot \frac{1}{T_{4i}} \quad \forall i = 1, 2, 3
\end{align*}
\]

where \( \Delta N_{i_1} \) and \( \Delta N_{i_2} \) are the state-variables associated with each PSS, \( T_{w_i} \) is the washout time constant, \( T_i, \ldots, T_4 \) are the phase-lead time constants and \( K_C \) is the stabilizer gain.

Equations (3.13) may be arranged in standard vector-matrix form as in (3.14), and represent the state-space model of the PSS at all machines:

\[
\frac{d}{dt} \mathbf{x}_{\text{PSS}}(t) = \mathbf{C} \cdot \mathbf{x}(t) + \mathbf{D} \cdot \mathbf{x}_{\text{PSS}}(t) + \mathbf{\Gamma}_1 \cdot \mathbf{p}
\]

where

\[
\mathbf{x}_{\text{PSS}}(t) = \begin{bmatrix} \Delta N_{11}(t) \Delta N_{21}(t) \Delta u_1(t) \Delta N_{12}(t) \Delta N_{22}(t) \Delta u_2(t) \Delta N_{13}(t) \Delta N_{23}(t) \Delta u_3(t) \end{bmatrix}^T
\]

\( \mathbf{C}, \mathbf{D} \) and \( \mathbf{\Gamma}_1 \) are the matrices associated with the PSS model with appropriate dimensions, and are given in the Section 8.3.2.

By defining an augmented state-vector \( \mathbf{x}_C(t) = [\mathbf{x}(t) \quad \mathbf{x}_{\text{PSS}}(t)]^T \), the state-space model of the closed-loop system becomes:

34
\[
\frac{d}{dt} \chi_C(t) = A_C \cdot \chi_C(t) + \Gamma_C \cdot p
\]  

(3.15)

where

\[A_C = \begin{bmatrix} A & B_1 \\ C & D \end{bmatrix}\]
and \[\Gamma_C = \begin{bmatrix} \Gamma \\ \Gamma_1 \end{bmatrix}\]

\[B_1\] is a re-defined control matrix given as:

\[B_1 = \begin{bmatrix} 0 & 0 & b_1 & 0 & 0 & b_2 & 0 & 0 & b_3 \end{bmatrix}\]

where \(b_1, b_2, b_3\) are the column vectors of the control matrix \(B\).

The perturbation term in (3.15) can be eliminated, by applying a coordinate transformation in the state-space, as follows:

\[\chi'(t) = \chi_C(t) - \chi_C(\infty)\]

(3.16)

Hence, (3.15) reduces to the standard state-variable form:

\[\frac{d}{dt} \chi'(t) = A_C \cdot \chi'(t)\]

(3.17)

where

\[\chi'(0) = -\chi(\infty) = -A_C^{-1} \cdot \Gamma_C \cdot p\]

is the initial state of \(\chi'(t)\), which is also the steady-state value of \(\chi(t)\).

### 3.3.2 Lyapunov's Method Based Optimization of Lead-lag PSS for Multi-machine Power Systems

Section 3.2.3 provides the details of the method of PSS parameter optimization using Lyapunov’s method. In this section, the analysis is extended in a similar manner, to find the optimal parameters of PSS in a multi-machine system.

One important aspect associated with PSS tuning in multi-machine systems is the problem of siting of PSS on appropriate machines. This is required in order to find those critical machines where a PSS optimally tuned would damp out specific modes. This helps reduce the computational burden, particularly in case of large
systems. A lot of work has been reported in the literature addressing the siting problem [8],[10],[11],[40] In this chapter, and in the thesis as well, this issue has not been addressed.

Conventionally, the PSS tuning methods used for multi-machine systems have either used a sequential approach or a simultaneous approach. The sequential tuning approach is computationally simple but introduces eigenvalue drift as the sequence progresses, while the simultaneous tuning approach though being very complex to handle, particularly for large systems, does provide the optimal solution.

In the following analysis, the Lyapunov’s method was applied to multi-machine PSS tuning using the sequential approach.

The weighing matrix $Q$ is now the sum of the squares of each machine’s power angle deviation from their respective steady-state value.

Thus, we have

$$Q = \text{diag}(0 1 0 0 0 1 0 0 1 0 0)$$

and

$$J = \int_{0}^{\infty} \sum_{i=1}^{3} \left( \Delta \delta_{i} - \Delta \delta_{i,ss} \right)^{2} dt$$

Using the approach described in Section 3.2.3 we obtain the optimal parameters of each PSS sequentially. Various combinations of tuning sequences were tried out and the system performance was found to be the best for a sequence: machine-1-machine-2-machine-3, and therefore we report the results obtained with this sequence only. Table 3.2 provides a detailed report of the behavior of the PSS parameters, tuned using the above sequence, and shows the corresponding system eigenvalues as the tuning sequence progresses.

It can be seen that the system eigenvalues keep changing as the sequence progresses and this is an undesirable phenomenon. Thus, there is a case for examining and determining the PSS parameters simultaneously while also looking for ways on how to address the problem of increased computational burden arising from such an approach.
Table 3.2 Sequential tuning of multi-machine power systems

| Machine | PSS settings | | | | | | | |
|---------|--------------|---|---|---|---|---|---|---|---|
|         | Step 1: sub-optimal PSS on all machines | Step 2: Machine 1 tuned | Step 3: Machine 1 and 2 tuned | Step 4: Machine 1, 2 and 3 tuned |
|         | $T_1$ [s] | $K_C$ | $T_1$ [s] | $K_C$ | $T_1$ [s] | $K_C$ | $T_1$ [s] | $K_C$ |
| 1       | 0.10 | 3 | 0.11 | 73 | 0.11 | 73 | 0.11 | 73 |
| 2       | 0.10 | 3 | 0.10 | 3 | 0.16 | 15 | 0.16 | 15 |
| 3       | 0.10 | 3 | 0.10 | 3 | 0.29 | 12.5 |

Eigenvalues and damping ratios

<table>
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<th>Eigenvalues</th>
<th>Damping</th>
<th>Eigenvalues</th>
<th>Damping</th>
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<th>Damping</th>
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<td>0.1956</td>
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</table>
Figure 3.9 shows the system dynamic performances with the initial PSS settings ($K_d = 3.0$, $T_p = 0.1$). This was chosen as a minimal PSS since, the open-loop system being unstable, it was required to obtain a stable system to proceed with the optimization (Table 3.2).

**Figure 3.9** System dynamic performance with the minimal PSS

The system dynamic performance obtained using the optimally tuned PSS is shown in Figure 3.10.

**Figure 3.10** System dynamic performance with the optimal PSS
3.4 Concluding Remarks

This chapter presents the development of a composite state-space model of the system including the lead-lag power system stabilizer (PSS). Phase characteristics of the system as well as the PSS have been investigated and a method for tuning of PSS parameters based on exact phase compensation has been presented. It is found that for desirable system performance, a full phase compensation may not be necessary.

Subsequently, a method based on Lyapunov’s parameter optimization has been presented for tuning of lead-lag PSS. This method makes use of the Integral of Squared Error criterion with an objective of minimizing the power angle deviation from its steady-state value. System dynamic performance for single machine to infinite bus, as well as a multi-machine system show that this method provides superior responses as compared to the phase-compensation technique based PSS.

Due to the eigenvalue drift phenomenon and the high computational burden required for simultaneous type of tuning approach, Genetic Algorithm based simultaneous tuning methods have a promising scope and could address many of the concerns raised in large scale PSS parameter tuning. This issue will be addressed in the following chapters.
4 Lyapunov Method Based Genetic Algorithm for PSS Tuning

4.1 General Aspects

Fixed-structure lead-lag type of power system stabilizers discussed in Chapter 3 have found practical applications and these generally provide acceptable dynamic performances [5]. There have been arguments that these controllers, being tuned for one nominal operating condition, provide sub-optimal performance when there are variations in system operating load or system configuration. To address this issue, PSS parameter tuning methods that incorporate robustness within the optimization scheme and are applicable to fixed structure stabilizers are desirable. To ensure a sufficiently robust behavior, a wide operating domain need be considered in the tuning process.

In the present chapter we propose the application of a Genetic Algorithm (GA) search based on the classical Integral of Squared Error (ISE) criterion for tuning of PSS, as discussed in [39]. The proposed method ensures that for any operating condition within a pre-defined domain, the system with the optimally tuned PSS remains stable when subjected to small perturbations. The optimization employs a quadratic performance index that measures the quality of system dynamic performance within the tuning process. The solution obtained is globally optimal and robust. The proposed method has been tested on two different PSS structures-the conventional lead-lag type and the derivative type.

The proposed method makes use of the classical Lyapunov parameter optimization technique within a GA search framework to determine the "degree of goodness" of an individual within a population of possible solutions. The performance of each individual in the population is then evaluated through an objective function dynamically modeling the problem to solve and assess a fitness value. The selected individuals are then modified through the application of genetic operators, in order
to obtain the next generation. The globally optimal PSS parameter set thus obtained, is robust over a wide operating range.

### 4.1.1 Genetic Algorithm Specifications and the Proposed Method

The proposed Lyapunov method based genetic algorithm is initiated by generating randomly an initial population of binary coded individuals, where each individual represents a possible solution for the PSS parameters.

![Lyapunov Method Based Genetic Algorithm for PSS Tuning](image)

**Figure 4.1** Lyapunov Method Based Genetic Algorithm for PSS Tuning
A basic requirement for obtaining a feasible solution to the Lyapunov equation is that the state-matrix $A$ should be stable. Fulfillment of this condition is ensured by "stability screening". The entire population of individuals in each generation is screened (Figure 4.1) in order to ensure that only those individuals (each of them representing a PSS parameter set) that provide a stable system over the whole operating domain $D$, are allowed to proceed further in the optimization process. This also brings about significant reduction in the computational burden. Individuals resulting in unstable systems for an operating point within the domain $D$ ("bad individuals") are assigned a very high value of $J_{BKG}$, where $J_{BKG}$ is given as the mean value of performance indexes over the operating domain $D$, and given by (4.1). The bad individuals are gradually phased out from the population within a few generations.

$$J_{BKG} = \frac{1}{N_{op}} \sum_{P,Q} \left( \int_{0}^{\infty} (Q^T \cdot \dot{x}) \, dt \right) \quad \forall P, Q \in D \quad (4.1)$$

Every individual (chromosome) of the current population is evaluated for $J_{BKG}$ and a basis for the biased selection process is then established. To avoid premature convergence and speeding up of the search when the convergence is approached, the objective values obtained for each individual are mapped into fitness values through a ranking process. The rank-based fitness assignment overcomes the scaling problems of the proportional fitness assignment. The individuals will be ranked in the population in descending order of their fitness with respect to the problem domain. The higher the individual’s fitness is, the higher is its chance to pass-on genetic information to successive generations.

