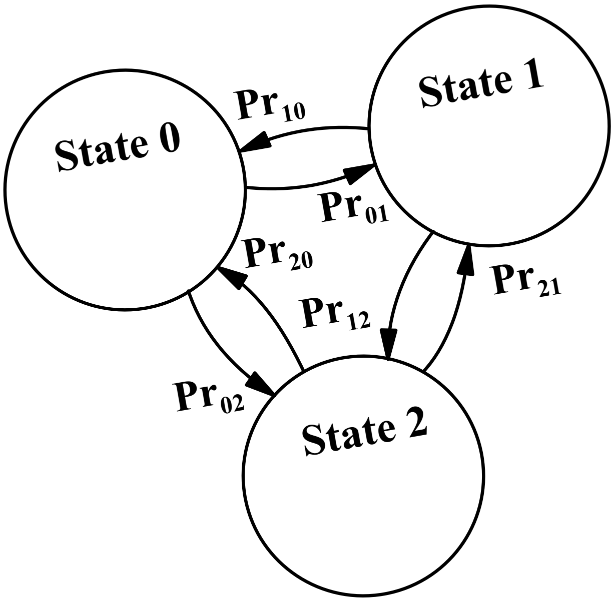

Power System Reliability Assessment using the Weibull-Markov Model

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by

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Dedicated to Wille

Preface

This thesis introduces the Weibull-Markov model as an alternative to the widely used homogenous Markov model. Such alternatives are likely to meet a lot of scepticism. This is understandable, because the homogenous Markov models are rooted in complex stochastic theory and have been used to great success in all areas of reliability calculations for a long time. To even think of developing a serious alternative in the course of a PhD project, and then even to do this as an electrical engineer, must seem to be at least naive.

Yet, the work presented here is not believed to be in vain. Although it took some time to get the mathematical equations in a form that seems to be bearable for real mathematicians, the principle idea behind them survived through the process of testing, checking and implementation. This gives good hope for the future of the Weibull-Markov model.

This Licenciate Thesis would not have been made possible without the constructive cooperation between the Chalmers University in Sweden and the DIgSILENT company in Germany.

Göteborg, Sweden, 27.02.2001,
Jasper van Casteren

Abstract

This Licenciate Thesis introduces an alternative stochastic model for performing reliability assessment calculations in electric power systems. This new model has been developed because the commonly used “homogenous Markov” model cannot be used to calculate cost parameters accurately. Yet, the current market developments lead to an increasing demand for cost-oriented reliability assessment.

The proposed alternative model, which was given the name “Weibull-Markov Model”, has been implemented and used in a commercial reliability assessment program with success. The use of the new model has proven not to cause any relevant slowing down of the calculation process, and yet to deliver reliability cost indices at the same time. Additional reliability calculations for cost calculations therefore are felt not to be required anymore.

A very important quality of the Weibull-Markov model is that it is 100% backwards compatible with the homogenous Markov model. This means that the reliability data that has been gathered to great costs and effort in the past, can still be used in the new calculations. The Weibull-Markov model provides for a so-called shape parameter with wich an existing homogenous Markov model can be adjusted to bring it closer to the measured data without actually changing the original data.

More research however will be needed to test the Weibull-Markov model further for its merits and limits.

Contents

Terms and Abbreviations

Terms

M_D^k	k^{th} central moment of D
V_D	variance of D
σ_D	standard variance of D
$G_D(t, \tau)$	remainder CDF for $D > t$
X_c	Stochastic component c
X_{c,n_c}	n_c'th successive state of X_c
T_{c,n_c}	n_c'th successive epoch at which X_c changes
$D_{c,i}$	duration of $X_c = i$
$\lambda_{c,ij}$	homogenous transition rate for $X_c = i$ to $X_c = j$
$\lambda_{c,i}$	homogenous state transition rate for $X_c = i$
S	Stochastic system
S_{n_s}	n_s'th successive state of S
T_{n_s}	n_s'th successive epoch at which S changes
X_{c,n_s}	X_c according to S_{n_s}
$X_{c,s}$	X_c according to $S = s$
$D_c(n_s)$	Remaining duration of X_{c,n_s} at T_{n_s}
$A_c(n_s)$	Passed duration of X_{c,n_s} at T_{n_s}
$\eta_{c,i}$	Weibull form factor for $X_c = i$
$\beta_{c,i}$	Weibull shape factor for $X_c = i$
$M_{c,i}$	Mean duration of $X_c = i$
$V_{c,i}$	Variance of the duration of $X_c = i$
$?_{c,n_s}$	= $?_{c,i}$ for $X_{c,n_s} = i$, i.e. η_{c,n_s}, β_{c,n_s}, etc.
$?_{c,s}$	= $?_{c,i}$ for $X_{c,s} = i$, i.e. $\eta_{c,s}$, $\beta_{c,s}$, etc.
$\mathrm{Fr}_{c,i}$	Frequency of occurrence of $X_c = i$
$\mathrm{Pr}_{c,i}$	Probability of occurrence of $X_c = i$
Fr_s	Frequency of occurrence of S=s
Pr_s	Probability of S=s

Abbreviations

TTF	Time To Failure
TTR	Time To Repair
MTBF	Mean Time Between Failures
TBM	Time Between Maintenance
CDF	Cumulative Density Function
PDF	Probability Density Function
SF	Survival Function

Chapter 1

Power System Reliability Assessment

The assessment of the reliability of a power system means the calculation of a set of performance indicators. An example of such an indicator is the average number of times per year that a certain load point cannot be supplied with electrical energy. Basically, there are two distinct sets of indicators: local indicators and system indicators. Local indicators are calculated for a specific point in the system. Examples are

- The average time per year during which a generator is not able to feed into the network
- The average duration of the interruptions at a busbar.
- The average interruption costs per year for a specific load.

System indicators express the overall system performance. Examples are

- The average amount of energy per year that cannot be delivered to the loads.
- the average number of interruptions per year, per customer.
- The average yearly interruption costs.

Power system reliability analysis is principally the analysis of a large set of unwanted system states which may occur in the future. The results of all these system state analyses is then used to calculate the various performance indicators. The basic diagram of the calculation procedure is depicted in Fig. 1.1. This diagram shows the healthy operational state of the system, in which all components behave properly. The reliability assessment creates events that will bring the system in an unhealthy state, which is then analyzed. When this analysis shows that the system is not longer able to meet all its demands, a set of intermediate results is send to a result analyzer. This is repeated for all relevant unhealthy system states. Finally, the result analyzer will post-process the gathered results in order to calculate the various performance indicators.

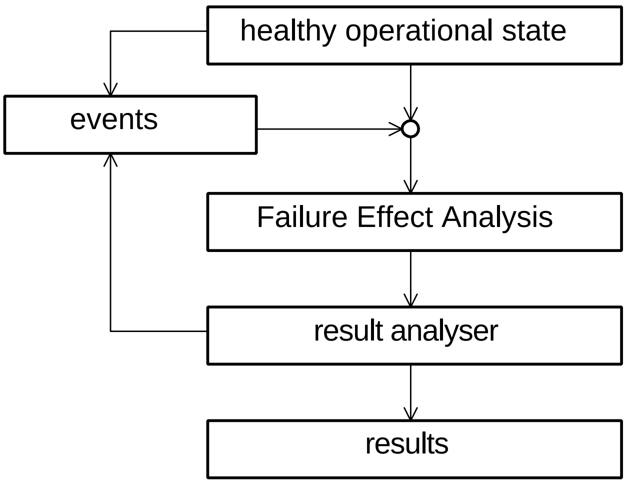

Figure 1.1: Basic reliability assessment scheme

The reliability assessment process thus starts with the creation of relevant system events. This must be done in such a way as to make it possible to weight the results of the failure effect analysis. Not every event is equally likely to happen, and the more likely events will have a greater impact on the performance indices than the less likely ones. Creating weighted events is made possible by the use of stochastic component models. These models are combined in system models and are used to create system states in specific orders and to calculate the frequency and probability of the occurrence of these system states.

This licenciate thesis introduces the basic methodologies for creating and using stochastic models, and will introduce the wide-spread and commonly used homogenous Markov model and the alternative Weibull-Markov model.

Chapter 2

Stochastic models

In order to calculate reliability performance indicators, the analyzed power system has to be represented by stochastic models first. An electrical power system is regarded as a collection of components. Each component is a typical part of the electric power system which is treated as one single object in the reliability analysis. Examples are a specific load, a line, a generator, etc., but also a complete transformer bay with differential protection, breakers, separators and grounding switches may be treated as a single component in a reliability assessment.

A component may exhibit different ’component states’, such as ’being available’, ’being repaired’, etc. In the example of a transformer, the following states could be distinguished:

1. the transformer performs to its requirements
2. the transformer does not meet all its requirements
3. the transformer is available, but not used
4. the transformer is in maintenance
5. the transformer is in repair
6. the transformer is being replaced by another transformer
7. the transformer behaves in such a way as to trigger its differential relay

For some reliability calculations, all these possible states may have to be accounted for. Normally, a reduction is made to two or three states. A two state model would, for instance, only distinguish between

1. the transformer is available
2. the transformer is not available

A three state model could further distinguish between repairs ("forced outages") and maintenance ("planned outages"), or between different levels of availability. Each of these states is described by

- an electrical model with electrical and operational constraints
- a duration distribution
- the possible transitions to the other states

The electric model for a transformer which is not available would be an infinite impedance, for instance. A model for a transformer which is only partly available would have a stuck tap changer, for instance, or would have a reduced capacity.

A stochastic component is a component with two or more states which have a random duration and for which the next state is selected randomly from the possible next states. A stochastic component changes abruptly from one state to another at unforeseen moments in time. If we would monitor such a stochastic component over a long period of time, while recognizing four distinct states – x_0, x_1, x_2 and x_3 –, a graph as depicted in Fig. 2.1 could be the result.
Because the behavior is stochastic, another graph will appear even if we would monitor an absolute exact copy of the component under exactly the same conditions.

For all stochastic models, only the state duration and the next state are stochastic quantities. Each distinct functional state of a component is therefore regarded as being completely deterministic, apart from its duration. Phenomena like randomly fluctuating impedances or random harmonic distortions are therefore not part of the stochastic behaviour of a component.

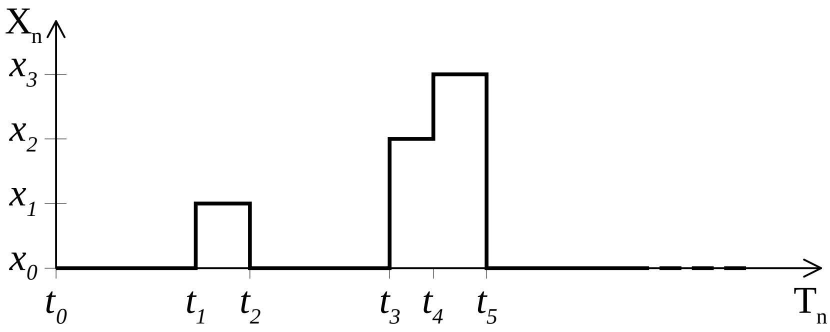

Figure 2.1: Example of monitored states of a component

If such phenomena are to be included in a reliability assessment, to assess the number of interruptions due to excessive harmonic distortion for example, the stochastic model must be extended by a number of states for which the fluctuating random quantity is considered constant.

This chapter introduces the stochastic models for electrical power system components. From these stochastic models, the model for a stochastic power system are then developed.

2.1 Stochastic Models

The basic quantity in reliability engineering is the duration D for which a component stays in the same state. This duration is a stochastic quantity, as its *precise* value is unknown. The word “precise” is emphasized here, as, although we don’t know the value of a stochastic quantity, we almost always know something about the possible values it could have. The time until the next unplanned trip of a generator, for example, is unknown, but nobody would expect a good generator to trip every day, as well as nobody would expect it to operate for 10 years continuously without tripping even once. This example range from 1 day to 10 years is too wide to be of practical use in a reliability assessment. However, for actual generators, a much smaller range of expected values can be justified by using measured data. The basic question about a stochastic quantity is thus about the range of its expected values, or ‘outcomes’: which outcomes can be expected and with what probability?

Both the outcome range and the outcome probabilities can be described by a single function; the "Cumulative Density Function" or CDF. This function is written as $F_D(\tau)$ and defines the probability of D being smaller than τ, which is written as:

$$F_D(\tau) = \Pr(D \leq \tau) \tag{2.1}$$

for reliability purposes, the probability for a negative duration is zero and the probability that the duration will be smaller than infinity is one:

$$\begin{aligned} F_D(0) &= 0 & (2.2) \\ F_D(\infty) &= 1 & (2.3) \end{aligned}$$

The "Probability Density Function", or PDF, for D, $f_D(\tau)$, is the derivative of the cumulative density function. The PDF gives a first insight into the possible values for D, and the likelihood of it taking a value in a certain range.

$$\begin{aligned} f_D(\tau) &= \frac{d}{d\tau} F_D(\tau) & (2.4) \\ f_D(\tau) &= \lim_{\Delta\tau \to 0} \frac{\Pr(\tau \leq D < \tau + \Delta\tau)}{\Delta\tau} & (2.5) \\ \int_0^\infty f_D(\tau) d\tau &= 1.0 & (2.6) \end{aligned}$$

The survival function (SF), $R_D(\tau)$, is defined as the probability that the duration D will be longer than τ.

$$R_D(\tau) = \Pr(D > \tau) = 1 - F_D(\tau) \tag{2.7}$$

For a specific component, i.e. a transformer, where D is the life time, which is also known as the time to failure (TTF), the survival function gives the probability for the component functioning for at least a certain amount of time without failures. For a large group of similar components, the SF is the expected fraction of components that will 'survive' up to a certain time without failures.

The hazard rate function (HRF), $h_D(\tau)$, is defined as the probability density for a component to fail, for a certain time τ, given the fact that the component is still functioning at τ,

$$h_D(\tau) = \lim_{\Delta\tau \to 0} \frac{\Pr(\tau \leq D < \tau + \Delta\tau \ \mid \ D > \tau)}{\Delta\tau} = \frac{f_D(\tau)}{R_D(\tau)} \tag{2.8}$$

The HRF is an estimate of the unreliability of the components that are still functioning without failures after a certain amount of time. An increasing HRF signals a decreasing reliability. An increasing HRF means that the probability of failure in the next period of time will increase with age. A decreasing HRF could, for instance, occur when only the better components survive.

The expected value of a function g of a stochastic quantity D is defined as

$$E(g(D)) = \int_0^\infty g(\tau) f_D(\tau) d\tau \tag{2.9}$$

The expected value of D itself is its mean E_D, which is defined as

$$E_D = E(D) = \int_0^\infty D f_D(\tau) d\tau \tag{2.10}$$

The k'th central moment, M_D^k, is defined as

$$M_D^k = E([D - E(D)]^k) \tag{2.11}$$

The variance V_D is defined as the second central moment

$$V_D = M_D^2 = E([D - E(D)]^2) = E(D^2) - (E(D))^2 \tag{2.12}$$

The standard variance σ_D is defined as the square of the variance

$$\sigma_D = \sqrt{V_D} \tag{2.13}$$

The remainder CDF is defined as the CDF of the remaining duration after an inspection moment t. Because the total duration D is a stochastic quantity, the remainder $D - t$ is also stochastic. The remainder CDF, $G_D(\tau, t)$, is defined as

$$\begin{aligned} G_D(\tau, t) &= \Pr(D \leq \tau \mid D > t) = \frac{\Pr(t < D \leq \tau)}{\Pr(t < D)} \\ &= \frac{F_D(\tau) - F_D(t)}{R_D(t)} \end{aligned} \tag{2.14}$$

2.1.1 The Exponential Distribution

The negative exponential distribution is defined by

$$f_D(\tau) = \lambda e^{-\lambda\tau} \tag{2.15}$$

which makes that

$$F_D(\tau) = 1 - e^{-\lambda\tau} \tag{2.16}$$

$$h_D(\tau) = \lambda \tag{2.17}$$

$$E_D = \frac{1}{\lambda} \tag{2.18}$$

$$V_D = \frac{1}{\lambda^2} \tag{2.19}$$

The HRF of the negative exponential distribution, $h_D(\tau)$, is not dependent of time which considerably simplifies many reliability calculations.

The remainder CDF, $G_T(\tau, t)$, for the negative exponential distribution equals

$$G_D(\tau, t) = 1 - e^{-\lambda(\tau - t)} = F_D(\tau - t) \tag{2.20}$$

The remainder of an exponential distributed duration thus has the same distribution as the total duration. The expected value, variance, etc. for the remainder thus equal those of the total duration, independent of the inspection time. This is a peculiarity unique to the negative exponential distribution. It also shows that the negative exponential distribution is a very abstract distribution. If, for instance, a repair duration would be negative exponentially distributed, then the expected time to finish the repair would be independent of the time already spend repairing. Such a type of repair is hard to imagine. An example of a duration that is negative exponentially distributed is the time it takes to throw 9 sixes with 9 dice, with a throw about every 3 seconds. In this way, a throw of 9 sixes will occur about once a year, on average, but the expected time until the next 9 sixes will always be about 1 year as the probability of 9 sixes in the next throw is independent of the number of trials done so far.

2.1.2 The Weibull distribution

Where the exponential distribution uses only one parameter (λ), the Weibull PDF uses a shape factor β and a scale factor η. It is defined as

$$f_D(\tau) = \frac{\beta}{\eta^\beta}\tau^{\beta-1}\exp\left[-\left(\frac{\tau}{\eta}\right)^\beta\right] \tag{2.21}$$

which makes that

$$F_D(\tau) = 1 - \exp\left[-\left(\frac{\tau}{\eta}\right)^\beta\right] \tag{2.22}$$

$$h_D(\tau) = \frac{\beta}{\eta^\beta}\tau^{\beta-1} \tag{2.23}$$

$$E_D = \eta\Gamma\left(1+\frac{1}{\beta}\right) \tag{2.24}$$

$$V_D = \eta^2\left\{\Gamma\left(1+\frac{2}{\beta}\right) - \left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2\right\} \tag{2.25}$$

where

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$$

is the normal gamma function.

The Weibull PDF equals the negative exponential distribution when the shape parameter $\beta = 1.0$. The gamma function which is needed for calculating the expected value and variance can be evaluated without much computational effort by standard numerical methods. Some examples of the Weibull PDF, for different means and variance are displayed in in Fig. 2.2 and Fig. 2.3. The conditional CDF, $G_T(\tau, t)$, for the Weibull distribution equals

$$G_D(\tau, t) = \exp\left[\left(\frac{t}{\eta}\right)^\beta - \left(\frac{\tau}{\eta}\right)^\beta\right] \tag{2.26}$$

which is dependent on the inspection time t.

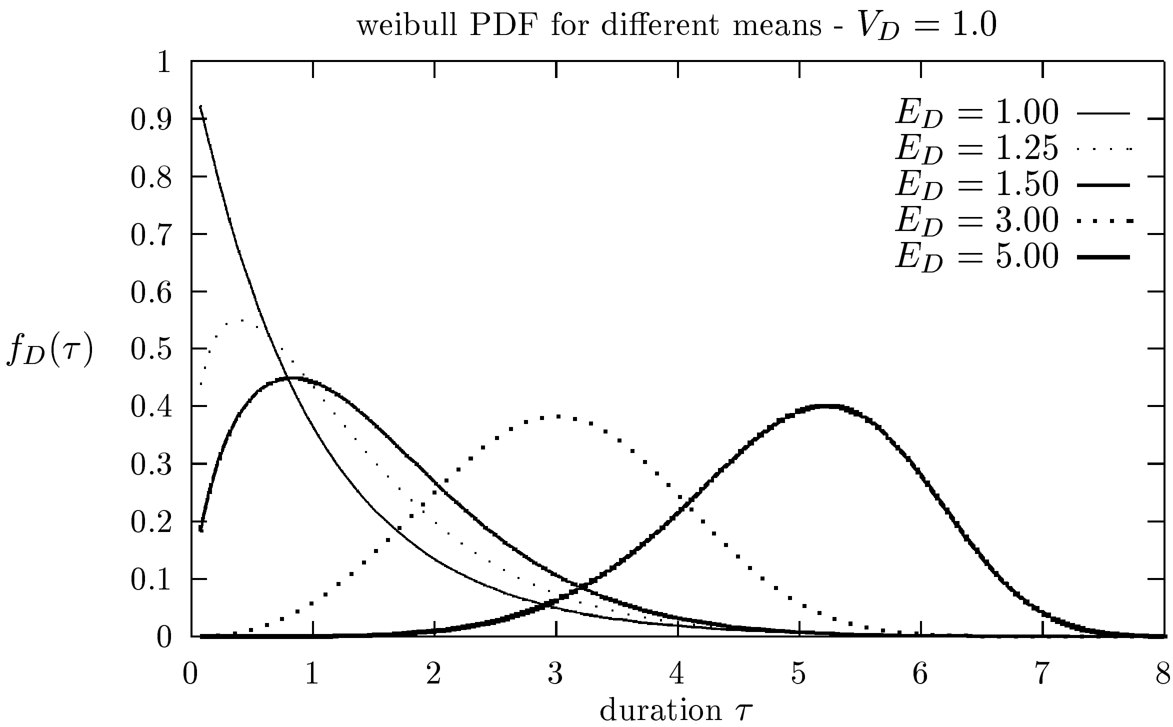

Figure 2.2: Some Weibull PDFs for different mean values

2.1.3 Weibull Probability Charts

The Weibull survival function can easily be transformed into straight lines. The transformation is made by taking the double logarithm of the reciprocal of the survival function

$$\ln(\frac{1}{R_D(\tau)}) = \left(\frac{\tau}{\eta}\right)^{\beta} \tag{2.27}$$

$$\ln(\ln(\frac{1}{R_D(\tau)})) = \beta\ln(\tau) - \beta\ln(\eta) \tag{2.28}$$

from which it is clear that a plot of $\ln(-\ln(R_D(\tau)))$ against $\ln(\tau)$ will produce straight lines. Because the transformation is independent of the shape and scale parameters, it is possible to draw plotting paper where the vertical axis is logarithmic and corresponds to the survival time τ, and the horizontal axis is transformed to correspond with $\ln(-\ln(1 - F_D(\tau)))$. Such “Weibull-probability charts” are a helpful tool in estimating shape and scale parameters without the risk of producing completely unrealistic parameters

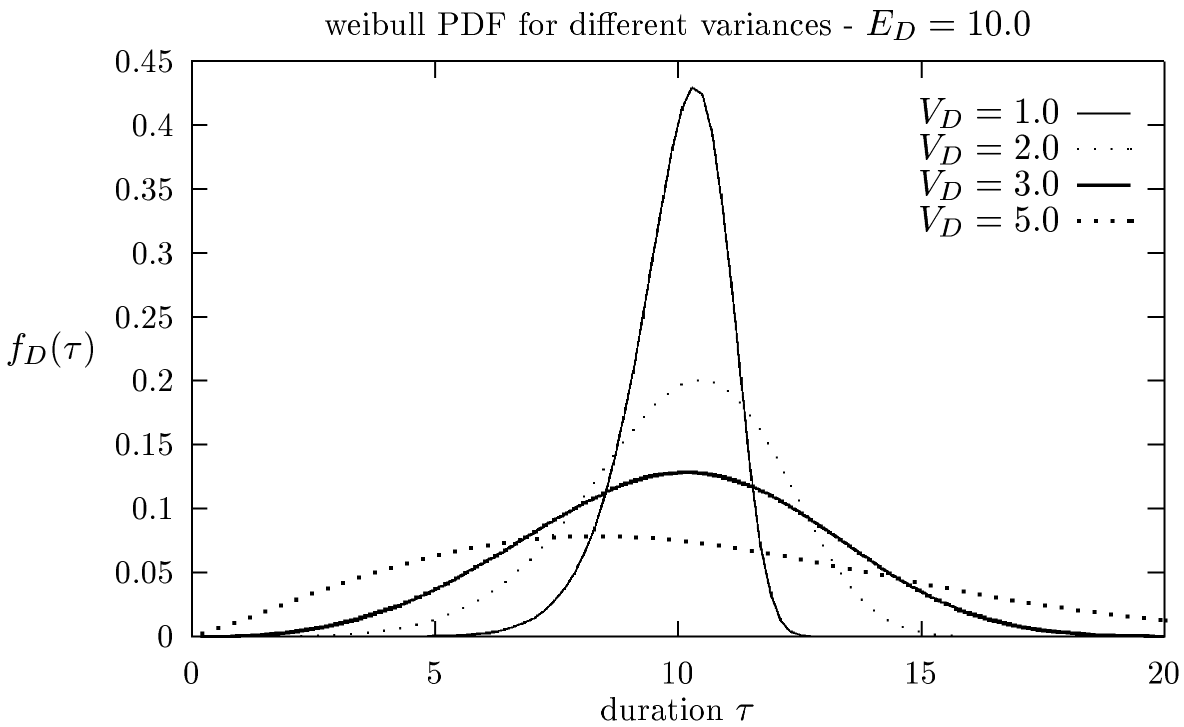

Figure 2.3: Some Weibull PDFs for different variance values

for measurements which do not fit a single Weibull distribution. Such unrealistic parameters would be produced without further warning by numerical parameter estimation algorithms.