The next generation will be populated with offspring, obtained from selected parents. The selection is a process used to determine the number of trials for one particular individual used in reproduction. The selection process uses the stochastic universal sampling method, a single-phase sampling algorithm with minimum spread, zero bias and time complexity in the order of the number of individuals ($N_{(ND)}$).

Recombination of the selected individuals is carried out with pairs of individuals from the current population using a multi-point crossover process having a certain probability. The individuals in the pairs will exchange genetic information with each other, thereby creating two new individuals, the offspring. After that, each individual in the population will be mutated with a given probability, through a random process of replacing one allele of a gene with another to produce a new genetic structure.

The GA employed in this study uses an elitist strategy, in which the offspring is created with a generation gap of 80% and reinserted in the old population by
replacing the least fit predecessors. Most fit individuals are allowed to propagate through successive generations and only a better individual may replace them.

The GA stops when a pre-defined maximum number of generations is achieved or when the value returned by the objective function, being below a threshold, remains constant for a number of iterations.

### 4.2 Mathematical Model of the System

Two types of PSS have been considered for analysis- (a) the conventional lead-lag network (4.2) with gain $K_c$, time-constants $T_j$ and $T_w$, and wash-out filter time-constant $T_w$, and (b) the derivative network (4.3) with gain $K_d$ and time constant $T$.

\[ u(s) = K_c \cdot \frac{sT_w}{1 + sT_w} \cdot \frac{1 + sT_1}{1 + sT_2} \cdot \Delta \omega(s) \]  
\[ \text{(4.2)} \]

\[ u(s) = K_d \cdot \frac{sT}{1 + sT} \cdot \frac{T}{1 + sT} \cdot \Delta T_e(s) \]  
\[ \text{(4.3)} \]

As discussed in Chapter 3, $T_w$ is the washout time-constant, which is used to washout dc signals and without it, steady changes in speed would modify the terminal voltage. To guarantee the lead characteristic of the control signal, $T_j$ is kept to minimum physically achievable ($T_j = 0.05$ seconds). Thus, the lead-lag PSS parameters to be optimized are $K_c$ and $T_j$.

It is important to note here the difference in input signals to the two PSS. The phase-lead PSS is based on the commonly used rotor speed deviation input signal as shown in (4.2). On the other hand, the derivative PSS (4.3) is based on electrical torque deviation signal $\Delta T_e$. The parameters to be tuned in the later case are the derivative gain, $K_d$, and the time constant, $T$.

The linear dynamic model of the composite system inclusive of excitation system and PSS on each generator, can be obtained in a similar manner to the one outlined in Chapter 3, and state-space representation is given in:

\[ \frac{d\dot{x}(t)}{dt} = A \cdot \dot{x}(t) + \Gamma \cdot p \]  
\[ \text{(4.4)} \]

$A$ and $\Gamma$ are the state and perturbation matrices and depend on the system configuration and operating conditions, while $\dot{x}$ and $p$ are state and perturbation vectors, respectively.
In order to eliminate the perturbation term in (4.4) and reduce the system model to the standard closed-loop state-space form, a coordinate transformation in the state-space is applied as given by:

\[ \dot{x}' = x - x(\infty) \]  \hspace{1cm} (4.5)

The resulting state-transformed system model thus obtained is given by:

\[ \frac{d}{dt} \dot{x} = A \cdot \dot{x}' \]  \hspace{1cm} (4.6)

where \( \dot{x}'(0) = -\dot{x}(\infty) = -A^{-1} \cdot \Gamma \cdot p \) is the steady state value of \( \dot{x}(t) \).

### 4.3 Single Machine Connected to Infinite Bus

The proposed genetic algorithm based approach applying the classical Lyapunov stability and parameter optimization technique is now used to determine the optimal parameters of a PSS that is robust over a wide operating domain. For the purpose of these study cases we consider the operating domain \( D \{ D \in [0.3, 1.3] \text{ and } Q \in [-0.3, 1.0], \text{ in per unit} \} \), with a step-size of 0.1 per-units in each case. Therefore, we have a total of 154 operating points on the \( P-Q \) plane.

![Diagram](image)

**Figure 4.2** Small perturbation transfer function model of SMIB system equipped with lead-lag type PSS
Figure 4.2 shows the small perturbation transfer function model of the single-machine infinite bus system with the lead-lag PSS.

The state variable vector $\mathbf{x}$ and state matrix $\mathbf{A}$ for the SMIB system equipped with the lead-lag PSS of (4.2) have been provided in Section 3.2.1.

Figure 4.3 shows the small perturbation transfer function model of the SMIB system equipped with a derivative type PSS.

The state vector of the system can be defined as:

$$
\mathbf{x}(t) = \begin{bmatrix}
\Delta \omega \\
\Delta \delta \\
\Delta E_q \\
\Delta E_d \\
\Delta u
\end{bmatrix}
$$

(4.7)

The system matrix $\mathbf{A}$ with the derivative type PSS of (4.3) is described by:
The optimal parameters for both lead-lag and derivative types of PSS obtained with the proposed GA based method are compared with the optimal parameters setting obtained by applying the ISE Technique earlier reported in [39], that was obtained considering one nominal operating condition.

Additionally, for the derivative PSS, the set of optimal parameters obtained with the proposed GA based method is also compared with an earlier reported eigenvalue shift based GA method [32].

\[
A = \begin{bmatrix}
0 & -\frac{K_1}{M} & -\frac{K_2}{M} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{K_4}{T_{do}} & -\frac{1}{K_{A} \cdot K_5} & \frac{1}{K_{A} \cdot K_6} & \frac{1}{T_{do}} & 0 & -\frac{K_A}{T_{A}} \\
0 & \frac{K_A \cdot K_5}{T_A} & \frac{K_4}{K_{A} \cdot K_5} & -\frac{1}{T_A} & 0 & -\frac{1}{T_A} \\
0 & \frac{K_1}{K_d} & \frac{K_d}{K_1} & 0 & -\frac{1}{T} & 0 \\
0 & -\frac{K_1}{K_d} & -\frac{K_d}{K_1} & 0 & -\frac{1}{T} & -\frac{1}{T}
\end{bmatrix} \quad (4.8)
\]

### 4.3.1 Analysis

The optimal parameters for both lead-lag and derivative types of PSS obtained with the proposed GA based method are compared with the optimal parameters setting obtained by applying the ISE Technique earlier reported in [39], that was obtained considering one nominal operating condition.

Additionally, for the derivative PSS, the set of optimal parameters obtained with the proposed GA based method is also compared with an earlier reported eigenvalue shift based GA method [32].

![Performance index as function of PSS gain $K_d$, for different time constants $T$](image)

**Figure 4.4** Performance index as function of PSS gain $K_d$, for different time constants $T$
Figure 4.4 depicts the tuning process of a derivative type PSS in a SMIB system performed using the ISE Technique, in a similar manner as described in Section 3.2.3.2 for the lead-lag PSS.

Table 4.1 provides a summary of the converged PSS parameters for the lead-lag PSS, along with the corresponding system eigenvalues, damping factors \( \xi \) and natural frequencies \( f_n \) for oscillatory modes. The ISE technique based PSS [39] achieves a fairly robust optimum parameter set that is very close to the global optimal set obtained using the GA based method. \( J_N \) and \( J_{AVG,N} \) are the corresponding values of \( J \) and \( J_{AVG} \), respectively, normalized with respect to the best, and provide a quantitative measure of the quality of dynamic performance with a particular type of PSS and PSS settings.

<table>
<thead>
<tr>
<th></th>
<th>No PSS</th>
<th>Proposed GA based method</th>
<th>ISE technique [39]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gain and Time constant</strong></td>
<td>–</td>
<td>33.98</td>
<td>38.65</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.12</td>
<td>0.11</td>
</tr>
<tr>
<td><strong>Eigenvalues at nominal operating point, ( \xi, f_n ) [Hz]</strong></td>
<td>-14.298, -6.371</td>
<td>-30.964, -3.888 ± j7.773</td>
<td>-30.461, -3.196 ± j8.471</td>
</tr>
<tr>
<td></td>
<td>0.103 ± j 5.5</td>
<td>0.43</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>-4.777</td>
<td>0.86</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>-0.102</td>
<td>1.566</td>
<td>1.695</td>
</tr>
<tr>
<td><strong>( J_N ) at nominal operating point</strong></td>
<td>–</td>
<td>1.008</td>
<td>1.0</td>
</tr>
<tr>
<td><strong>( J_{AVG,N} )</strong></td>
<td>–</td>
<td>1.0</td>
<td>1.01</td>
</tr>
</tbody>
</table>

Table 4.2 shows the optimal settings for derivative PSS and the corresponding-eigenvalues and performance indices. The performance indices are normalized to the best (see Table 4.1). The proposed GA based method, though with \( J_{AVG,N} \) of 4.72, does provide a fairly satisfactory dynamic performance, while the eigenvalue shifting method based PSS with \( J_{AVG,N} \) of 24 provides a much worsened response. Note that when tuned with classical ISE Technique, the system is not stable for the entire operating domain \( D \) considered, although for the nominal operating point behaves better than the genetically tuned systems.
Table 4.2 Optimal parameter setting for derivative PSS obtained with three different methods

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain and Time constant</td>
<td>-</td>
<td>2.03</td>
<td>6.47</td>
<td>1.55</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.30</td>
<td>0.34</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>0.103 ± j 5.5</td>
<td>-1.357 ± j5.33</td>
<td>-1.959 ± j9.21</td>
<td>-0.877 ± j5.759</td>
</tr>
<tr>
<td></td>
<td>-2.756 ± j4.62</td>
<td>-2.081</td>
<td>-0.726 ± j2.892</td>
<td>-2.981 ± j3.248</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-1.56</td>
<td>-1.7857</td>
</tr>
<tr>
<td>$J_N$ at nominal operating point</td>
<td>-</td>
<td>4.97</td>
<td>21.33</td>
<td>4.69</td>
</tr>
<tr>
<td>$J_{AVG,N}$</td>
<td>-</td>
<td>4.72</td>
<td>24</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 4.5 and Figure 4.6 show the distribution of $J_{AVG}$ for individuals in the population and their convergence to the global optimum during the search process, for two different PSS types, i.e., the conventional lead-lag PSS and the derivative PSS, respectively.

Figure 4.5  Distribution of the solution during the search process for conventional lead-lag type PSS

It is seen that the best individual from a generation progresses towards convergence, corresponding to the minimum of $J_{AVG}$; and thereby providing the
optimal values of PSS parameters. The figures provide an image of the distribution of possible solutions cumulatively obtained during the genetic search, thus emphasizing the algorithm's convergence towards the global optimum and intuitively pinpointing the same.

It should be noted that, the vertical axis denoting the performance $J_{j_{AVG}}$, appears with a different scale for the two PSS cases- the derivative PSS scale being of the order of $10^{-4}$, while the conventional lead-lag PSS is of the order of $10^{-5}$.