2.1.4 Lifetime, Repair and Failure Density

In many reliability textbooks, a distinction is made between repairable and non-repairable components. While in many systems the latter are of great importance, non-repairable components do not exist in electric power systems or are of no importance. Electric power systems are not build to perform once or twice, but to perform continuously, 24 hours a day, 365 days per year. Therefore, there is no such thing as a ‘mission time’ for electric power systems, nor for their components.

However, in speaking of a repair of a power system component, an abstract definition of the word ‘repair’ is used. What actually is meant by repairable components is that the functionality of those components can always be restored, even when this means that the physical hardware (the appliance) is completely replaced.

A repair can restore the component to a condition as if it was brand new again. Repair by replacement is the trivial example. Such a perfect repair is called a repair-as-good-as-new. If the repair restores the component to about the same condition in which it was directly before the failure, it is called a repair-as-bad-as-old. Normally, repairs in between maintenance, which are often performed under great time pressure, are repairs-as-bad-as-old. Maintenance is normally considered to be a repair-as-good-as-new.

Components will normally start their life ready for use, i.e. with available functionality, and as-good-as-new. This is called their 'NEW' state. As soon as they are taken into service they will age and they will no longer be in their 'NEW' state. However, as long as they perform to expectation they are said to be in their 'UP' state. A component may fail at some time after it has been taken into service. This time is called the Time To Failure or TTF.

Ideal Maintenance

Non-repairable components will function until they fail. After that, their life is over. Although an electrical power system will never exhibit non-repairable components, their behaviour is of theoretical interest.

The life-time of non-repairable components may be prolonged by preventive maintenance. A preventive maintenance restores the component to the state 'as good as new', if it was still functioning. If we schedule maintenance at fixed intervals T_M, we can calculate the new PDF for the time to failure for the maintained component.

The probability density for the period until the first maintenance, $f_1(t)$, has the same shape as the original PDF because maintenance has not changed anything yet:

$$f_1(t) = \begin{cases} f_T(t) & \text{if } 0 < t \leq T_M \\ 0 & otherwise \end{cases} \tag{2.29}$$

Note the fact that $f_1(t)$ is strictly speaking not a PDF, because it only describes the distribution in the first period and $\int_0^\infty f_1(t)dt$ will therefore be smaller than one.

The probability of surviving until $t = T_M$ is $R_T(T_M)$. This is also the probability of the component to fail at $t > T_M$ if the maintenance would not have been performed. When the duration of the maintenance is neglected here, which is acceptable because it will normally be much smaller than T_M, then this remaining probability is distributed over all moments $t > T_M$. The shape of the distribution is again the same as the original because the maintenance is a repair-as-good-as-new. The probability distribution for the second period will therefore be

$$f_2(t) = \begin{cases} R_T(T_M) f_1(t - T_M) & \text{if } T_M < t \leq 2T_M \\ 0 & otherwise \end{cases} \tag{2.30}$$

The general solution is achieved by repeating this for $t = kT_M$ with $k \in \mathcal{N}^+$ (see [36, p.14])

$$f_T^*(t) = \sum_{k=0}^{\infty} R_T^k(T_M) f_1(t - kT_M) \tag{2.31}$$

This is an important result, because the geometric expression $R_T^k(T_M)$ forces the PDF to fluctuate between two negative exponential curves. Therefore, we would expect the failure rate to become more or less constant for higher t, regardless of the original shape of the PDF. This is an important result, because it shows that measurements of the time to failure should measure the time from the end of the last maintenance to the moment of failure, and not as the time between failures. Interpreting time between failures as a TTF will lead to a stochastic model with negative exponentially distributed durations, which would not represent the component itself, but the combination of the component and the planned maintenance. Such models can not be used for determining the effects of changed maintenance planning.

2.2 Homogenous Markov Models

One of the important qualities of the homogenous Markov model is that it causes each stochastic system build from homogenous Markov models to be a homogenous Markov model again, only much larger. This enables the calculation of state probability, frequency and duration by analytic matrix operations. Although this seems to be a great advantage, this is only partly so, because the calculation of system state indices is often much easier by adding the contributions of the system components. The one exception is the calculation of system state duration distributions, but these distributions, although easily obtained for a homogenous Markov system model, will be unrealistic in such a model. This is so because the one important disadvantage of the homogenous Markov model is the exclusive use of the negative exponential distribution for all stochastic durations in the system. In the case of repair of maintenance durations, these distributions are already highly unlikely, but in the case of life time, they cannot be other than incorrect. Using a negative exponentially distributed lifetime will always cause the model to react to preventive maintenance by a lowered overall availability, which is surely not the case for normal power system equipment.

Nevertheless, the homogenous Markov model is very important due to its computational elegance. A good understanding of the basic properties of the homogenous Markov model is required for understanding other models and methods used in power system reliability assessment.

2.2.1 The Homogenous Markov Component

The monitored stochastic behaviour of a component can be described completely by a set of state and epoch combinations $(x_n, t_n)_{n=0}^{\infty}$. This is illustrated by Fig. 2.4 which shows a possible graph of the monitored states of a component with four distinct states; x_0, x_1, x_2 and x_3.

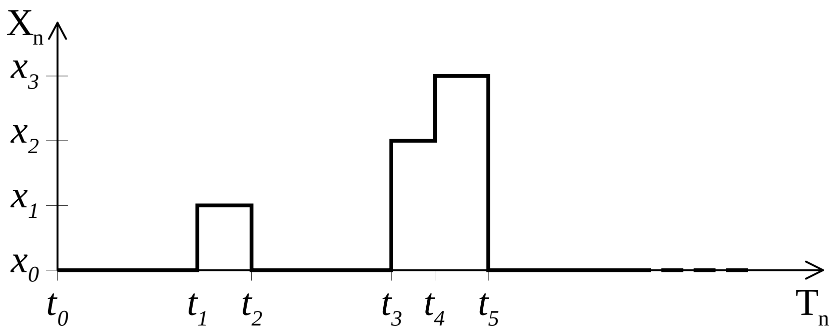

Figure 2.4: Example of monitored states of a component

If we would monitor the exact same component under exact the same conditions, another set $(x_n, t_n)_{n=0}^{\infty}$ will be the result, as each next state and each state duration $(t_{n+1} - t_n)$ are stochastic quantities. Each set $(x_n, t_n)_{n=0}^{\infty}$ is an outcome from a infinite number of possible outcomes, and is called a "component history". The stochastic history for the component with index "c", is written as "$(X_{c,n_c}, T_{c,n_c})_{n_c=0}^{\infty}$". Both X_{c,n_c} and T_{c,n_c} are stochastic quantities and we may talk, for example, about "the probability of $X_{c,23} = x_1$" or "the probability density function of $(T_{c,45} - T_{c,44})$".

The homogenous Markov model is now defined by:

- the set of possible states $\overline{x_c} = \{1, 2, \cdots, N_c\}$ where N_c is the number of possible states
- the stochastic history $(X_{c,n_c}, T_{c,n_c})_{n_c=0}^{\infty}$, where
  - $\forall_{n_c}(X_{c,n_c} \in \overline{x_c}, X_{c,n_c} \neq X_{c,n_c+1})$
  - $T_{c,0} = 0$ and $\forall_{n_c}(T_{c,n_c+1} > T_{c,n_c})$
- the set of continuous probability distribution functions $F_{c,ij}(t)$ for the conditional state durations $D_{c,ij}$

$$
\begin{aligned}
F_{c,ij}(t) &= \Pr(D_{c,ij} \leq t) \\
&= \Pr(X_{c,n_c} = i \;,\; (T_{c,n_c+1} - T_{c,n_c}) \leq t \mid X_{c,n_c+1} = j) \\
&= 1 - \exp\left(-\frac{t}{\lambda_{c,ij}}\right)
\end{aligned}
$$

From the homogenous Markov model, the stochastic process $X_c(t)$ can be defined as

$$
T_{c,n_c} \leq t < T_{c,n_c+1} \Rightarrow X_c(t) = X_{c,n_c} \tag{2.32}
$$

The conditional state durations $D_{c,ij}$ are the stochastic durations for state i, given the fact that the next state will be j. An outcome of a history is obtained by drawing outcomes $d_{c,ik}$ for all conditional state durations $D_{c,ik}$ in each state, and selecting the smallest value. Then, with j such that $d_{c,ij} = \min(d_{c,ik})$, $X_{c,n_c+1} = j$ and $T_{c,n_c+1} = T_{c,n_c} + d_{c,ij}$.

The probability distribution for the duration $D_{c,i}$ of state $x_{c,i}$ is thus the distribution of the minimum of the conditional durations:

$$
\begin{aligned}
F_{c,i}(t) &= \Pr(D_{c,i} \le t) = \Pr(\min_{j=1}^{N_c}(D_{c,ij}) \le t) \\
&= 1 - \prod_{j=1}^{N_c} \Pr(D_{c,ij} > t) = 1 - \prod_{j=1}^{N_c} \exp\left(-\frac{t}{\lambda_{c,ij}}\right) \\
&= 1 - \exp\left(-\frac{t}{\lambda_{c,i}}\right) \qquad (2.33)\\
\frac{1}{\lambda_{c,i}} &= \sum_{j=1}^{N_c} \frac{1}{\lambda_{c,ij}} \qquad (2.34)
\end{aligned}
$$

The state duration is thus again exponentially distributed and is characterized by the single ‘state transition rate’ $\lambda_{c,i}$. The state transition rate is expressed in ‘per time’ units, and may thus be interpreted as a frequency. This frequency, however, expresses the number of transitions out of the state per time spend in the state, and not per total time. The state transition rate thus only equals the state frequency for a component with just one state. For a component with two identical states, the state frequency will be half the state transition rate. The expected state duration can be directly calculated from the state transition rate as

$$
E(D_{c,i}) = \frac{1}{\lambda_{c,i}} \qquad (2.35)
$$

For the transition probability $P_c(i,j) = \Pr(X_{c,n_c+1} = j \mid X_{c,n_c} = i)$ it follows that

$$
\begin{aligned}
P_c(i,j) &= \Pr(D_{c,ij} = \min_{k=1}^{N_c}(D_{c,ik})) \\
&= \int_0^\infty \Pr(\min_{k \ne j}(D_{c,ik}) \ge \upsilon)\ \frac{1}{\lambda_{c,ij}}\ e^{-\upsilon/\lambda_{c,ij}}\ d\upsilon \\
&= \int_0^\infty \frac{1}{\lambda_{c,ij}}\ e^{-\upsilon/\lambda_{c,i}}\ d\upsilon \\
&= \frac{\lambda_{c,i}}{\lambda_{c,ij}} \qquad (2.36)
\end{aligned}
$$

Both the state duration distribution and the transition probabilities are thus independent of time and independent of the history of the system. By

(2.36), a constant transition probability matrix $P_c = [P_c(i,j)]$ is defined for the Markov model. The fact that the transition probabilities are constant means that the sequence of states in a history of the component is independent of the time spend in those states. This sequence of states, $(X_{c,n_c})_{n_c=0}^{\infty}$, is called the "embedded Markov chain". For Markov chains with stationary transition probabilities the so-called Markov-property holds, which can be written as

$$\begin{aligned} &\Pr(X_{c,n_c+m} = j \mid X_{c,0} = k, X_{c,1} = l, \cdots, X_{c,n_c} = i) \\ &\qquad = \Pr(X_{c,n_c+m} = j \mid X_{c,n_c} = i) = P_c^{(m)}(i,j) \end{aligned} \tag{2.37}$$

where $P_c^{(m)}(i,j)$ is the value on the i,j position in the m^{th} power of P_c. For all $P_c^{(m)}$, $\forall_i \sum_{j=1}^{N_c} P^{(m)}(i,j) = 1.0$.