![Figure 4.6](image)

**Figure 4.6** Distribution of the solution during the search process for derivative type PSS

Figure 4.7 shows that $J_{j_{AVG}}$ decreases monotonously with time, and in approximately 50 generations, the optimization process finds a solution that remains unchanged thereafter, and $J_{j_{AVG}}$ reaches a steady minimum value. These results were obtained with a genetic process over 160 generations and having a population of 50 individuals.
Figure 4.7  $J_{HG}$ of best individual for each generation

Figure 4.8 shows the corresponding PSS parameters variation during the genetic process, and indicates their convergence to the optimal value.

Figure 4.8  PSS parameter variation during genetic process, for each generation

Figure 4.9 and Figure 4.10 shows the area covered by the imaginary positive part of the system eigenvalues over the entire operating domain $D$, for optimum parameter settings of lead-lag PSS and derivative type PSS, respectively. It can be
seen that the conventional lead-lag PSS provides a more stable system since its eigenvalues are further away from the unstable axis vis-à-vis the derivative PSS.

**Figure 4.9** Distribution of system eigenvalues with optimal lead-lag PSS over the operating domain

**Figure 4.10** Distribution of system eigenvalues for optimal derivative-type PSS over the operating domain

The optimum parameters obtained using our proposed method, which is based on measurement of system dynamic performance in the time domain, is now compared with an earlier reported GA based PSS [32] where the principle is to use a frequency-domain approach that applies eigenvalue shifting technique.
Understandably, the addition of an optimally tuned PSS enlarges the stability region on the P-Q plane considerably. The conventional lead-lag PSS tuned using the proposed GA based method provides the largest stability region (the entire marked region) (Figure 4.11). It is to be noted that this stability region is considerably larger than the domain actually considered for the GA based optimization (shown by the rectangular box). Also shown in Figure 4.11 are the corresponding stability regions with the derivative type PSS tuned using the proposed GA based method (shown by o-marks) and when tuned using the eigenvalue shifting method of [32] (shown by x-marks).

![Figure 4.11 Stability regions on the P-Q plane with different PSS](image)

### 4.3.2 System Dynamic Performances

Performance of the PSS with optimum parameters obtained using the proposed method was examined through dynamic analysis for various system loading conditions (heavy, nominal and light), small perturbations, as well as large faults. Figure 4.12 and Figure 4.13 show the comparative performances of conventional lead-lag and derivative type PSS, respectively, for different load conditions when subjected to one per cent change in mechanical torque. The system behaved satisfactorily with both PSS for light and nominal load conditions. However, during heavy load, the settling time of the oscillations is considerably shorter and the overall performance is better with the conventional lead-lag PSS.
System performance under small perturbation for lead-lag PSS tuned using the proposed GA based method

Figure 4.12

System performance under small perturbation for derivative type PSS tuned using the proposed GA based method

Figure 4.13

Further, comparing dynamic performance of the lead-lag PSS obtained using the proposed GA based method with two different PSS parameter sets of derivative
type PSS, one obtained using our proposed method (PSS 1 in Figure 4.14) and the other obtained in [32] (PSS 2 in Figure 4.14), we note that the proposed GA based lead-lag PSS provides considerably superior responses. This behavior was also explained through the comparison of $J_{AVG,N}$ values in Table 4.2.

![Figure 4.14](image)

**Figure 4.14** Dynamic behavior of conventional and derivative type PSS at nominal operating point

Subsequent studies on dynamic performance analysis reported in this chapter are carried out considering the lead-lag PSS only. The system is now tested for a combination of events, commencing with a small perturbation at time $t=0$, followed by a three-phase short circuit on one of the two parallel lines, very close to the generator bus, at time $t=3$ seconds. The short circuit is cleared by the protection system after 0.1 seconds, *i.e.*, at time $t=3.1$ seconds, by disconnecting the faulted line.

The system dynamic response and the associated PSS output signal with the lead-lag PSS tuned using the proposed GA based method are shown in Figure 4.15, Figure 4.16 and Figure 4.17. It is seen that following the disturbance, the system recovers very satisfactorily, while reaching a new steady state in approximately 6 seconds. It is also to be noted that in order to avoid large variations in terminal voltages, a signal limiter has now been applied to restrict the PSS output within certain pre-decided limits, [-0.1 p.u., 0.2 p.u.] in the present case.
Figure 4.15  Rotor angle deviation for the system with optimal GA based lead-lag PSS during a small perturbation, followed by a three-phase short-circuit and removal of fault

Figure 4.16  Angular speed deviation for the system with optimal GA based lead-lag PSS during a small perturbation, followed by a three-phase short-circuit and removal of fault
Optimal GA based lead-lag PSS output signal during a small perturbation, followed by a three-phase short-circuit and removal of fault

### 4.4 Multi-machine Power System

The proposed GA based technique incorporating Lyapunov’s parameter optimization criterion is now used to determine the optimal parameters of PSS on the three-machine system described in Section 2.6. In order to bring about robustness, a set of six operating points (OP-1 to OP-6) is considered by varying the system loads, Load-A, Load-B and Load-C (refer Figure 2.3) in steps. The set of system load conditions considered for the GA based method is presented in Table 4.3.

<table>
<thead>
<tr>
<th></th>
<th>LC-1</th>
<th>LC-2</th>
<th>LC-3</th>
<th>LC-4</th>
<th>LC-5</th>
<th>LC-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load-A</td>
<td>0.83+j0.55</td>
<td>0.91+j0.61</td>
<td>0.99+j0.67</td>
<td>1.1+j0.73</td>
<td>1.21+j0.81</td>
<td>1.33+j0.97</td>
</tr>
<tr>
<td>Load-B</td>
<td>0.44+j0.33</td>
<td>0.48+j0.36</td>
<td>0.53+j0.40</td>
<td>0.59+j0.44</td>
<td>0.64+j0.48</td>
<td>0.71+j0.59</td>
</tr>
<tr>
<td>Load-C</td>
<td>0.55+j0.39</td>
<td>0.61+j0.42</td>
<td>0.67+j0.47</td>
<td>0.73+j0.51</td>
<td>0.81+j0.56</td>
<td>0.89+j0.68</td>
</tr>
</tbody>
</table>

Based on the above load conditions, an optimal power flow (OPF) with "minimization of losses" as the criterion, is run for each load configuration. The optimal generation schedule so obtained for each unit for each load configuration considered is shown in Table 4.4. This OPF solution is used as the initial operating condition for the multi-machine system for PSS tuning using the proposed GA based method.
The OPF solution providing the initial operating conditions

<table>
<thead>
<tr>
<th>Gen-1</th>
<th>OP-1</th>
<th>OP-2</th>
<th>OP-3</th>
<th>OP-4</th>
<th>OP-5</th>
<th>OP-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.93+j0.16</td>
<td>1.02+j0.24</td>
<td>1.12+j0.33</td>
<td>1.23+j0.43</td>
<td>1.36+j0.54</td>
<td>1.49+j0.67</td>
<td></td>
</tr>
<tr>
<td>0.51-j0.04</td>
<td>0.56+j0.02</td>
<td>0.62+j0.08</td>
<td>0.68+j0.14</td>
<td>0.76+j0.22</td>
<td>0.84+j0.31</td>
<td></td>
</tr>
<tr>
<td>0.38-j0.14</td>
<td>0.42-j0.10</td>
<td>0.47-j0.05</td>
<td>0.52+j0.01</td>
<td>0.57+j0.07</td>
<td>0.63+j0.15</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.5 shows the optimal PSS parameters obtained using the proposed GA based method. For the sake of comparison, the optimal parameters obtained using the ISE technique described in [41] are also presented. The eigenvalues of the closed loop matrix A, for the optimal PSS parameter settings, are also provided. It should be noted that the ISE technique uses a sequential approach to tune the parameters while in the present GA based approach, all PSS have been tuned simultaneously.

Table 4.5 Optimal PSS parameters using the proposed GA based technique as compared to those obtained using ISE Technique

<table>
<thead>
<tr>
<th>GA-PSS</th>
<th>ISE-PSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSS Parameters (Kc, Ti)</td>
<td>Eigenvalues</td>
</tr>
<tr>
<td>Generator-1: (45.06, 0.17)</td>
<td>-5.93 ± j16.37</td>
</tr>
<tr>
<td></td>
<td>-1.54 ± j15.14</td>
</tr>
<tr>
<td></td>
<td>-5.78 ± j12.63</td>
</tr>
<tr>
<td></td>
<td>-2.19 ± j8.753</td>
</tr>
<tr>
<td>Generator-2: (45.52, 0.06)</td>
<td>-10.72 ± j2.78</td>
</tr>
<tr>
<td></td>
<td>-4.52 ± j5.215</td>
</tr>
<tr>
<td></td>
<td>-35.834</td>
</tr>
<tr>
<td></td>
<td>-34.191</td>
</tr>
<tr>
<td>Generator-3: (2.13, 0.44)</td>
<td>-24.167</td>
</tr>
<tr>
<td></td>
<td>-18.238</td>
</tr>
<tr>
<td></td>
<td>-5.0434</td>
</tr>
<tr>
<td></td>
<td>-3.1556</td>
</tr>
<tr>
<td></td>
<td>-0.1024</td>
</tr>
<tr>
<td></td>
<td>-0.1000</td>
</tr>
<tr>
<td></td>
<td>-0.1009</td>
</tr>
</tbody>
</table>

The average performance index, Javg (given by (4.1)), of the "best" individual in each generation is selected and plotted over generations to show its convergence.
rate. Figure 4.18 is an accurate representation of all GA based optimization processes performed during this study, and presents the convergence rate evolution of a population of 40 individuals, during a genetic process of 460 generations. The values of the solution and performance index are presented in Table 4.6, case 7.

![Figure 4.18](image)

**Figure 4.18** Plot of $J_{AVG}$ for the ‘best’ individual for each generation

The corresponding PSS (a) time constants and (b) gains’ variations during the genetic process are depicted in Figure 4.19, for 460 generations.

![Figure 4.19](image)

**Figure 4.19** Parameters ($T_{ii}$ and $K_{Ci}$, $i = 1, 2, 3$) of best individual, for each generation

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The convergence rate of the performance index is also reflected in the variation of the best individual of each generation during the entire genetic process.

Table 4.6 shows the performance indexes $J_{AVG}$ of the Lyapunov’s optimization method based GA search solutions for different configurations and in different stages of the genetic search process. It can be seen that the GA search provides the best solution in Case 8, for $N_{IND} = 50$ over 500 generations.