The Markov property tells us that the probability to find the component in a certain state after a certain number of transitions only depends on the number of transitions and on the starting state.

The homogenous Markov model may be graphically depicted as shown in Fig. 2.5, which shows a Markov Model with three states. Because the state duration PDF and the transition probabilities can both be calculated from the transition rates, the only data needed to completely define a homogenous Markov model is the set of these transition rates λ_{ij}.
Both the transition probabilities and the conditional state durations in a homogenous Markov model are independent of the history of the system. This means that when the component is found to change to a certain state at a certain time, the probabilities for the next states and the distribution of the duration of the current state, are always known.

By calculating the state duration rates and the state transition probabilities from the transition rates, an alternative representation of the homogenous Markov model results. This representation is graphically depicted in Fig. 2.6. Both representations are analogous. The transition rates can be calculated from the alternative model data as

$$\lambda_{ij} = \Pr_c(i,j) \cdot \lambda_i \tag{2.38}$$

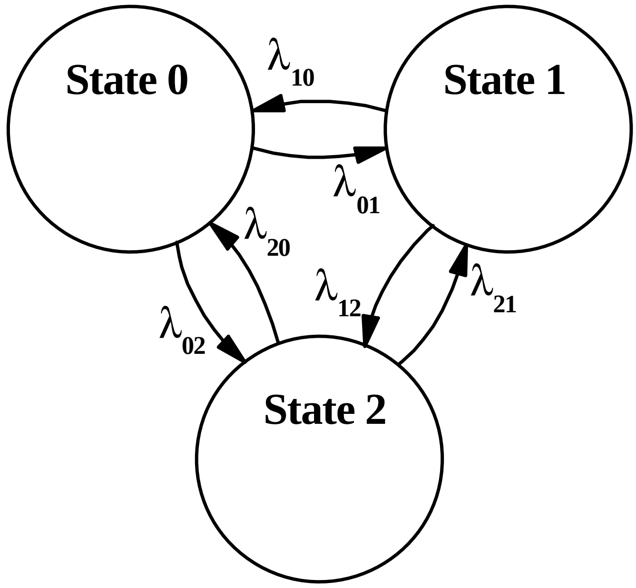

Figure 2.5: The homogenous Markov model

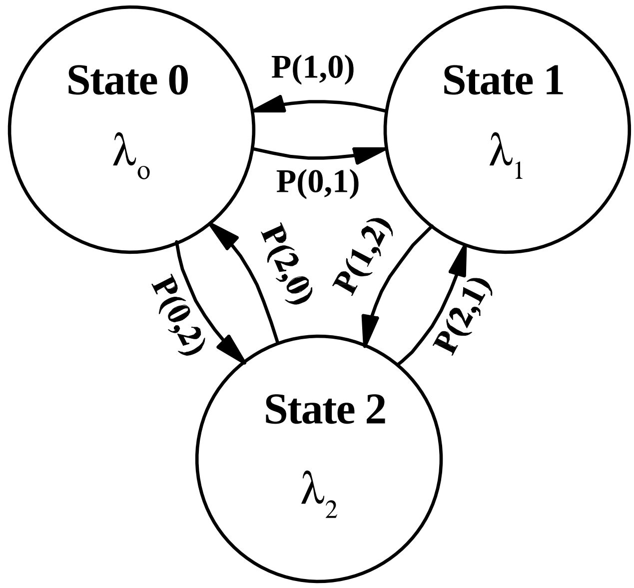

Figure 2.6: Alternative representation of the homogenous Markov Model

2.2.2 The Homogenous Markov System

A homogenous Markov system is a stochastic model of a power system for which all stochastic components are homogenous Markov components. The homogenous Markov system is nothing more than a combination of those components.

The homogenous Markov system is defined by

- the number of homogenous Markov components N
- the set of homogenous Markov components $\left((X_{c,n_c}, T_{c,n_c})_{n_c=0}^{\infty}\right)_{c=1}^{N}$
- the resulting stochastic system history $(S_{n_s}, T_{n_s})_{n_s=0}^{\infty}$, where
  - $S_{n_s} = (X_{1,n_s}, X_{2,n_s}, \cdots, X_{N,n_s})$ and $X_{c,n_s} = X_c(T_{n_s})$
  - $(T_{n_s})_{n_s=0}^{\infty} = \bigcup_{c=1}^{N} (T_{c,n_c})_{n=0}^{\infty}$

The system state $S(t)$ at any time t is thus the vector of component states $X_c(t)$ at that time. Because the number of possible states is limited for all components, the system state space is also limited. However, because the stochastic components are assumed to be stochastically independent, the number of possible system states is the product of the number of possible component states. For a moderate system of 100 components, each of which has two states, the size of the system state space is 2^{100} possible states.

The system changes state when at least one of its component changes its state. However, because all components are assumed to be stochastically independent, the probability of two of them changing state at the very same moment is zero:

$$\neg\exists_{a,b,n_a,n_b}(T_{a,n_a} = T_{b,n_b}) \tag{2.39}$$

For each system epoch T_{n_s}, there is thus exactly one component with the same epoch,

$$\forall_{n_s}\exists_c^1\exists_{n_c}(T_{n_s} = T_{c,n_c}) \tag{2.40}$$

The system thus changes state because one component changes state, and that component is therefore called the “causing component”. Each system state S_{n_s} has a single causing component. Two succeeding system states may have the same causing component.

For a system which changes to state S_{n_s} at time T_{n_s}, the remaining state duration for component c is defined as

$$D_c(n_s) = T_{c,n_{sc}+1} - T_{n_s} \tag{2.41}$$
$$n_{sc} = \sup\{n_c \in \mathbb{N}^+ \mid T_{c,n_c} \leq T_{n_s}\} \tag{2.42}$$

and the age of the component state as

$$A_c(n_s) = T_{n_s} - T_{c,n_{sc}} \tag{2.43}$$

These basic relations are illustrated by Fig. 2.7.

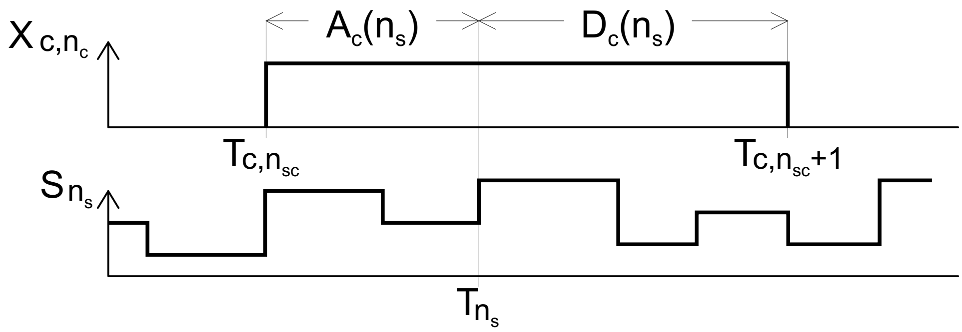

Figure 2.7: Component Age and Remaining Duration

The age of a component state is the time it has already spend in its current state at the start of a new system state, and the remaining duration is the time for which it will continue to stay in that state. The duration of the system state is the minimum of all remaining component state durations. According to (2.34), the state durations for the homogenous component are negative exponentially distributed. Because the conditional density function of a exponentially distributed duration for all inspection times equals the original distribution, the distribution of the remaining state duration is independent of the age of the state and equals the total component state duration distribution. Therefore,

$$F_{D_c(n_s)}(t) = 1 - \exp\left(-\frac{t}{\lambda_{n_{sc}}}\right) \tag{2.44}$$

where $\lambda_{n_{sc}}$ is the state transition rate for the state $X_{c,n_{sc}}$.

Because the system state duration is the minimum of the remaining component durations, the distribution of the system state duration can be calculated as:

$$\begin{aligned} F_{D_{n_s}}(t) &= \Pr(D_{n_s} < t) = \Pr(\min(D_c(n_s)) \leq t) \\ &= 1 - \exp\left(-\frac{t}{\lambda_{n_s}}\right) \end{aligned} \qquad (2.45)$$

$$\frac{1}{\lambda_{n_s}} = \sum_{c=1}^{N} \frac{1}{\lambda_{n_{sc}}} \qquad (2.46)$$

where $D_{n_s} = T_{n_s+1} - T_{n_s}$ is a stochastic system state duration.

From (2.45), it is clear that all system state durations are negative exponentially distributed. Because the minimum outcome for the remaining state durations for all components also determines the next system state, the same expressions as used in (2.36) can be used to show that the system state transition probabilities are independent of time. The conclusion is that the homogenous Markov system is itself again a homogenous Markov model. This is the most important quality of the homogenous Markov model.

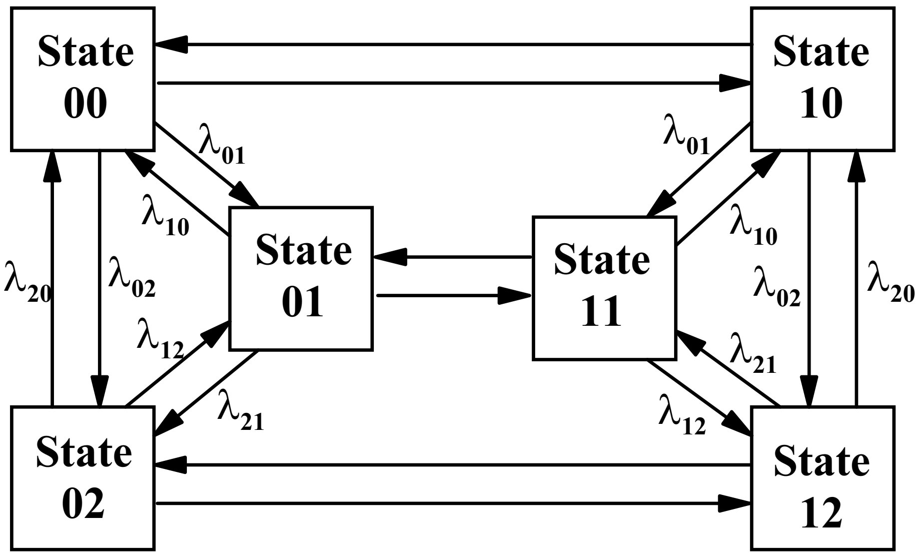

Figure 2.8: A homogenous Markov system

An example of a homogenous system is graphically depicted in Fig. 2.8.

This example system consists of two components, one with three and one with two states. The horizontal transitions in this picture are caused by a change of the two-state component, the other transitions are caused by the three-state component. From (2.44), it follows that the transition rates of the Markov system equal the corresponding component transition rates. In Fig. 2.8, the transition rates of the three state component are shown. If the three state component would be the one shown in Fig. 2.5, then the transition rates shown in Fig. 2.8 and Fig. 2.5 would be the same.

2.2.3 Interruption Costs Calculations

Stochastic models are used to create specific system states which are then analyzed. Such analysis may include load flow calculations and topological analysis, but may also include power system protection algorithms, power restoration procedures and optimization methods, during which the network is reconfigured and generation may be rescheduled.