### Table 4.6 Performance of the proposed GA based PSS tuning method for different genetic configurations

<table>
<thead>
<tr>
<th>Case</th>
<th>$N_G$</th>
<th>$N_{IND}$</th>
<th>PSS Parameters $(K_C, T_I)$</th>
<th>Performance Index $J_{AVG} \times 10^6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>50</td>
<td>50</td>
<td>(44.63, 0.18) (40.7, 0.06) (2.32, 0.45)</td>
<td>3.1606</td>
</tr>
<tr>
<td>2.</td>
<td>100</td>
<td>30</td>
<td>(44.29, 0.18) (49.38, 0.06) (2.48, 0.39)</td>
<td>3.1546</td>
</tr>
<tr>
<td>3.</td>
<td>100</td>
<td>50</td>
<td>(45.2, 0.17) (42.66, 0.06) (2.64, 0.37)</td>
<td>3.1546</td>
</tr>
<tr>
<td>4.</td>
<td>150</td>
<td>30</td>
<td>(44.84, 0.17) (46.15, 0.06) (3.16, 0.35)</td>
<td>3.1538</td>
</tr>
<tr>
<td>5.</td>
<td>200</td>
<td>30</td>
<td>(43.21, 0.17) (49.89, 0.06) (3.61, 0.35)</td>
<td>3.1545</td>
</tr>
<tr>
<td>6.</td>
<td>200</td>
<td>50</td>
<td>(42.03, 0.18) (43.94, 0.07) (3.42, 0.297)</td>
<td>3.1639</td>
</tr>
<tr>
<td>7.</td>
<td>460</td>
<td>40</td>
<td>(44.56, 0.17) (46.24, 0.06) (1.96, 0.46)</td>
<td>3.1526</td>
</tr>
<tr>
<td>8.</td>
<td>500</td>
<td>50</td>
<td>(45.06, 0.17) (45.52, 0.06) (2.13, 0.44)</td>
<td>3.1524</td>
</tr>
</tbody>
</table>

Usually, the solution is not reached in 50 generations and for reliable result, approximately 100 generations are required. A higher number of individuals in population will increase the probability of finding the optimum solution in a smaller number of generations, but it will also increase the computational time needed to complete the evaluation of one generation. The simulations performed show that, very often, a population with 30 individuals would suffice to find an optimum within 150 generations (e.g. Table 4.6, case 4).

However, of a big importance in the GA based optimization process, is the convergence criterion, whose inadequate setting may cause premature termination of the process, far away from the global optimum.

In order to test the robustness of the GA based PSS, three different operating conditions were considered: a light load, a nominal load and a heavy load, as given in Table 4.7. It might be noted that these load conditions at buses #5, #6 and #8, are same as those used in [34], except that the corresponding generation levels are obtained here using an OPF simulation with "minimizing losses" as objective.
Table 4.7 Three different loading conditions for examining the performance of the GA based PSS

<table>
<thead>
<tr>
<th>Loading</th>
<th>Gen-1</th>
<th>Gen-2</th>
<th>Gen-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>Q</td>
<td>P</td>
<td>Q</td>
</tr>
<tr>
<td>Nominal</td>
<td>0.71</td>
<td>0.28</td>
<td>1.63</td>
</tr>
<tr>
<td>Heavy</td>
<td>2.77</td>
<td>1.20</td>
<td>1.38</td>
</tr>
<tr>
<td>Light</td>
<td>0.81</td>
<td>0.14</td>
<td>0.44</td>
</tr>
</tbody>
</table>

The dynamic responses are plotted for rotor speed deviation of generator-1 following a 0.01 per unit step change in mechanical torque on the same generator. The responses are plotted for both the GA based PSS and the ISE technique based PSS.

Figure 4.20 shows the plot for the nominal operating condition. The GA based PSS has a lower peak off-shoot and smaller oscillations and an overall better damped response.

Figure 4.20  Rotor speed deviation on generator-1 with GA based PSS and ISE technique based PSS for nominal load condition

Figure 4.21 shows the plot of rotor speed deviations for the heavy operating load condition. It is evident that the GA based PSS performs distinctly better compared to the ISE technique based PSS.
Figure 4.21  Rotor speed deviations on generator-1 with GA based PSS and ISE technique based PSS for heavy load condition

Figure 4.22  Rotor speed deviations on generator-1 with GA based PSS and ISE technique based PSS for light load condition.
Figure 4.22 shows the comparison of dynamic performances under a light load operating condition. In this case, the GA based PSS and the ISE technique based PSS both do provide satisfactory responses.

Evidently, it can be said that the conventionally tuned PSS provides satisfactory performance at light loads and up to the nominal operating point, at which it is tuned. However, when the system load increases beyond the nominal point, the performance deteriorates. The GA based PSS, on the other hand, continues to perform well for all operating loads and hence has a higher level of robustness.

4.5 Concluding Remarks

This chapter presents a novel approach to tuning of Power System Stabilizers (PSS) using Genetic Algorithms (GA) based search process that incorporates the classical Lyapunov optimization criterion. The advantage of using a GA based search is that within these global search techniques a wide operating range can be taken in consideration in the tuning process. In contrast, the conventional tuning approaches are based on one nominal operating condition. Furthermore, the problems associated with eigenvalue drift arising from sequential tuning in multi-machine PSS is avoided.

The advantage of the proposed Lyapunov method based GA over other earlier reported GA methods is that the proposed method takes into consideration the dynamics of the system in the time-domain and is hence much more convenient to understand. The Integral of Squared Error criterion also provides an exact quantification of the system performance as against other methods, which primarily use the eigenvalue shift approach and measuring the damping factors. The optimal PSS obtained using the proposed method provides considerably superior dynamic performances under a wide range of operating conditions. The computational burden of the proposed method is within practical limits.

Genetic algorithms represent a useful tool for large-scale optimization problems, but inappropriate selection of genetic search parameters may lead to premature termination, or even to the divergence of genetic process. However, during the investigations reported here, the final solution was always found to be in vicinity of the same location within the search space, depending on the desired accuracy (pre-specified in the convergence criterion).

The proposed method has been tested on two different PSS structures- the conventional lead-lag and the derivative type, and two different system models- the single machine to infinite bus (SMIB) and a multi-machine model. Investigations reveal that the conventional lead-lag PSS provides performances superior to the derivative PSS.
The simulations and tests performed showed that the operating range for which the system proposed GA based PSS withstands small perturbations is much bigger than the range considered within the objective function.

The dynamic responses were satisfactory for large variations in system load conditions, for different system topologies and even for transient phenomena occurred due to severe faults in the system.
5 Tuning of Proportional Plus Integral Plus Derivative (PID) PSS Using Genetic Algorithm

5.1 Introduction
In this chapter, the tuning of fixed structure proportional plus integral plus derivative (PID) PSS for the single-machine infinite bus and multi-machine power systems has been considered. PID controllers have found applications in power system control problems for their simplicity and ease of realization. In [42] a pole-shifting self-tuning PID controller has been designed for damping of low frequency oscillations in multi-machine systems. The PID controller gains are adapted in real-time to track the system conditions in order to provide robustness to the system. In [43], a fuzzy rule-base is used to tune the gain settings of a PID stabilizer. The introduction of fuzzy logic to tune the PID gains makes the PID control structure inherently non-linear. On-line tracking of the error signal and their time derivative (difference) is used to evaluate the gains. Genetic algorithm based PID controllers have been proposed in [44] for controller design to improve the transient stability of ac-dc lines after faults. The PID controller is applied to the HVDC control system, both on the rectifier side as well as the inverter side and the gains are tuned such that the disturbance from a fault is minimum.

The tuning scheme proposed in this chapter uses a genetic algorithm (GA) based search that integrates a classical parameter optimization criterion based on Integral of Squared Error (ISE). This method succeeds in achieving a robust,
simultaneously tuned and globally optimal PID-PSS parameter set, while maintaining the simplicity of the classical optimization method. The tuning method implicitly builds-in an increased robustness through an objective function, that depends on the operating domain.

PID controllers have been used for power system stabilization for their simplicity and ease of realization. They are feedback controllers whose output is generally based on the error between a user-defined set point, \( \omega_s \), and the measured variable \( \omega_t \). Each element of the PID controller refers to a particular action taken on the error for example, the proportional gain \( K_p \) is an adjustable amplifier that is usually responsible for system stability. The integral gain \( K_i \) is responsible for driving the error to zero, while the derivative gain \( K_d \) is responsible for system damping.

In all practical implementations of PSS, the input signals are available in discrete form since digital instruments are used to measure the system variables such as speed, voltages, terminal power, current, etc. Therefore, it is important to capture the effect of discrete inputs on PSS parameter settings, which understandably will be affected. A proper selection of the sampling time is important because, though a small sampling time would be desirable, it would nevertheless increase the computational burden significantly. On the other hand, a large sampling time will miss significant system information on the dynamics while achieving fast computation. A proper compromise selection of \( T_s \) is thus critical and shall be discussed later in this chapter.

Tuning of a PID controller involves the adjustment of its gains \( K_p, K_i, \) and \( K_d \) to achieve some user-defined "optimal" character of system response. The structure of a PID PSS with rotor speed deviation as input can be represented as

\[
u(t) = K_p \cdot \Delta \omega(t) + K_p \cdot \frac{d}{dt} \Delta \omega(t) + K_i \cdot \int \Delta \omega \cdot dt \tag{5.1}
\]

The above control logic can be expressed in discrete-mode as follows:

\[
u_k = K_p \cdot \Delta \omega_k + K_d \cdot (\Delta \omega_k - \Delta \omega_{k-1}) + K_i \cdot \sum_{p=1}^{N} \Delta \omega_k \cdot T_s \tag{5.2}
\]

In equation (5.2), the PSS parameters to be optimized are \( K_p, K_d, \) and \( K_i \).

The small perturbation dynamic model of the multi-machine system without PSS was discussed in detail in Chapter 2, and the transfer function block diagram
representation is given in Figure 2.1. The PSS output signal \( u(t) \) shall be acting on the voltage regulator summing junction of each machine.

The general structure of the PID-PSS is shown in Figure 5.1 where \( K_p, K_d \) and \( K_i \) are the proportional, derivative and integral gains respectively. Note that the input to the PSS comprises discrete samples of the speed deviation signal \( \Delta \omega \) obtained with a sampling time \( T_s \).