The principle objective of the system state analysis is to express the ability of the system to meet the load demands in preliminary performance indicators. These preliminary indicators are then used, at the end of the system state creation and analysis phase, to calculate overall performance indicators. The interruption costs indicators express the expected costs per year due to load interruptions. For each load point, the LPEIC (load point expected interruption costs, $/a) indicator expresses the total expected costs per year due to the interruptions of the loads connected to the load point.

By drawing outcomes for the stochastic conditional state durations for each component, the duration of the current state and the number of the next state are determined. When the current state duration has passed, a transition to the next state is made and new outcomes can be drawn to determine the new duration and the following state again. In this way, a possible history for the component can be simulated in time. By performing a parallel simulation of all components in the system, the whole system can be simulated in time. This way of generating system states in a chronological order is called “Monte-Carlo simulation”.

Each time a new system state is reached during the Monte-Carlo simulation, it is analyzed for performance. Because the simulation of one single period of, or instance, 5 years, would not make it possible to derive exact overall performance indicators, it is necessary to simulate and analyze the same period a number of times. The LPEIC is then calculated as

$$LPEIC = \frac{\sum_{i=1}^{I} c(d_i)}{K \cdot \text{analyzed period}} \tag{2.47}$$

where $c(d_i)$ is a single outcome of the interruption cost function c, for the simulated duration d_i, I is the total number of interruptions of the considered load point during the whole Monte-Carlo analysis, and K is the number of times the analyzed period was simulated.

The Monte-Carlo simulation technique is very powerful because it allows for the simulation of about any possible event in the system and it does not require any restriction on the stochastic component models. Its big disadvantage, however, is its high computational demand. The technique of system state enumeration is therefore often used in stead.

In a system state enumeration, all relevant system states are created and analyzed one by one. In this case, no outcomes are drawn for the stochastic durations, but the probabilities and frequencies of the system states are calculated directly. The big advantage of the state enumeration is that each possible system state is analyzed only once, and exact results for the performance indicators can be calculated analytically.

For the reliability costs indicators, such as the LPEIC, the state enumeration methods, however, are problematic. In many textbooks and articles (e.g. [16],[43]), the following equation is used for calculating the LPEIC.

$$LPEIC = \sum_{i=1}^{M} f_i \cdot c(d_i) \tag{2.48}$$

where f_i is the frequency of the i^{th} system state that causes an interruption of the load point and M is the number of different system states which lead to such interruption. This equation, however, is principally wrong, as it assumes that all interruptions caused by the i^{th} system state are of duration d_i. This is an assumption that can not be justified, as the durations of repairs or maintenance are stochastic.

The use of (2.48) is often defended by the assumption that power to an interrupted load will be restored by network reconfiguration, and not by repair. Network reconfiguration can normally be modeled by using switching durations with a small standard deviation. Three arguments against the use of (2.48) in a state enumerated reliability assessment are formulated here.

1. It is often unknown if power can be restored to all interrupted loads by network reconfiguration alone, in all cases. One objective of a reliability assessment may be to find cases where such restoration methods fail.

2. The financial risk related to interruption costs cannot be assumed to be determined by the majority of cases in which the restoration procedures work as planned. Load interruption during unusual system conditions may lead to unexpected high restoration durations. Such may happen in the case of failing protection devices, stuck breakers, unavailable (backup) transmission lines, peak load situations, etc.

3. Wide area power systems, or systems in rural areas, may lack the needed network reconfiguration options or may ask for the modeling of switching times as stochastic quantities.

4. Statistical data shows a considerable spread in interruption duration ([21], [98], [70])

For a correct calculation of interruption costs or other reliability cost indicators, it is therefore necessary to assess the duration distribution of all interruptions during a state enumerated reliability assessment. If these distributions are known, the LPEIC should be calculated as

$$LPEIC = \sum_{i=1}^{M} f_i \cdot E_i(c(d)) \tag{2.49}$$

where $E_i(c(d))$ is the expected value of the interruption costs, given the duration PDF for the i^{th} system state and the load interruption cost function.

2.2.4 Device of Stages

The homogenous Markov model makes it possible to quickly calculate the distribution of the duration of any system state, because all components use negative exponential distributions exclusively. This distribution, however, has not been selected because it was a good model for the actually measured component state duration, but only because it simplified the calculations. The distribution of the Markov system state duration therefore cannot be used to calculate reliability worth indices.

The only alternative to using homogenous Markov models is using non-homogenous models. The problem with these models, however, is that it is normally very hard, if not impossible, to calculate a system state duration distribution. The Weibull-Markov model, which will be introduced in the next chapters, forms an exception to this rule.

For non-homogenous models, it is commonly tried to convert the model into a Markovian model, preferably a homogenous one. One of the possible ways for such conversion is the method of the device of stages. Because this method is considered a general solution for the case of having non-homogenous component models, it is introduced here. It will be shown, however, that this method is not a solution to the problem of assessing interruption costs.

The method of the device of stages represents each state which has a non-exponential duration distribution by a combination of ‘virtual’ states that are exponentially distributed. The series and/or parallel combinations and the transitions rates of those virtual states are chosen so as to make the duration distribution of the transitions through the group of virtual states as good an approximation as possible of the original non-exponential distribution. The representation of a single non-exponential state by a combination of exponential states is illustrated by Fig. 2.9, for a two-state component. The state “1” of this component is non-exponential and is therefore converted to a series-parallel combination of 5 exponentially distributed states. The transition rates for these 5 virtual states should now be chosen so that the distribution of the duration between entering state “a” or “b” and leaving state “e” should equal, or approximate, the duration distribution of state “1”.
The method of the device of stages can be used to calculate time depen-

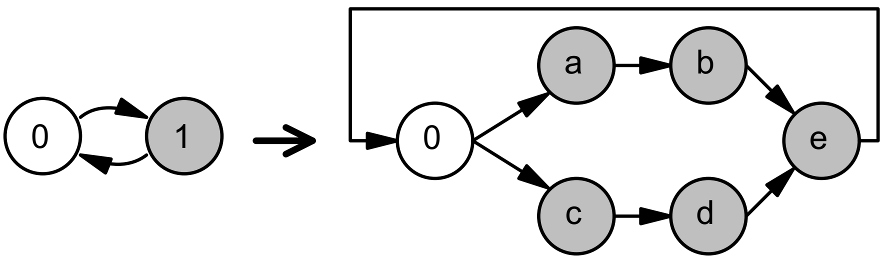

Figure 2.9: Example of a device of stages

dent state probabilities, and may thus be used for addressing ageing effects, effects of preventive maintenance, or other time-dependent behaviour of the component. However, it is not a solution to the problem of calculating system state duration distributions. This is best illustrated by an example. In Fig. 2.10, a system with two components is depicted, each of which has two states. Assumed is that the UP state of these components (“0” and “a”) are negative-exponentially distributed, but the DOWN states (“1” and “b”) are Weibull distributed. The system is supposed to function when at least one component is in the UP state. The question therefore is to find an expression of the distribution of the system down time, which is the distribution of the duration of system state “1b”, which is shown in grey in Fig. 2.10.

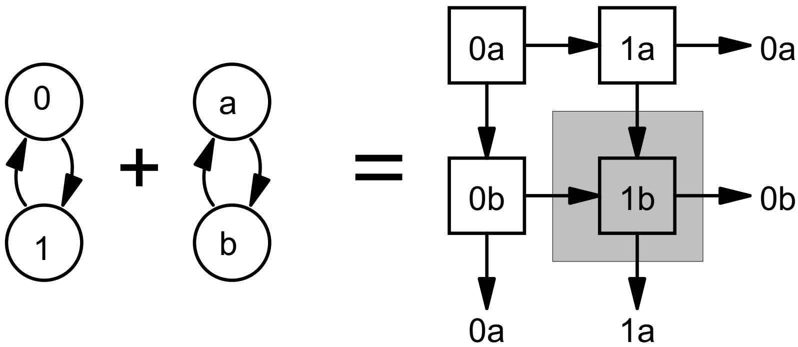

Figure 2.10: System with two two-state components

Even for this very simple system, the expression for the system down time distribution is not simple because it is the distribution of the minimum of a Weibull distribution with the remaining distribution of another Weibull distribution, for which the age is unknown.

In Fig. 2.11, the two down states of the two components have been converted to a series of two and three virtual states: ‘1’, ‘2’ and ‘b’, ‘c’, ‘d’. The resulting system now has 12 possible states, of which 6 are down states. The system has now become homogenous Markovian, and for each of the states in Fig. 2.11, the durations are negative exponentially distributed. However, in order to develop an expression of the duration distribution of the combination of the six system down states, we would have to account not only for the six states, but also for every way in which the system could change from an up state to a down state and back again, and for every possible route between entering and leaving the part of the state space with down states. For a combination of a number of virtual states in series, or a number of virtual states in parallel, such overall distributions can be found. For more complex virtual state spaces, such as the one in Fig. 2.11, no general way of calculating the overall duration distribution could be found.

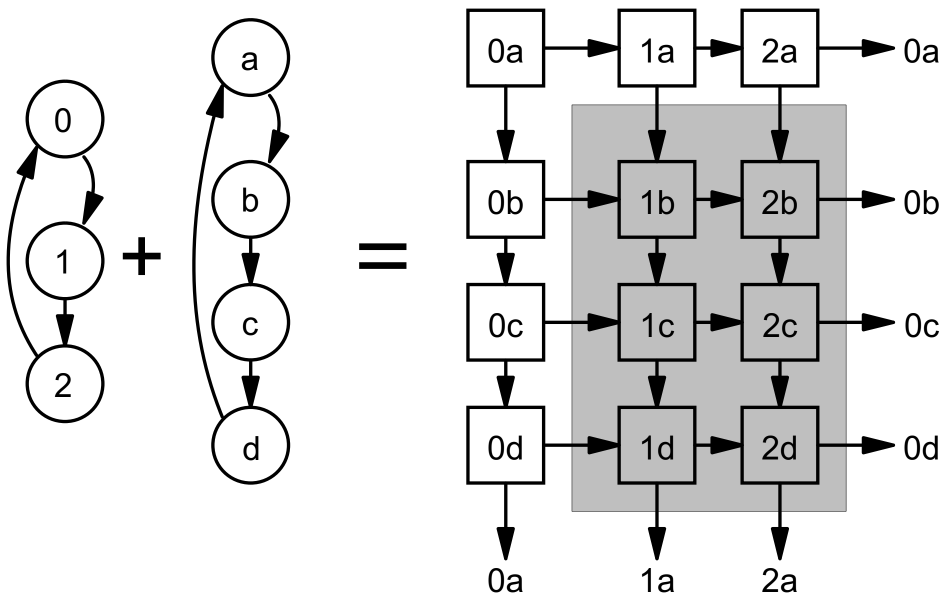

Figure 2.11: Converted components and resulting system

2.3 Weibull-Markov Models

The Weibull-Markov model is a non-homogenous Markov model. Although the mathematics involved in defining and using the Weibull-Markov model are somewhat more complex than those used for the homogenous Markov model, it will be shown that this model lends itself for all types of reliability calculations possible with the homogenous Markov model, and yet enables a correct analytical calculation of interruption costs.

The definition of the Weibull-Markov component starts by altering the homogenous component by using not a negative exponential distribution, but a Weibull distribution for the conditional state durations. The result is a stochastic component, defined by

- the set of possible states $\overline{x_c} = \{1, 2, \cdots, N_c\}$ where N_c is the number of possible states
- the stochastic history $(X_{c,n_c}, T_{c,n_c})_{n_c=0}^{\infty}$, where
  - $\forall_{n_c}(X_{c,n_c} \in \overline{x_c}, X_{c,n_c} \neq X_{c,n_c+1})$
  - $T_{c,0} = 0$ and $\forall_{n_c}(T_{c,n_c+1} > T_{c,n_c})$
- the stochastic process $X_c(t) = X_{c,n_c}$ for $T_{c,n_c} \leq t < T_{c,n_c+1}$
- the set of continuous probability distribution functions $F_c(t)$ for the conditional state durations $D_{c,ij}$

$$
\begin{aligned}
F_{c,ij}(t) &= \Pr(D_{c,ij} \leq t) \\
&= \Pr(X_{c,n_c} = i\ ,\ (T_{c,n+1} - T_{c,n_c}) \leq t \mid X_{c,n+1} = j) \\
&= 1 - \exp\left(-\left(\frac{t}{\eta_{c,ij}}\right)^{\beta_{c,ij}}\right)
\end{aligned}
$$

This model with Weibull distributions, which has independent duration distributions defined for each transition separately, equals the homogenous Markov model when all shape factors $\beta_{c,ij}$ equal one.

The above model is mathematically problematic in the sense that it it is even hard to derive a useful expression for the component state duration distribution. As with the homogenous model, the next state and the current state duration are determined by drawing outcomes for all conditional state durations and selecting the lowest one. The probability distribution for the duration $D_{c,i}$ of state $x_{c,i}$ is therefore the distribution of the minimum of the conditional durations:

$$\begin{aligned} F_{c,i}(t) &= \Pr(D_{c,i} \leq t) = \Pr(\min_{j=1}^{N_c}(D_{c,ij}) \leq t) \\ &= 1 - \prod_{j=1}^{N_c} \Pr(D_{c,ij} > t) = 1 - \prod_{j=1}^{N_c} \exp\left(-\left(\frac{t}{\eta_{c,ij}}\right)^{\beta_{c,ij}}\right) \\ &= 1 - \exp\left(\sum_{j=1}^{N_c} - \left(\frac{t}{\eta_{c,ij}}\right)^{\beta_{c,ij}}\right) \end{aligned} \tag{2.50}$$

Expression (2.50) can be simplified drastically by taking a same shape factor for all conditional state durations in a same state:

$$\beta_{c,ij} = \beta_{c,i} \tag{2.51}$$

For such "same shape" models,

$$F_{c,i}(t) = 1 - \exp\left(\left(-\frac{t}{\eta_{c,i}}\right)^{\beta_{c,i}}\right) \tag{2.52}$$

$$\left(\frac{1}{\eta_{c,i}}\right)^{\beta_{c,i}} = \sum_{i=1}^{N_c} \left(\frac{1}{\eta_{c,ij}}\right)^{\beta_{c,i}} \tag{2.53}$$

The state duration for a "same-shape" Weibull distributed component is thus again Weibull distributed with the scale factor given by (2.53). The stochastic component as defined above, together with (2.51) is called a **"Weibull-Markov"** component.

With the expression for the state duration distribution, the transition probability matrix for the Weibull-Markov component can be derived as

$$\begin{aligned} P_c(i,j) &= \Pr(D_{c,ij} = \min_{k=1}^{N_c}(D_{c,ik})) \\ &= \int_0^\infty \Pr(\min_{k\neq j}(D_{c,ik}) \geq \upsilon) \, \frac{\beta_{c,i}\upsilon^{\beta_{c,i}-1}}{\eta_{c,ij}^{\beta_{c,i}}} \, e^{-(\upsilon/\eta_{c,ij})^{\beta_{c,i}}} \, d\upsilon \\ &= \int_0^\infty \frac{\beta_{c,i}\upsilon^{\beta_{c,i}-1}}{\eta_{c,ij}^{\beta_{c,i}}} \, e^{-(\upsilon/\eta_{c,i})^{\beta_{c,i}}} \, d\upsilon \\ &= \frac{\eta_{c,i}}{\eta_{c,ij}} \end{aligned} \tag{2.54}$$

The transition probabilities for the Weibull-Markov component are thus independent of time and independent of the history of the component. The Weibull-Markov model is thus a semi-Markov model. With the stationary transition probabilities, it is clear that X_{c,n_c} is again an embedded Markov-chain for which

$$\begin{aligned}\Pr(X_{c,n_c+m} = j \mid X_{c,0} = k, X_{c,1} = l, \cdots, X_{c,n_c} = i) \\ = \Pr(X_{c,n_c+m} = j \mid X_{c,n_c} = i) = P_c^{(m)}(i,j)\end{aligned}$$

where $P_c^{(m)}(i,j)$ is the value on the i,j position in the m^{th} power of P_c. For all $P_c^{(m)}$, $\forall_i \sum_{j=1}^{N_c} P^{(m)}(i,j) = 1.0$.

Expressions (2.53) and (2.54) are equivalent to their homogenous counterparts (2.36) and (2.34). A graphical representation of the Weibull-Markov component is shown in Fig. 2.12.

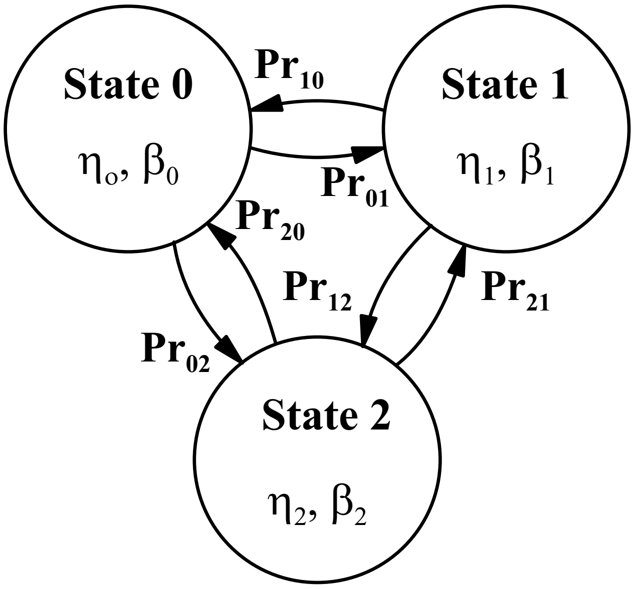

Figure 2.12: The Weibull-Markov Model

The definition and basic equations of the Weibull-Markov model look very much alike those of the homogenous Markov model, except for the additional distribution shape parameter β. The requirement that the new model should be compatible with the homogenous Markov model is met: all homogenous Markov models can be transformed into Weibull-Markov models without a need for new reliability data, by using $\beta = 1$ in all states. Existing measurements of component reliability can be used at any convenient time to calculate more realistic shape factors. It is possible to gradually substitute homogenous data by a more realistic Weibull-Markov data.

2.3.1 Component State Probability and Frequency

The fact that the Weibull-Markov model is fully compatible with the homogenous Markov model is very important. Even more important is the requirement that it should be possible to analyze Weibull-Markov components by fast and exact methods. These component analyses are needed to calculate component state probabilities and frequencies, which are needed to speed up a system analysis. The assessment of system reliability indices does not ask for ‘solving’ a large Weibull-Markov model. As will be shown, a system build from Weibull-Markov models is very hard to analyze as a whole with analytic methods. However, by ‘pre-processing’ the components, the system properties can be assessed by adding the contributions of the components.

The state probabilities and frequencies of a Weibull-Markov component can be calculated by first regarding the embedded Markov chain. For any Weibull-Markov model, leaving out the component index, the chain state probability vector is defined as

$$Q_n = [\Pr(X_n = 1), \Pr(X_n = 2), \cdots, \Pr(X_n = N_c)] \tag{2.55}$$

The chain state probability vector can be calculated from the initial state probability after n state transitions as

$$Q_n = P^{(n)} \cdot Q_0 \tag{2.56}$$

where

$$P = [P(i,j)] = \begin{bmatrix} P(1,1) & P(1,2) & P(1,3) & \cdots & P(1,N) \\ P(2,1) & P(2,2) & P(2,3) & \cdots & \\ \vdots & & & & \vdots \\ P(N,1) & \cdots & & & P(N,N) \end{bmatrix}$$

and $P(i,j) = \Pr(X_{n+1} = j \mid X_n = i)$

In [88] it is shown that for a Markov chain with stationary transition probabilities, the long term state probabilities can be found by solving

$$Q = P \cdot Q \tag{2.57}$$

where

$$Q = [Q(1), Q(2), \cdots, Q(N)] = \lim_{n\to\infty} Q_n \tag{2.58}$$

With the Weibull-Markov component, there are no self-transitions, and therefore $\forall_i (P(i,i) = 0)$. The long term Markov chain state probabilities can be solved by using

$$\forall_i \left(\sum_{j=1}^{N_c} P(i,j) = 1.0 \right) \tag{2.59}$$

$$\sum_{i=1}^{N_c} Q(i) = 1.0 \tag{2.60}$$

and by taking

$$\begin{aligned} A_{ij} &= \begin{cases} P(i,N)+1 & \text{if } i = j \\ P(i,n) - P(i,j) & \text{if } i \neq j \end{cases} \\ b_i &= P(i,N) \\ Q' &= [Q(1), Q(2), \cdots, Q(N-1)] \end{aligned}$$

A solution for Q' can then be found by solving

$$A \cdot Q' = b \tag{2.61}$$

and $Q(N)$ is found by using (2.60) again.

The state probabilities for the Weibull-Markov component can be calculated from the embedded state probabilities as

$$\Pr(i) = \frac{Q(i) * E(D_i)}{\sum_{i=1}^{N} Q(i) * E(D_i)} \tag{2.62}$$

where $E(D_i) = \eta_i \Gamma\left(1 + \frac{1}{\beta_i}\right)$ is the expected duration of state i.

With the known Weibull-Markov state probabilities $\mathrm{Pr}(i)$, the state frequencies $\mathrm{Fr}(i)$ can be calculated as

$$\mathrm{Fr}(i) = \frac{\mathrm{Pr}(i)}{E(D_i)}$$

2.4 The Weibull-Markov System

The Weibull-Markov system is defined in the same way as the homogenous Markov system; as a stochastic model of a power system for which all stochastic components are Weibull-Markov components.

The Weibull-Markov system is thus defined by

- the number of Weibull-Markov components N
- the set of Weibull-Markov components $\left((X_{c,n_c}, T_{c,n_c})_{n_c=0}^{\infty}\right)_{c=1}^{N}$
- the resulting stochastic system history $(S_{n_s}, T_{n_s})_{n_s=0}^{\infty}$, where
  - $S_{n_s} = (X_{1,n_s}, X_{2,n_s}, \cdots, X_{N,n_s})$ and $X_{c,n_s} = X_c(T_{n_s})$
  - $(T_{n_s})_{n_s=0}^{\infty} = \bigcup_{c=1}^{N} (T_{c,n_c})_{n=0}^{\infty}$

Except for the different component state duration distributions, the Weibull-Markov system behaves in the same way as the homo/-genous Markov system. Again, the probability of two components changing state at the very same moment is zero:

$$\neg\exists_{a,b,n_a,n_b}(T_{a,n_a} = T_{b,n_b}) \tag{2.63}$$

and thus

$$\forall_{n_s}\exists_c^1\exists_{n_c}(T_{n_s} = T_{c,n_c}) \tag{2.64}$$

Identical definitions for the remaining state duration and the age of the components c for all system states are used.

$$\begin{aligned} D_c(n_s) &= T_{c,n_{sc}+1} - T_{n_s} && (2.65)\\ n_{sc} &= \sup\{n_c \in \mathbb{N}^+ \mid T_{c,n_c} \leq T_{n_s}\} && (2.66)\\ A_c(n_s) &= T_{n_s} - T_{c,n_{sc}} && (2.67) \end{aligned}$$

However, where the distribution of the remaining duration of the state of a homogenous component equals the distribution of the complete duration, this is generally not the case for a Weibull-Markov component. For a Weibull-Markov component, the distribution of the remaining duration normally depends on the history of the system. As the system state duration is the minimum of the remaining component state durations, this dependency makes it very hard, if not impossible, to derive exact expressions for the system state duration distribution as a whole. Even more important is that also the probabilities for the following system state become history dependent, as they are too determined by the smallest outcome of the remaining component state durations.

From this it is clear that the Weibull-Markov system is not only a non-homo/-genous model, but even not a Markov model. It will however be shown that the Weibull-Markov system will becomes a semi-Markov system again when it is assumed to be stationary.

2.4.1 Weibull-Markov System State Probability and Frequency

The Weibull-Markov model can only be an alternative to the homogenous Markov model if it is possible to calculate state probabilities and frequencies analytically with comparable computational efforts.