![Figure 5.1](image)

**Figure 5.1** The general structure of a PID-PSS with discrete input signal

The small perturbation transfer function model for the above system representations can be expressed in state-space form as follows:

\[
\frac{d}{dt} x(t) = A \cdot x(t) + B \cdot u + \Gamma \cdot p
\]  

In (5.3), \( A \) is the state matrix, \( B \) is the control matrix and \( \Gamma \) is the perturbation matrix and depend on the system parameters and operating conditions, \( x(t) \) is the state-vector defined in (5.4) and \( p \) is the perturbation vector. \( N_G \) is the number of generators.

\[
x(t) = [\Delta \omega_i \Delta \delta_i \Delta E'_{hi} \Delta E_{ij,i}] \quad \forall \; i \in N_G
\]  

The linear dynamic model of the composite system inclusive of excitation system and the PID-PSS can be represented in state-space form as in (5.5). \( A_c \) is the corresponding composite system matrix.

\[
\frac{d}{dt} x(t) = A_c \cdot x(t) + \Gamma \cdot p
\]  

The discrete mode equivalent of (5.3) can be expressed as:
\[ x_{k+1} = G \cdot x_k + H \cdot u_k + \Gamma_\text{D} \cdot \mathcal{P} \] (5.6)

\( G, H \) and \( \Gamma_\text{D} \) are discrete-mode equivalents of \( A, B \) and \( \Gamma \) respectively and are defined as follows:

\[
\begin{align*}
G &= e^{ATS} \\
H &= \left( e^{ATS} - I \right) \cdot A^{-1} \cdot B \\
\Gamma_\text{D} &= \left( e^{ATS} - I \right) \cdot A^{-1} \cdot \Gamma
\end{align*}
\] (5.7)

5.1.1 Genetic Algorithm Specifications

The GA employed in this study uses an *elitist strategy*, in which the offspring is created with a *generation gap* of 90% and reinserted in the old population by replacing the least fit predecessors. Most fit individuals are allowed to propagate through successive generations and only a better individual may replace them.

Each individual of a generation is a Gray coded binary string of search variables, each variable using a 30-bit representation. The selection process uses the *stochastic universal sampling* method, a single-phase sampling algorithm with minimum spread, zero bias and time complexity in the order of the number of individuals \( N_{\text{IND}} \). Recombination is performed using a *multi-point crossover* process with a probability of 0.7 and mutation is applied with a low probability of 0.03.

Within the genetic search, the evaluation process is performed by an objective function, which is a measure of the system’s behavior under a small perturbation. The average performance index \( J_{\text{AIG}} \) is calculated as follows:

\[
J_{\text{AIG}} = \frac{1}{N_{\text{op}}} \cdot \sum_{p,q} \left( \sum_{k=1}^{N_{\text{op}}} \Delta \delta_k^2 \cdot T_S \right) \quad \forall P, Q \in D
\] (5.8)

where \( N_{\text{op}} \) is the number of operating points in the considered domain \( D \).
\( T_S \) is the sampling time
\( \Delta \delta \) is the rotor angle deviation

Simulations with population sizes ranging between 30 to 200 individuals have been performed. Very often, a population with 30 individuals would suffice to find an optimum, and the number of generations required being proportional to the number of variables. As the population size increases, the probability of finding the global optimum increases, while also increasing the simulation time required for each generation. The results presented in this chapter have been obtained with
population sizes of 40 and 60 individuals for single- and multi-machine systems respectively.

The convergence criterion is of critical importance and it determines the required number of generations to complete the genetic process. Improper setting of the criterion may lead to premature termination of the process, far away from the global optimum.

Figure 5.2 shows the working scheme of the proposed GA based method for tuning of PID PSS using the ISE criterion.

**Figure 5.2** Proposed GA based tuning scheme for PID - PSS
5.2 Single Machine Infinite Bus System Analysis

The proposed GA based tuning scheme is applied to a single-machine infinite bus system operating over a wide operating domain. For this study case an operating domain $D (D \in P = [0.3, 1.3] \text{ and } Q = [-0.3, 1.0])$ comprising 154 operating points was considered. Figure 5.3 shows the variation of the performance index of the best individual in current generation of the GA based search process and Figure 5.4 shows the variation of the corresponding PID controller gains over the generations and their convergence towards the optimal solution.

![Figure 5.3 Genetic search performance for SMIB PID-PSS tuning](image)

![Figure 5.4 PID-PSS parameter variation and convergence during genetic process for SMIB](image)
It can be noted that the process reaches the optimum solution in about 25
generations, after which the performance index reaches a value that remains steady
over the remaining search process, and genetic operators do not affect the best
individual in consequent generations.

The optimal gains of the PID-PSS are shown in Table 5.1. Also shown in the table,
are the corresponding optimum parameters of a lead-lag PSS (Table 4.1) tuned
using the GA based scheme discussed in Chapter 4. The values of $J_{AVG}$ for PID-
PSS and lead-lag PSS are very close, indicating that both provide very good
performance for the single-machine system considered.

Table 5.1 Optimum PSS parameters and performance index for GA based PID and lead-lag
PSS

<table>
<thead>
<tr>
<th>$K_p$</th>
<th>$K_d$</th>
<th>$K_f$</th>
<th>$J_{AVG}$</th>
<th>$K_c$</th>
<th>$T_i$</th>
<th>$J_{AVG}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>38.28</td>
<td>473.84</td>
<td>-0.3</td>
<td>$1.557 \times 10^{-6}$</td>
<td>33.98</td>
<td>0.12</td>
<td>$1.598 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

A comparison of the dynamic behavior of the SMIB system equipped with
optimally tuned PID and lead-lag PSS is shown in Figure 5.5. The system is
subjected to a 1% step change in mechanical torque under heavy load operating
conditions ($P = 1.5$ p.u., $Q = 1.1$ p.u.). We can see that both the PSS show good
dynamic performance even for a load condition that is outside the operating
domain $D$ that was considered for the PSS tuning.

![Figure 5.5](image-url)  

**Figure 5.5**  Rotor angle deviation of SMIB equipped with PID and Lead-lag PSS
5.2.1 Effect of sampling period on PID-PSS tuning and dynamic performance

For the design of the PID-PSS, the sampling time $T_S$ plays an important role and Table 5.2 shows the dependence of optimum PSS setting and corresponding performance index on $T_S$. We observe from Table 5.2 that a gradual deterioration in performance takes place as $T_S$ increases. In order to achieve a high degree of accuracy, a very small sampling time is desirable, which however increases the computational burden. The performance index is lowest for $T_S = 0.001$ seconds which however will have a very high computing burden. As we progressively increase $T_S$ the performance index deteriorates gradually while reducing the computational burden. In our perspective, the best trade-off between accuracy and computational burden is at $T_S = 0.01$ seconds.

Now we examine if the chosen sampling period of $T_S = 0.01$ sec is the optimal choice. The last column of Table 5.2 shows the performance index for different $T_S$ with nominal PSS$^2$. The nominal PSS works well up to $T_S = 0.05$ sec while beyond this, the performance index is very high thus implying that re-tuning is required.

![Table 5.2 Effect of $T_S$ on GA based PID-PSS design](image)

<table>
<thead>
<tr>
<th>$T_S$</th>
<th>Optimum GA based PID-PSS at nominal operating point</th>
<th>$J_{nom}$ ($\times 10^5$) with nominal PSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>sec</td>
<td>$K_P$</td>
<td>$K_D$</td>
</tr>
<tr>
<td>0.001</td>
<td>38</td>
<td>4592</td>
</tr>
<tr>
<td>0.005</td>
<td>40.60</td>
<td>957</td>
</tr>
<tr>
<td>0.01</td>
<td>38.28</td>
<td>473.84</td>
</tr>
<tr>
<td>0.05</td>
<td>34.28</td>
<td>126.36</td>
</tr>
<tr>
<td>0.1</td>
<td>23.26</td>
<td>76.64</td>
</tr>
</tbody>
</table>

In all further investigations in this paper, we consider a sampling time of $T_S = 0.01$ seconds.

5.3 Multi-machine Power System Analysis

The proposed GA based technique is now used to determine the optimal parameters of the PSS on the three-machine system (refer Figure 2.3). In order to bring about robustness, a set of six operating points, as discussed in Section 4.4 is considered by varying the system loads, Load-A, Load-B and Load-C, in steps. Based on these load conditions, an optimal power flow (OPF) with minimizing losses as the criterion is run for each load configuration. Thus, the generation level for each unit

$^2$ Nominal PSS is the optimum PSS obtained with $T_S = 0.01$ sec (i.e. $K_P=38.28$, $K_D=473.84$, $K_I=0.30$)
in each load configuration considered is obtained and represents the initial operating point for the system.

As in the case of SMIB system, Figure 5.6 shows the current generation best individual’s performance index and Figure 5.7 shows the convergence of PID-PSS parameters.

![Genetic search performance for multi-machine PID PSS tuning](image)

**Figure 5.6** Genetic search performance for multi-machine PID PSS tuning

![Solution variation during genetic search for multi-machine PID-PSS tuning](image)

**Figure 5.7** Solution variation during genetic search for multi-machine PID-PSS tuning
We note that the optimum solution is achieved in approximately 115 generations when $J_{AVG}$ attains a steady-state. However, the derivative and integral gains change slightly even afterwards, while on the other hand, $K_r$ remains unchanged. This indicates a strong relationship between $K_r$ and $J_{AVG}$, and a weak dependence between the other two gains ($K_p$ and $K_i$) and $J_{AVG}$.

Table 5.3 shows the optimal solutions obtained with the proposed technique for PID and lead-lag types of PSS (4.1). In this case, just by comparing the values of the corresponding performance indices it can be concluded that the PID-PSS performs better.

**Table 5.3 Optimum PSS solution and performance index for GA based PID and Lead-lag types of stabilizer**

<table>
<thead>
<tr>
<th>Machine</th>
<th>GA Based PID PSS</th>
<th>J$_{AVG}$ [x10$^6$]</th>
<th>GA Based lead-lag PSS</th>
<th>J$_{AVG}$ [x10$^6$]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K_p$</td>
<td>$K_d$</td>
<td>$K_i$</td>
<td>$K_c$</td>
</tr>
<tr>
<td>1</td>
<td>56.11</td>
<td>44.01</td>
<td>-0.24</td>
<td>45.06</td>
</tr>
<tr>
<td>2</td>
<td>12.92</td>
<td>2.75</td>
<td>4.91</td>
<td>45.52</td>
</tr>
<tr>
<td>3</td>
<td>29.93</td>
<td>29.81</td>
<td>28.92</td>
<td>2.13</td>
</tr>
</tbody>
</table>

In order to test the robustness of the GA based PSS we consider three different operating conditions, a light load, a nominal load and a heavy load, as given in Table 5.4. It might be noted that these load conditions at buses #5, #6 and #8, are same as those used in [34], except that the corresponding generation levels are obtained here using an OPF simulation with minimizing losses as objective.

**Table 5.4 Loading conditions used to test the robustness of GA based PID-PSS**

<table>
<thead>
<tr>
<th>Operating conditions (in per unit)</th>
<th>Gen-1</th>
<th>Gen-2</th>
<th>Gen-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loading</td>
<td>P</td>
<td>Q</td>
<td>P</td>
</tr>
<tr>
<td>Nominal</td>
<td>0.71</td>
<td>0.28</td>
<td>1.63</td>
</tr>
<tr>
<td>Heavy</td>
<td>2.77</td>
<td>1.2</td>
<td>1.38</td>
</tr>
<tr>
<td>Light</td>
<td>0.81</td>
<td>0.14</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Figure 5.8, Figure 5.9 and Figure 5.10 show the dynamic responses corresponding to PID and lead-lag PSS plotted for rotor speed deviation of generator-1 following a 1% step change in mechanical torque on the same generator for nominal, light and heavy load conditions, respectively. The PID-PSS has a lower peak offshoot and a well-damped response for all cases investigated.
Rotor speed deviation on generator 1 for multi-machine system with PID or lead-lag stabilizer under nominal load conditions

Figure 5.8

Rotor speed deviation on generator 1 for multi-machine system with PID and lead-lag stabilizer under light load conditions

Figure 5.9
5.4 Concluding Remarks

This chapter presents the design of a proportional plus integral plus derivative (PID) power system stabilizer (PSS) that uses the Integral of Squared Error (ISE) criterion to determine the performance of each individual during the search process.