The probability for a system state s is defined as the probability to find the system in that state at time t and is written as $\Pr(s,t) = \Pr(S(t) = s)$. For any moment in time:

$$\sum_{s=1}^{N_s} \Pr(S(t) = s) = 1 \qquad (2.68)$$

where N_s is the number of possible system states.

As all component are assumed to be statistically independent, the system state probability is the product of the component state probabilities:

$$\mathrm{Pr}(s,t) = \prod_{c=1}^{N} \mathrm{Pr}(X_c(t) = x_c) \tag{2.69}$$

The frequency of a system state s is defined as the density of the number of transitions into the system state per unit of time, for a certain moment in time.

$$\mathrm{Fr}(s,t) = lim_{\Delta t \to 0} \frac{E(\text{number of transitions to s in}[t, t+\Delta t])}{\Delta t} \tag{2.70}$$

If we write a system with only two components as S^2, the upper index being the number of components, then the frequency of the system state $s^2 = (x_1, x_2)$ at t is the frequency of $X_1(t) = x_1$ times the probability of $X_2(t) = x_2$, plus the frequency of $X_2(t) = x_2$ times the probability of $X_1(t) = x_1$.

$$\mathrm{Pr}(s^2,t) = \mathrm{Pr}(x_1,t) \cdot \mathrm{Pr}(x_2,t) \tag{2.71}$$

$$\mathrm{Fr}(s^2,t) = \mathrm{Fr}(x_1,t) \cdot \mathrm{Pr}(x_2,t) + \mathrm{Fr}(x_2,t) \cdot \mathrm{Pr}(x_1,t) \tag{2.72}$$

This can be repeated for a third component and the two-component system:

$$\mathrm{Pr}(s^3,t) = \mathrm{Pr}(s^2,t) \cdot \mathrm{Pr}(x_3,t) \tag{2.73}$$

$$\mathrm{Fr}(s^3,t) = \mathrm{Fr}(s^2,t) \cdot \mathrm{Pr}(x_3,t) + \mathrm{Fr}(x_3,t) \cdot \mathrm{Pr}(s^2,t) \tag{2.74}$$

and, by induction, for the whole system:

$$\mathrm{Pr}(s,t) = \mathrm{Pr}(s^{N-1},t) \cdot \mathrm{Pr}(x_N,t) \tag{2.75}$$

$$\mathrm{Fr}(s,t) = \mathrm{Fr}(s^{N-1},t) \cdot \mathrm{Pr}(x_N,t) + \mathrm{Fr}(x_N,t) \cdot \mathrm{Pr}(s^{N-1},t) \tag{2.76}$$

These recursive equations for probability and frequency are independent of the state duration distributions. The recursive equation for the system state frequency can be rewritten into

$$\mathrm{Fr}(s,t) = \sum_{c=1}^{N} \left(\mathrm{Fr}(x_c,t) \cdot \prod_{d=1, d\neq c}^{N} \mathrm{Pr}(x_d,t) \right) \tag{2.77}$$

or into

$$\mathrm{Fr}(s,t) = \mathrm{Pr}(s,t) \cdot \sum_{c=1}^{N} \frac{\mathrm{Fr}(x_c,t)}{\mathrm{Pr}(x_c,t)} \tag{2.78}$$

The equality (2.77) is known as the state frequency balance.

For the stationary system, for $t \to \infty$, the component state probabilities and frequencies become time independent and are written as $\Pr(x_c)$ and $\mathrm{Fr}(x_c)$. The system state probability and frequency will then also become time independent as

$$\Pr(s) = \lim_{t\to\infty} \Pr(s,t) = \prod_{c=1}^{N} \Pr(x_c) \tag{2.79}$$

$$\mathrm{Fr}(s) = \lim_{t\to\infty} \mathrm{Fr}(s(t)) = \Pr(s) \cdot \sum_{c=1}^{N} \frac{\mathrm{Fr}(x_c)}{\Pr(x_c)} \tag{2.80}$$

For the stationary system, the expected state duration, or ‘state expectancy’, in units per time per unit of time, equals the state probability. The expected system state duration is then calculated by dividing the system state expectancy by the system state frequency:

$$E(D_s) = \Pr(s)/\,\mathrm{Fr}(s) \tag{2.81}$$

In most reliability assessment calculations, the unit of frequency is taken as $1/a$, where a = annum = 8760 hours, and the expectancy is expressed in hours. In that case,

$$E(D_s) = \Pr(s) \cdot 8760/\,\mathrm{Fr}(s) \text{ hours} \tag{2.82}$$

2.4.2 Weibull-Markov System State Duration Distribution

For homogenous systems, the system state duration distribution is found without problems. Because the system too is a homogenous Markov model, the system state duration will be distributed according to a negative exponential distribution, with a duration rate which is the reciprocal sum of all corresponding component transition rates.

To find an expression for the state duration distribution of a Weibull-Markov system, it is important that we consider stationary systems only. This means that all component models in the system are stationary, and the history for each component beyond the last state change is irrelevant.

Suppose a stationary system with two components. If we would monitor such a system for a short period, a graph as depicted in Fig. 2.13 could be the result. It is now possible to regard the epochs of the one component as

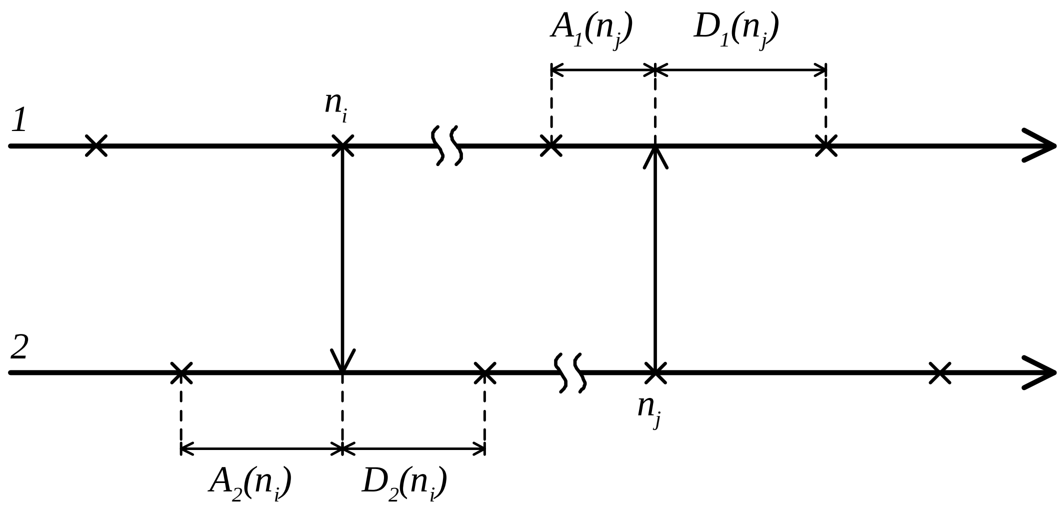

Figure 2.13: A monitored period for a two-component system

inspection times for the other, and vice versa, as is depicted in Fig. 2.13. As long as each system state is treated separately, and because both components are stationary and stochastically independent, each separate epoch of the one is a random inspection time for the other.

The derivation of the expression for the system state duration distribution starts with a well known result from renewal theory, according to which a component X_c, when found in state i at a random inspection time τ, has a remaining state duration distribution according to

$$\Pr(D_{c,i} - \tau < D \mid D_{c,i} > \tau) = \frac{1}{M_{c,i}} \int_0^D [1 - F_{c,i}(t)]dt \qquad (2.83)$$

where $M_{c,i}$ is the mean duration of $X_c = i$ and $F_{c,i}(t) = \Pr(D_{c,i} < t)$.

By using the random epochs of the one component as an inspection time for the other, the remaining state duration $D_1(n_j)$ and $D_2(n_i)$ in Fig. 2.13 will thus respect (2.83).

According to (2.83), the distribution of the remaining state duration is independent of the passed state duration, $D_{c,i} - \tau$, at the moment of inspection τ, for all types of distributions for the total state duration $F_{c,i}(t)$. From this, it follows that the probability of X_c, when found in state i at time τ, to not change state in the interval $\tau \leq t \leq \tau + D$ is

$$\Pr(X_c(\tau + D) = i \mid X_c(\tau) = i) = \frac{1}{M_{c,i}} \int_D^\infty [1 - F_{c,i}(t)]dt \tag{2.84}$$

Because a system will change as soon as one of its components changes, the probability of a system, consisting of N components, which is found in state s at time τ, with $T_{n_s} < \tau < T_{n_s+1}$, to not change state in the interval $\tau \leq t \leq \tau + D$ is

$$\Pr(S(\tau + D) = s \mid S(\tau) = s) = \prod_{c=1}^{N} \frac{1}{M_{c,n_s}} \int_D^\infty [1 - F_{c,n_s}(t)]dt \tag{2.85}$$

where M_{c,n_s} is the mean duration of X_{c,n_s} and $F_{c,n_s}(t) = \Pr(D_{c(n_s)} < t)$.

Expression (2.85) is an important first result, because it shows that it is possible to express the remaining system state duration distribution in terms of component state properties. However, we do not want to calculate the distribution of the remaining system state duration for arbitrary inspection times, but the distribution for the whole state duration. The inspection time in (2.85) then equals the moment at which the system state starts, which is the moment at which the causing component changes its state. The distribution of the 'remaining' state duration for that one component will thus equal the distribution of the total state duration. For all other components, we have to use (2.84), as they have already spend some time in their state at the moment the new system state starts.

If we suppose that component X_c is the causing component for system state S_{n_s}, then it follows that he probability of that system state, when its starts at epoch T_n because component X_c changed state at T_n, to last longer than

D, is

$$\Pr(S(T_{n_s}+D)=S_{n_s} \mid X_{c,n_s} \neq X_{c,n_s-1}) =$$
$$= [1-F_{c,n_s}(D)] \prod_{k\neq c}^{N} \left(\frac{1}{M_{k,n_s}} \int_D^{\infty} [1-F_{k,n_s}(t)]dt \right) \qquad (2.86)$$

All that is left now to do is to find an expression for the probability for each component to be the causing component for system state S_{n_s}. Equation (2.86) can then be weighted by that probability and the distribution of the system state duration can then be found by summing the weighted expressions for each component.

The probability that X_c is the causing component of system state S is the fraction of occurrences of that system state which happen due to a change of X_c. That fraction of occurrences is the relative system state frequency due to X_c, which is written as $\mathrm{Fr}_s(X_c)$. If $s=(x_{1,s},\cdots,x_{c,i},\cdots,x_{N,s})$, then s can be reached by any transition of component X_c from $x_{c,j\neq i}$ to $x_{c,i}$. Possible previous states of s are therefore all $(x_{1,s},\cdots,x_{c,j\neq i},\cdots,x_{N,s})$. This is depicted in Fig. 2.14.

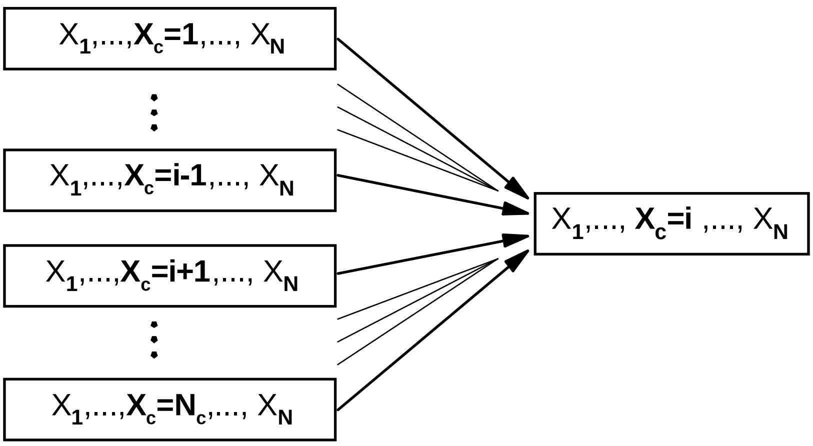

Figure 2.14: Transitions due to changing X_c

For each of the transitions shown in Fig. 2.14, the absolute transition frequency equals

$$\mathrm{Fr}(X_{c,n_s-1}=j \mid X_{c,n_s}=i) = \mathrm{Fr}_c(j,i) \cdot \prod_{k\neq c}^{N} \Pr(x_{k,s}) \qquad (2.87)$$

and the total transition frequency for any of these states to S thus equals

$$\begin{aligned}\mathrm{Fr}(X_{c,n_s-1} \neq i \mid X_{c,n_s} = i) &= \sum_{j\neq i}^{N_c} \mathrm{Fr}(X_{c,n_s-1} = j \mid X_{c,n_s} = i) \\ &= \mathrm{Fr}_{c,i} \prod_{k\neq c}^{N} \mathrm{Pr}(x_{k,s}) \qquad (2.88)\end{aligned}$$

For (2.88), it was used that $\mathrm{Fr}_{c,i} = \sum_{j\neq i}^{N_c} \mathrm{Fr}_c(j,i)$, which expresses the fact that a component state frequency is the sum of the absolute transition frequencies into that state.