The advantage of using PID-PSS is that these are easy to realize and being in discrete mode, their associated computations are lesser. Genetic algorithms applications to such PID-PSS provide an increased benefit, that of robustness.

Analytical studies show that the proposed GA based method for tuning PID-PSS provides very satisfactory dynamic performances over a wide operating domain and that their performances are comparable with the conventional lead-lag PSS.
6 Final Remarks

This thesis attempts to apply the powerful properties of a genetic algorithm (GA) based search and optimization method to tuning of power system stabilizers (PSS). One of the primary requirements of a good tuning method is that the resulting PSS is robust enough to wide variations in system parameters, while also being computationally manageable. In this respect, the proposed GA based tuning method provides satisfactory results.

The work presented here deals with the design of lead-lag, derivative type and proportional plus integral plus derivative (PID) types of PSS. Parameter optimization based on Lyapunov’s method incorporating Integral of Squared Error (ISE) criterion has been used within the GA process in the proposed PSS tuning approaches.

The thesis also examines classical approaches to tuning of lead-lag and derivative PSS that consider one nominal operating condition. Investigations reveal that the classical approach does provide satisfactory performances for operating conditions up to the nominal but deteriorated responses when the load increases. Moreover, the classically tuned PSS fails to stabilize the system at certain operating conditions. The proposed GA based method on the other hand, provides the option of including any operating point within its tuning domain, thus ensuring system stability over a large domain, and in particular, the tuning domain.

Genetic Algorithms represent a powerful tool to solving optimization problems. They possess an intrinsic flexibility and the freedom to choose desirable optima according to design specifications. Therefore the objective function, being the part of the algorithm that models the actual dynamic problem we attempt to solve, plays a crucial role towards finding the global optimum.
Although the design method is meant to merely cope with small signal stability phenomena, when tested for transients, the system behaved satisfactorily.

6.1 Salient features of the present work

- The thesis provides a broad-ranging overview of the research work carried out in the area of PSS tuning over the past two decades and brings out the main research issues that have been addressed.

- The thesis provides a detailed description of the development of the system mathematical models, both for single-machine infinite bus as well as the multi-machine system under small perturbations. These models are generic enough and can be applied to large sized power systems.

- The thesis provides an exhaustive analysis of classical tuning methods applicable to lead-lag and derivative type PSS and brings out a comparison of performances achieved by systems having PSS designed using these methods.

- The thesis proposes a novel approach to tune lead-lag PSS using GA by applying the Lyapunov's method of parameter optimization. The main feature of this method is that it is a time-domain approach and uses a performance criterion that accurately quantifies the dynamic performance of the system under perturbation. The genetic process further uses this in the individuals’ fitness assignment stage as a measure of one's quality, thereafter creating a sound basis to finding the best individual in the population.

- The thesis develops a novel GA based optimization approach for tuning of PID PSS. The main feature of this approach is that, the PID PSS acts in discrete mode and thus, the system model has been developed in discrete domain. An optimal sampling period has been determined considering the conflicting requirements of computation time vis-à-vis accuracy of information on system dynamics, due to discretization.

6.2 Scope for future work in this area

Tuning of PSS for large interconnected power systems has been a challenging problem for power engineers and though a lot of work has been reported in this area, several issues remain unresolved. Based on the work reported in this thesis, we briefly layout some of the issues that need to be addressed within the same framework discussed here, in order to gain an exhaustive understanding of the problem of PSS tuning and its characteristics.
The system investigated has been limited up to a three generator, nine bus system. It would be desirable to examine GA based PSS tuning for larger and more realistic systems. Based on the experience accumulated during simulations and due to the development of both the system model and the GA program in a generic manner, the extension of the work could be done without difficulties.

As mentioned in one of the earlier chapters, siting of PSS is an important issue, more so, when the system size increases considerably. It is thus important to examine the GA based PSS tuning method while incorporating the PSS siting issues.

The systems considered in the thesis assume that the loads are constant impedance loads. It would be of interest to the designer to understand how the dynamics of the system will be affected by the load dependence on voltage and consequently, how the optimal PSS parameters will be affected.

The powerful properties of GA based optimization can be further exploited to examine various other controller structures and determine their globally optimal settings.

Test and implement different genetic algorithm strategies (e.g. multi-population, multi-objective) in an attempt to achieve a less time consuming process and gain better understanding of genetic algorithms applicability to various power system phenomena.
7 References

References


8 Appendices

8.1 Appendix I

8.1.1 Network Reduction
This property is used to obtain the network reduction as shown below.

Let

\[ \bar{I} = \bar{Y} \cdot \bar{V} \]  
\[ \text{(8.1)} \]

where \( \bar{V} \) is the bus voltage vector, \( \bar{Y} \) is the admittance matrix of the system and \( \bar{I} \) is the bus current vector. Therefore,

\[ \bar{I} = \begin{bmatrix} I_n \\ 0 \end{bmatrix} \]  
\[ \text{(8.2)} \]

Now the matrices \( \bar{Y} \) and \( \bar{V} \) are partitioned accordingly to get

\[ \begin{bmatrix} I_n \\ 0 \end{bmatrix} = \begin{bmatrix} \bar{Y}_{nn} & \bar{Y}_{nr} \\ \bar{Y}_{rn} & \bar{Y}_{rr} \end{bmatrix} \begin{bmatrix} \bar{V}_n \\ \bar{V}_r \end{bmatrix} \]  
\[ \text{(8.3)} \]

where the subscript \( n \) is used to denote generator nodes and the subscript \( r \) is used for the remaining nodes (Figure 8.1). Thus for the analyzed system \( \bar{V}_n \), which now
is the generator terminal voltage vector, has the dimension \((n \times 1)\) and \(\bar{V}_r\) has the dimension \((r \times 1)\).

Expanding equation (8.3),
\[
\bar{I}_n = \bar{V}_nn \cdot \bar{V}_n + \bar{V}_rr \cdot \bar{V}_r \\
0 = \bar{V}_nn \cdot \bar{V}_n + \bar{V}_rr \cdot \bar{V}_r
\]  
(8.4)

from which we eliminate \(\bar{V}_r\), to find
\[
\bar{I}_n = (\bar{V}_nn - \bar{V}_rr \cdot \bar{V}_rr^{-1} \cdot \bar{V}_rn) \cdot \bar{V}_n
\]  
(8.5)

If \((\bar{V}_nn - \bar{V}_rr \cdot \bar{V}_rr^{-1} \cdot \bar{V}_rn) = \bar{Y}_m\), then (8.5) becomes:
\[
\bar{I}_n = \bar{Y}_m \cdot \bar{V}_n
\]  
(8.6)

The matrix \(\bar{Y}_m\) is the desired reduced matrix of \(\bar{Y}\). It has the dimensions \((n \times n)\), where \(n\) is the number of generators.
8.1.2 Small-signal Stability Models of Power Systems

The phasor diagram of the \( i^{th} \) machine of a multi-machine system may be shown as in Figure 8.2. While \( d_i \) and \( q_i \) are the coordinates for the \( i^{th} \) machine alone, D and Q are the common coordinates for all machines in the system. The phase-angle difference between \( d_i \) and D, or \( q_i \) and Q, is denoted by \( \delta_i \), which is constantly changing and could be positive or negative.

![Figure 8.2 Phasor diagram of the \( i^{th} \) machine](image)

Note: The upper bar stands for complex values. Since we look only into the reduced network (having only the generator nodes), we leave out the subscript \( n \) standing for generator nodes. Therefore, the terminal voltage \( \vec{V}_i \) of the \( i^{th} \) machine of the system in common coordinates becomes:

\[
\vec{V}_i = E_{q_i}^* \cdot e^{(90-\delta_i)} - j \cdot x_{d_i}^* \cdot \vec{I}_i + (x_{q_i} - x_{d_i}^*) \cdot I_{q_i}^* \cdot e^{-j\delta_i}
\]  

(8.7)

Note that

\[
\vec{E}_{q_i}^* = E_{q_i}^* \cdot e^{(90-\delta_i)}, \quad \vec{I}_{q_i} = j \cdot I_{q_i} \cdot e^{-j\delta}
\]

(8.8)

For \( n \) machines of an \( n \)-machine system, equation (8.7) may be written in matrix form
\[ \mathbf{V} = [e^{j(90 - \delta)}] \mathbf{E}'_q - j \cdot [x'_q] \mathbf{I} + [x'_q - x_d'] \mathbf{I} \cdot e^{-j\delta} \mathbf{I}_q \]  

(8.9)

where the coefficients \([e^{j(90 - \delta)}] \mathbf{x}'_d \mathbf{I} \mathbf{x}'_q \mathbf{I}_q\) and \([e^{-j\delta}]\) should be read as diagonal matrices and \(\mathbf{V}, \mathbf{E}'_q, \mathbf{I}, \) and \(\mathbf{I}_q\) are column vectors of size \(n\).

### 8.1.3 Armature Current Components

Substituting the solution of \(\mathbf{V}\) of (8.9) in (8.6) and solving for \(\mathbf{I}\) gives

\[ \mathbf{I} = \mathbf{V} \cdot (e^{j(90 - \delta)} \mathbf{E}'_q - j \cdot [x'_d] \mathbf{I} + [x'_q - x_d'] \mathbf{I} \cdot e^{-j\delta} \mathbf{I}_q) \]  

(8.10)

where

\[ \mathbf{V} = (\mathbf{V}^{-1} + j \cdot [x'_d])^{-1} \]  

(8.11)

Note that the admittance matrix \(\mathbf{V}\) in equation (8.11) refers to the reduced network, therefore differs from \(\mathbf{Y}\) matrix from (8.1).

For the \(i^{th}\) machine of an \(n\)-machine system in \(D-Q\) coordinates, the current has \(n\) terms

\[ \mathbf{I}_i = \sum_{j=1}^{n} \mathbf{Y}_j \left[ e^{j(90 - \delta_j)} \mathbf{E}'_j + (x_q - x_d') \cdot e^{-j\delta} \cdot I_q \right] \]  

(8.12)

including the term of \(j = i\).

In \(d_q - q_i\) coordinates,

\[ \mathbf{I}_i = \mathbf{I}_i \cdot e^{j\delta_i} = \sum_{j=1}^{n} \mathbf{Y}_j \left[ e^{j(\beta_j + \delta_j + 90)} \mathbf{E}'_j + (x_q - x_d') \cdot e^{-j(\beta_j + \delta_j)} \cdot I_q \right] \]  

(8.13)

where

\[ \mathbf{Y}_j = Y_j \cdot e^{j\beta_j}, \quad \delta_j = \delta_i - \delta_j \]  

(8.14)

Therefore
\[ i_{di} = \text{Re}(\vec{i}_i) = \sum_{j=1}^{n} Y_{ij} \left[ -S_{ij} \cdot E_{q_j}^{\cdot} + \left( x_{q_j} - x_{d_j}^{\cdot} \right) \cdot C_{ij} \cdot I_{q_j} \right] \]

\[ i_{qi} = \text{Im}(\vec{i}_i) = \sum_{j=1}^{n} Y_{ij} \left[ C_{ij} \cdot E_{q_j}^{\cdot} + \left( x_{q_j} - x_{d_j}^{\cdot} \right) \cdot S_{ij} \cdot I_{q_j} \right] \]

(8.15)

where

\[ C_{ij} = \cos(\beta_{ij} + \delta_{ij}) \quad S_{ij} = \sin(\beta_{ij} + \delta_{ij}) \]

(8.16)

Let the deviation of \( \vec{i}_i \) be defined by

\[ \Delta I_{di} + j \cdot i_{qi} = \Delta I_{d,i} + j \cdot \Delta I_{q,i} \]

(8.17)

Here \( I \)'s stand for currents in individual machine coordinates. Same rule will be applied for \( V \)'s.

From (8.15) and for \( n \) machines, we will have

\[ \Delta I_{d} = P_{d} \cdot \Delta \delta + Q_{d} \cdot \Delta E_{q}^{\cdot} + M_{d} \cdot \Delta I_{q} \]

\[ L_{q} \cdot \Delta I_{q} = P_{q} \cdot \Delta \delta + Q_{q} \cdot \Delta E_{q}^{\cdot} \]

(8.18)

where

\[ P_{d_{ij}} = Y_{ij} \left[ C_{ij} \cdot E_{q_j}^{\cdot} + \left( x_{q_j} - x_{d_j}^{\cdot} \right) \cdot S_{ij} \cdot I_{q_j} \right] \]

\[ P_{q_{ij}} = Y_{ij} \left[ S_{ij} \cdot E_{q_j}^{\cdot} - \left( x_{q_j} - x_{d_j}^{\cdot} \right) \cdot C_{ij} \cdot I_{q_j} \right] \]

\[ P_{d_{ij}} = -\sum_{j \neq i} P_{d_{ij}}, \quad P_{q_{ij}} = -\sum_{j \neq i} P_{q_{ij}} \]

\[ Q_{d_{ij}} = -Y_{ij} \cdot S_{ij}, \quad Q_{q_{ij}} = Y_{ij} \cdot C_{ij}, \quad j = \ldots, n \]

(8.19)

\[ L_{q_{ij}} = -Y_{ij} \cdot \left( x_{q_j} - x_{d_j}^{\cdot} \right) \cdot S_{ij} \quad j \neq i \]

\[ L_{ii} = 1 - Y_{ii} \cdot \left( x_{q_i} - x_{d_i}^{\cdot} \right) \cdot S_{ii} \]

\[ M_{q_{ij}} = Y_{ij} \cdot \left( x_{q_j} - x_{d_j}^{\cdot} \right) \cdot C_{ij} \quad j = \ldots, n \]

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The solutions for $\Delta I_d$ and $\Delta I_q$ of (8.18) become

$$\Delta I_d = Y_d \cdot \Delta E'_q + F_d \cdot \Delta \delta$$

$$\Delta I_q = Y_q \cdot \Delta E'_q + F_q \cdot \Delta \delta$$

where

$$Y_d = Q_d + M_d \cdot Y_q, \quad F_d = P_d + M_d \cdot F_q$$

$$Y_q = L_q^{-1} \cdot Q_q, \quad F_q = L_q^{-1} \cdot P_q$$

(8.21)

8.1.4 K-constant Derivation

Having found the d and q current components including the transmission relation, $K_1, \ldots, K_6$ will be expressed using the electrical torque expression, internal voltage equation, and from the terminal voltage relation, as it will be shown below.

An electric torque approximately equals an electric power when the synchronous speed is chosen as the base speed. For the $i$th machine,

$$T_{ei} \equiv \Re(\overline{T}_{ni} \cdot \overline{F}_{ni}) = I_{q_i} \cdot E'_{q_i} + I_{q_i} \cdot \left( x_{q_i} - x_{d_i} \right) \cdot I_{d_i}$$

(8.22)

After linearization and for $n$ machines, (8.22) becomes:

$$\Delta T_e = K_1 \cdot \Delta \delta + K_2 \cdot \Delta E'_q$$

(8.23)

where

$$K_1 = Q_1 \cdot F_q + D_t \cdot F_d$$

$$K_2 = Q_t \cdot Y_q + D_t \cdot Y_d + I_{q0}$$

(8.24)

in which $D_t, Q_t$ and $I_{q0}$ are diagonal matrices with the elements as it follows

$$D_{t_{ii}} = \left( x_{q_i} - x_{d_i} \right) \cdot I_{q0_i}$$

$$Q_{t_{ii}} = \left( x_{q_i} - x_{d_i} \right) \cdot I_{d0_i} + E'_{q0_i}$$

$$I_{q0_{ii}} = I_{q0_i}$$

(8.25)
The component of torque given by $K_1$ is in phase with $\Delta \delta$, hence representing a synchronizing torque component. The second term of $()$ represents the component of torque resulting from variations in field flux linkage.

The internal voltage equation for $n$ machines may be written

$$\left( I + s \cdot T_{do} \right) \cdot \Delta \tilde{E}_q = \Delta \tilde{E}_{sd} - \left[ x_d - x_{d*} \right] \cdot \Delta I_d$$  \hspace{1cm} (8.26)

where $I$ is the unity matrix and $T_{do}$ a diagonal matrix. Substituting $\Delta I_d$ of (8.20) in (8.26) and shifting terms gives

$$\left( I + s \cdot T_{do}^* \cdot K_{3y} \right) \cdot \Delta \tilde{E}_{q_i} = K_{3y} \cdot \left( \Delta E_{f_{ij}} + \sum_{j=1}^{n} \frac{1}{K_{3y}} \Delta E_{q_j} - \sum_{j=1}^{n} K_{3y} \cdot \Delta \delta_j \right)$$ \hspace{1cm} (8.27)

where

$$K_{3y} = \left( I + \left( x_{d_{ij}} - x_{d*} \right) \cdot Y_{d_{ij}} \right)^{-1}$$
$$K_{3y} = \left( x_{d_{ij}} - x_{d*} \right) \cdot Y_{d_{ij}}$$ \hspace{1cm} (8.28)
$$K_{4y} = \left( x_{d_{ij}} - x_{d*} \right) \cdot F_{d_{ij}}$$

The terminal voltage relation could be written as

$$\Delta \tilde{V}_d = \left[ x_q \right] \cdot \Delta I_q$$
$$\Delta \tilde{V}_q = \Delta \tilde{E}_{q} - \left[ x_d \right] \cdot \Delta I_d$$ \hspace{1cm} (8.29)

Furthermore

$$\Delta \tilde{V} = \tilde{V}_0^{-1} \cdot \tilde{V}_{do} \cdot \Delta \tilde{V}_{do} + \tilde{V}_0^{-1} \cdot \tilde{V}_{q0} \cdot \Delta \tilde{V}_{q0}$$ \hspace{1cm} (8.30)

which can be written as

$$\Delta \tilde{V} = K_5 \cdot \Delta \delta + K_6 \cdot \Delta \tilde{E}_q$$ \hspace{1cm} (8.31)

where
\[
K_5 = D_v \cdot \left[ x_q \right] \cdot F_q - Q_v \cdot \left[ x_d' \right] \cdot F_d
\]
\[
K_6 = D_v \cdot \left[ x_q \right] \cdot Y_q - Q_v \cdot \left[ x_d' \right] \cdot Y_d + Q_v
\]

(8.32)

and

\[
D_v = V_0^{-1} \cdot V_d \cdot V_q \cdot V_0^{-1} \cdot V_q
\]

In these equations, \( V_0 \), \( V_d \), \( V_q \), \( D_v \), \( Q_v \), \( [x_q] \) and \( [x_d'] \) should be read as diagonal matrices.
8.2 Appendix II - System Data

8.2.1 System data for SMIB
All data are in p.u., except when specified.

\[
\begin{align*}
P &= 0.8 \\
Q &= 0.6 \\
V &= 1.0 \\
r &= 0.0 \\
x &= 0.2 \\
B &= 0.0 \\
G &= 0.0 \\
x_d &= 1.60 \\
x_q &= 1.55 \\
x_d' &= 0.32 \\
M &= 10.0 \text{ [s]} \\
T_{do}' &= 6.0 \text{ [s]} \\
f &= 50 \text{ [Hz]} \\
K_4 &= 50 \\
T_4 &= 0.05 \text{ [s]} \\
dT_M &= 0.01 \\
T_2 &= 0.05 \text{ [s]} \\
T_i &= 0.05 \text{ [s]} \\
T_{iw} &= 10.0 \text{ [s]} 
\end{align*}
\]
### 8.2.2 System data for 3-machine system (WSCC 3-machine, 9-bus)

#### Table 8.1 Generator data

<table>
<thead>
<tr>
<th>Generator</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated MVA</td>
<td>247.5</td>
<td>192.0</td>
<td>128.0</td>
</tr>
<tr>
<td>Voltage, kV</td>
<td>16.5</td>
<td>18.0</td>
<td>13.8</td>
</tr>
<tr>
<td>Power factor</td>
<td>1.0</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>Type</td>
<td>hydro</td>
<td>steam</td>
<td>steam</td>
</tr>
<tr>
<td>Speed, r/min</td>
<td>180</td>
<td>3600</td>
<td>3600</td>
</tr>
<tr>
<td>$x_d$, pu</td>
<td>0.1460</td>
<td>0.8958</td>
<td>1.3125</td>
</tr>
<tr>
<td>$x_d'$, pu</td>
<td>0.0608</td>
<td>0.1198</td>
<td>0.1813</td>
</tr>
<tr>
<td>$x_{q}$, pu</td>
<td>0.0969</td>
<td>0.8645</td>
<td>1.2578</td>
</tr>
<tr>
<td>$x_{q}'$, pu</td>
<td>0.0969</td>
<td>0.1969</td>
<td>0.250</td>
</tr>
<tr>
<td>$x_p$, pu</td>
<td>0.0336</td>
<td>0.0521</td>
<td>0.0742</td>
</tr>
<tr>
<td>$T_{dit}$, sec</td>
<td>8.96</td>
<td>6.00</td>
<td>5.89</td>
</tr>
<tr>
<td>$T_{qit}$, sec</td>
<td>0.0</td>
<td>0.535</td>
<td>0.60</td>
</tr>
<tr>
<td>H, sec</td>
<td>23.64</td>
<td>6.40</td>
<td>3.01</td>
</tr>
</tbody>
</table>

#### Table 8.2 Network data

<table>
<thead>
<tr>
<th>Bus</th>
<th>Type</th>
<th>Impedance [pu]</th>
<th>Admittance [pu]</th>
<th>Tap ratio [kV/kV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-4</td>
<td>trafo</td>
<td>0.0 + j0.0576</td>
<td>0.0 + j0.0</td>
<td>16.5/230</td>
</tr>
<tr>
<td>2-7</td>
<td>trafo</td>
<td>0.0 + j0.0625</td>
<td>0.0 + j0.0</td>
<td>18/230</td>
</tr>
<tr>
<td>3-9</td>
<td>trafo</td>
<td>0.0 + j0.0586</td>
<td>0.0 + j0.0</td>
<td>13.8/230</td>
</tr>
<tr>
<td>4-5</td>
<td>line</td>
<td>0.010 + j0.085</td>
<td>0.0 + j0.088</td>
<td>-</td>
</tr>
<tr>
<td>4-6</td>
<td>line</td>
<td>0.017 + j0.092</td>
<td>0.0 + j0.079</td>
<td>-</td>
</tr>
<tr>
<td>5-7</td>
<td>line</td>
<td>0.032 + j0.161</td>
<td>0.0 + j0.153</td>
<td>-</td>
</tr>
<tr>
<td>6-9</td>
<td>line</td>
<td>0.039 + j0.170</td>
<td>0.0 + j0.179</td>
<td>-</td>
</tr>
<tr>
<td>7-8</td>
<td>line</td>
<td>0.0085 + j0.072</td>
<td>0.0 + j0.0745</td>
<td>-</td>
</tr>
<tr>
<td>8-9</td>
<td>line</td>
<td>0.0119 + j0.1008</td>
<td>0.0 + j0.1045</td>
<td>-</td>
</tr>
</tbody>
</table>
8.3 Appendix III

8.3.1 Matrices of Multi-machine System without PSS

\[
\begin{bmatrix}
0 & K_{11} & K_{12} & 0 & 0 & -K_{12} & K_{22} & 0 & 0 & -K_{22} & K_{32} & 0
\\
\frac{\Delta M_1}{M_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\\
-2\pi & \frac{K_{411}}{M_1} & -1 & \frac{K_{11}}{M_1} & 1 & 0 & -K_{12} & -1 & 0 & 0 & -K_{13} & 1 & 0
\\
\frac{T_{d01}}{M_1} & \frac{T_{d01} K_{411}}{M_1} & \frac{T_{d01}}{M_1} & 1 & 0 & \frac{T_{d01} K_{12}}{M_1} & \frac{T_{d01} K_{13}}{M_1} & \frac{T_{d01} K_{22}}{M_1} & 0 & \frac{T_{d01} K_{32}}{M_1} & \frac{T_{d01} K_{43}}{M_1} & 0
\\
\frac{T_{s1}}{M_2} & \frac{T_{s1} M_2}{M_2} & 0 & \frac{T_{s1} M_2}{M_2} & 0 & \frac{T_{s1} M_2}{M_2} & 0 & \frac{T_{s1} M_2}{M_2} & 0 & \frac{T_{s1} M_2}{M_2} & 0 & 0
\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\\
0 & \frac{K_{421}}{M_2} & -1 & \frac{K_{12}}{M_2} & 1 & 0 & -K_{13} & -1 & 0 & 0 & -K_{22} & 1 & 0
\\
0 & \frac{T_{d02}}{M_2} & \frac{T_{d02} K_{421}}{M_2} & \frac{T_{d02}}{M_2} & 1 & 0 & \frac{T_{d02} K_{13}}{M_2} & \frac{T_{d02} K_{23}}{M_2} & \frac{T_{d02} K_{33}}{M_2} & 0 & \frac{T_{d02} K_{43}}{M_2} & \frac{T_{d02} K_{53}}{M_2} & 0
\\
0 & \frac{T_{s2}}{M_2} & \frac{T_{s2} M_2}{M_2} & 0 & \frac{T_{s2} M_2}{M_2} & 0 & \frac{T_{s2} M_2}{M_2} & 0 & \frac{T_{s2} M_2}{M_2} & 0 & \frac{T_{s2} M_2}{M_2} & 0 & 0
\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\\
0 & \frac{K_{431}}{M_3} & -1 & \frac{K_{13}}{M_3} & 1 & 0 & -K_{14} & -1 & 0 & 0 & -K_{23} & 1 & 0
\\
0 & \frac{T_{d03}}{M_3} & \frac{T_{d03} K_{431}}{M_3} & \frac{T_{d03}}{M_3} & 1 & 0 & \frac{T_{d03} K_{14}}{M_3} & \frac{T_{d03} K_{24}}{M_3} & \frac{T_{d03} K_{34}}{M_3} & 0 & \frac{T_{d03} K_{44}}{M_3} & \frac{T_{d03} K_{54}}{M_3} & 0
\\
0 & \frac{T_{s3}}{M_3} & \frac{T_{s3} M_3}{M_3} & 0 & \frac{T_{s3} M_3}{M_3} & 0 & \frac{T_{s3} M_3}{M_3} & 0 & \frac{T_{s3} M_3}{M_3} & 0 & \frac{T_{s3} M_3}{M_3} & 0 & 0
\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\\
\end{bmatrix}
\]

\[A = \begin{bmatrix}
0 & 0 & 0 & \frac{K_{A1}}{T_{A1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\\
\frac{1}{2H_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\\
0 & 0 & 0 & 0 & 0 & \frac{1}{2H_2} & 0 & 0 & 0 & 0 & 0 & 0
\\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2H_3} & 0 & 0 & 0 & 0 & 0
\\
\end{bmatrix}
\]
8.3.2 Matrices Multi-machine System with PSS

Let us consider the following notations:

\[ r_1 = \frac{K_{C_1} \cdot T_{11}}{T_{21}}; \quad m_1 = \frac{r_1 \cdot T_{31}}{T_{41}} \]

\[ r_2 = \frac{K_{C_2} \cdot T_{12}}{T_{22}}; \quad m_2 = \frac{r_2 \cdot T_{32}}{T_{42}} \]

\[ r_3 = \frac{K_{C_3} \cdot T_{13}}{T_{23}}; \quad m_3 = \frac{r_3 \cdot T_{33}}{T_{43}} \]

Thus, the matrices of the multi-machine power system introduced in Chapter 3 become:

\[
\begin{bmatrix}
A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} & A_{17} & A_{18} & A_{19} & A_{110} & A_{111} & A_{112} \\
r_{1}A_{11} & r_{1}A_{12} & r_{1}A_{13} & r_{1}A_{14} & r_{1}A_{15} & r_{1}A_{16} & r_{1}A_{17} & r_{1}A_{18} & r_{1}A_{19} & r_{1}A_{110} & r_{1}A_{111} & r_{1}A_{112} \\
m_{1}A_{11} & m_{1}A_{12} & m_{1}A_{13} & m_{1}A_{14} & m_{1}A_{15} & m_{1}A_{16} & m_{1}A_{17} & m_{1}A_{18} & m_{1}A_{19} & m_{1}A_{110} & m_{1}A_{111} & m_{1}A_{112} \\
A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} & A_{27} & A_{28} & A_{29} & A_{210} & A_{211} & A_{212} \\
r_{2}A_{21} & r_{2}A_{22} & r_{2}A_{23} & r_{2}A_{24} & r_{2}A_{25} & r_{2}A_{26} & r_{2}A_{27} & r_{2}A_{28} & r_{2}A_{29} & r_{2}A_{210} & r_{2}A_{211} & r_{2}A_{212} \\
m_{2}A_{21} & m_{2}A_{22} & m_{2}A_{23} & m_{2}A_{24} & m_{2}A_{25} & m_{2}A_{26} & m_{2}A_{27} & m_{2}A_{28} & m_{2}A_{29} & m_{2}A_{210} & m_{2}A_{211} & m_{2}A_{212} \\
A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} & A_{37} & A_{38} & A_{39} & A_{310} & A_{311} & A_{312} \\
r_{3}A_{31} & r_{3}A_{32} & r_{3}A_{33} & r_{3}A_{34} & r_{3}A_{35} & r_{3}A_{36} & r_{3}A_{37} & r_{3}A_{38} & r_{3}A_{39} & r_{3}A_{310} & r_{3}A_{311} & r_{3}A_{312} \\
m_{3}A_{31} & m_{3}A_{32} & m_{3}A_{33} & m_{3}A_{34} & m_{3}A_{35} & m_{3}A_{36} & m_{3}A_{37} & m_{3}A_{38} & m_{3}A_{39} & m_{3}A_{310} & m_{3}A_{311} & m_{3}A_{312}
\end{bmatrix}
\]

where \( A_{ij} \) are the elements of the \( A \) matrix of the multi-machine system described in Section 8.3.1.

\[
\Gamma_i = \begin{bmatrix}
\frac{1}{2H_1} & \frac{K_{C_1} T_{11}}{2H_1 T_{21}} & \frac{K_{C_1} T_{11} T_{31}}{2H_1 T_{21} T_{41}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2H_2} & \frac{K_{C_2} T_{12}}{2H_2 T_{22}} & \frac{K_{C_2} T_{12} T_{32}}{2H_2 T_{22} T_{42}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2H_3} & \frac{K_{C_3} T_{13} T_{33}}{2H_3 T_{23} T_{43}} & 0 & 0 & 0
\end{bmatrix}
\]
$$\mathbf{D} = \begin{bmatrix}
\frac{1}{\tau_{m}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\frac{\tau_{f}}{T_{1}}}{-1} & -\frac{1}{\tau_{m}} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\frac{\tau_{f}}{T_{1}}}{-1} \left(1 - \frac{\tau_{f}}{T_{1}}\right) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{\tau_{w}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\frac{\tau_{f}}{T_{2}}}{-1} & -\frac{1}{\tau_{2}} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\tau_{f}}{T_{2}} \frac{\tau_{f}}{T_{2}} & 0 & -\frac{1}{\tau_{2}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau_{w}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\frac{\tau_{f}}{T_{3}}}{-1} & -\frac{1}{\tau_{3}} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\tau_{f}}{T_{3}} \frac{\tau_{f}}{T_{3}} & 0 & -\frac{1}{\tau_{3}} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\tau_{f}}{T_{3}} \frac{\tau_{f}}{T_{3}} & 0 & -\frac{1}{T_{23}} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\tau_{f}}{T_{3}} \frac{\tau_{f}}{T_{3}} & 0 & \frac{\tau_{f}}{T_{3}} \frac{\tau_{f}}{T_{3}}
\end{bmatrix}$$