With (2.88), the probability that X_c causes S is now expressed as

$$\begin{aligned}\mathrm{Pr}(X_{c,n_s-1} \neq X_{c,n_s}) &= \frac{\mathrm{Fr}(x_{c,s}) \prod_{k\neq c}^{N} \mathrm{Pr}(x_{k,s})}{\sum_{c=1}^{N} \left(\mathrm{Fr}(x_{c,s}) \prod_{k\neq c}^{N} \mathrm{Pr}(x_{k,s})\right)} \qquad (2.89) \\ &= \frac{\mathrm{Fr}(x_{c,s}) / \mathrm{Pr}(x_{c,s}))}{\sum_{c=1}^{N} \mathrm{Fr}(x_{c,s}) / \mathrm{Pr}(x_{c,s})} \qquad (2.90)\end{aligned}$$

which results in

$$\mathrm{Pr}(X_{c,n_s-1} \neq X_{c,n_s}) = \frac{1/M_{c,s}}{\sum_{c=1}^{N} 1/M_{c,s}} \qquad (2.91)$$

For a Weibull-Markov system, (2.86) takes the form of

$$\mathrm{Pr}(S(T_{n_s} + D) = S_{n_s} \mid X_{c,n_s} \neq X_{c,n_s-1})$$

$$= e^{-\left(\frac{D}{\eta_{c,s}}\right)^{\beta_{c,s}}} \prod_{k\neq c}^{N} \frac{1}{M_{k,s}} \int_{k}^{\infty} e^{-(\frac{t}{\eta_{k,s}})^{\beta_{k,s}}} dt \qquad (2.92)$$

$$= e^{-\left(\frac{D}{\eta_{c,s}}\right)^{\beta_{c,s}}} \prod_{k\neq c}^{C} \frac{\eta_{k,s}}{M_{k,s}\beta_{k,s}} \left[\Gamma\left(\frac{1}{\beta_{k,s}}\right) - \Gamma\left(\frac{1}{\beta_{k,s}}, (\frac{D}{\eta_{k,s}})^{\beta_{k,s}}\right)\right] \qquad (2.93)$$

where $\Gamma(x, y)$ denotes the incomplete gamma function for x from 0 to y.

The combination of (2.91) and (2.93) leads to the following expression for the probability for the duration of system state S to last longer than D:

$$\Pr(T_{n_s+1} - T_{n_s} > D) =$$
$$\sum_{c=1}^{N}(\Pr(X_{c,n_s-1} \neq X_{c,n_s}) \cdot \Pr(S(T_{n_s}+D) = S_{n_s} \mid X_{c,n_s} \neq X_{c,n_s-1}))$$
$$= \frac{\prod_{c=1}^{N} \gamma_{c,s}(D)/M_{c,s}}{\sum_{c=1}^{N} 1/M_{c,s}} \sum_{c=1}^{N} \frac{\exp\left(-(D/\eta_{c,s})^{\beta_{c,s}}\right)}{\gamma_{c,s}(D)} \quad (2.94)$$

where

$$\gamma_{c,i}(D) = \frac{\eta_{c,i}}{\beta_{c,i}}\left[\Gamma\left(\frac{1}{\beta_{c,i}}\right) - \Gamma\left(\frac{1}{\beta_{c,i}}, \left(\frac{D}{\eta_{c,i}}\right)^{\beta_{c,i}}\right)\right] \quad (2.95)$$

From equation 2.94 and 2.95, it is clear that the system state duration distribution is independent of the previous system states in the steady state case. The $\gamma_{c,i}(D)$ function only depends on the component state duration distribution parameters β and η. The $\gamma_{c,i}(D)$ values for each component can thus be calculated prior to the actual reliability assessment.

2.5 Basic Power System Components

This chapter shows a possible implementation of a Weibull-Markov model for modeling stochastic power system components. The proposed methods are introduced on the basis of a model for the synchronous generator.

2.5.1 Defining a Weibull-Markov Model

The Weibull-Markov model is determined by the following set of parameters.

- N, the number of states
- $\{\beta_i\}$, the set of form-factors, one for each state
- $\{\eta_i\}$, the set of characteristic times, one for each state
- P, the transition probability matrix

- Electrical parameters, which define the electrical model for the component for each state

It is however possible to enter the state duration parameters other than by β and η, as in many cases, these are unknown. Alternatively,

- $\{\mu_i\}$, the mean state durations

- $\{\sigma_i\}$, the state duration variances

is possible too.

Any two of the resulting possible state duration parameters, β_i, η_i, μ_i, σ_i, will determine the other two. The following conversion formulas can be used:

$$
\begin{aligned}
\mu(\beta,\eta) &= \eta\Gamma_1(\beta) \\
\mu(\beta,\sigma) &= \Gamma_1(\beta)\sqrt{\frac{\sigma^2}{\Gamma_2(\beta)}} \\
\sigma(\beta,\eta) &= \sqrt{\eta^2\Gamma_2(\beta)} \\
\sigma(\beta,\mu) &= \frac{\mu\sqrt{\Gamma_2(\beta)}}{\Gamma_1(\beta)} \\
\beta(\eta,\mu) &= \Gamma_1^{\text{inv}}\left(\frac{\mu}{\eta}\right) \\
\beta(\eta,\sigma) &= \Gamma_2^{\text{inv}}\left(\frac{\mu}{\eta}\right) \\
\beta(\mu,\sigma) &= \Gamma_3^{\text{inv}}\left(\frac{\mu^2}{\eta^2}\right) \\
\eta(\beta,\mu) &= \frac{\mu}{\Gamma_1(\beta)} \\
\eta(\beta,\sigma) &= \sqrt{\frac{\sigma^2}{\Gamma_2(\beta)}}
\end{aligned}
$$

where

$$\begin{aligned}\Gamma_1(\beta) &= \lambda\Gamma(1+\frac{1}{\beta})\\ \Gamma_2(\beta) &= \Gamma(1+\frac{2}{\beta}) - \Gamma(1+\frac{1}{\beta})^2\\ \Gamma_3(\beta) &= \frac{\Gamma_1(\beta)^2}{\Gamma_2(\beta)}\end{aligned}$$

The inversion of $\Gamma_1(\beta)$, $\Gamma_2(\beta)$ and $\Gamma_3(\beta)$ can be performed by newton methods. De values for $\mu(\eta,\sigma)$, $\sigma(\eta,\mu)$ and $\eta(\mu,\sigma)$ can be calculated by first calculating the corresponding β.

The above set of conversion formulas enables an easy definition of a Weibull-Markov model. However, it is also possible to leave all form-factors $\{\beta_i\}$ to their default values of one, and enter the Weibull-Markov model as a homogenous Markov model. Such would be needed if no other than homogenous model data is available. In stead of $\{\beta_i\}$, $\{\eta_i\}$ and P, the homogenous Markov model requires the input of:

- $\{\lambda_{ij}\}$, the matrix of state transition rates

This asks for the conversion of the state transition matrix to the transition probability matrix P. Such conversion can be done by using (2.36).

The back transformation from the state probability matrix to a state transition matrix can be performed by using the calculated state duration means and (2.38). This back transformation makes it possible to switch between 'homogenous input mode' and 'Weibull-Markov input mode', which may be used to check the validity of the model or to check the correct transformation of a homogenous model into a more realistic Weibull-Markov model.

2.5.2 A Generator Model

The basic stochastic generator model is fairly primitive, as it only defines states for the generator being available or not available. In many applications, such a two-state model is sufficient. However, there are several reasons to include more states to account for partial outages of the generator. Such a partial outage is a condition in which the generator is still connected to the net and is still producing power, but in which the maximum output is limited. Such a situation may occur when

- A run-of-the-river hydro turbine is reduced in capacity due to a low river level.
- A large, multi-machine thermal power plant is reduced in capacity due to the outage of one or more generator units.
- The available power is reduced due to the outage of a sub-component, such as a pulverizer, a fan, a feed water or cooling water pump, etc.

A state that models a partial outage is called a “derated state”. In order to account for derated states, which number may differ from generator to generator, it must be possible to freely define new states.

The implementation of the dialog which is used to define a stochastic generator is shown in Fig. 2.15. This example shows a generator with two derated states which has been defined as a Weibull-Markov model after which the dialog was changed to the homogenous Markov mode. It therefore shows the transition rate matrix and it would be possible to edit this matrix. This might be useful when stochastic data is available in several different formats, which are all to be translated into state transition rate models. The entry for the “dependent state” in the figure is needed to calculate the remaining transition probability, in order to make sure that the transition probabilities add up to one for each state.
The electrical models for the different states of the synchronous generator only differ in the maximum available active power and in the number of available machines. The latter is only of importance when the stochastic model is used for a multi-machine power plant. The dialog for defining the electrical model is shown in Fig. 2.16.
Because the Weibull-Markov model allows for defining the shape of the state duration distribution, the dialog has ‘graph’ page, which is shown in Fig. 2.17. This page shows the duration distribution and a set of state parameters, such as the frequency and the MTBS (mean time between states). The state parameters are calculated from the entered model and are shown for reference and checking purposes. The shown graph page shows the distribution and state parameters for the ‘out of service’ state. The state probability informs the user that this generator is out of service for more

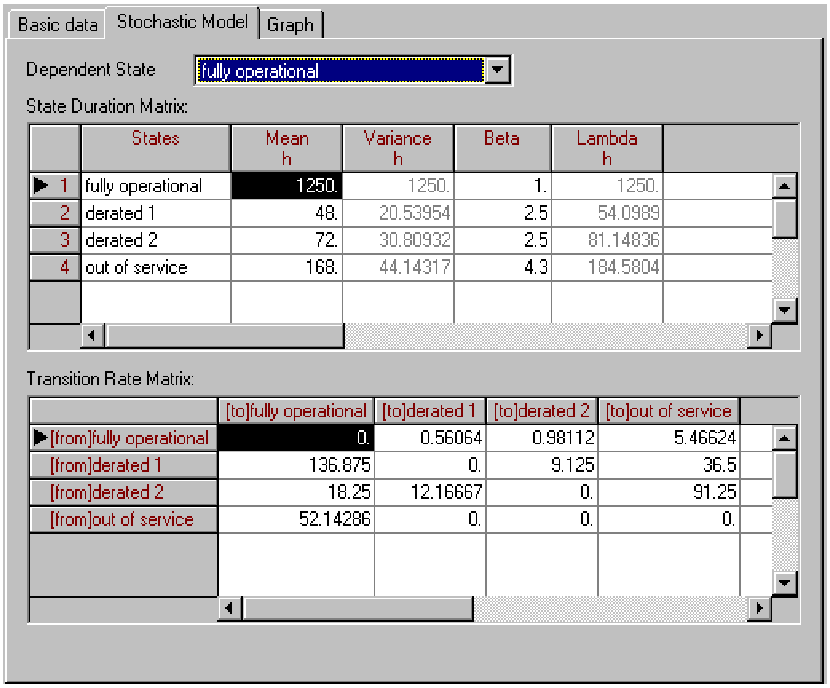

Figure 2.15: The stochastic model for a synchronous generator

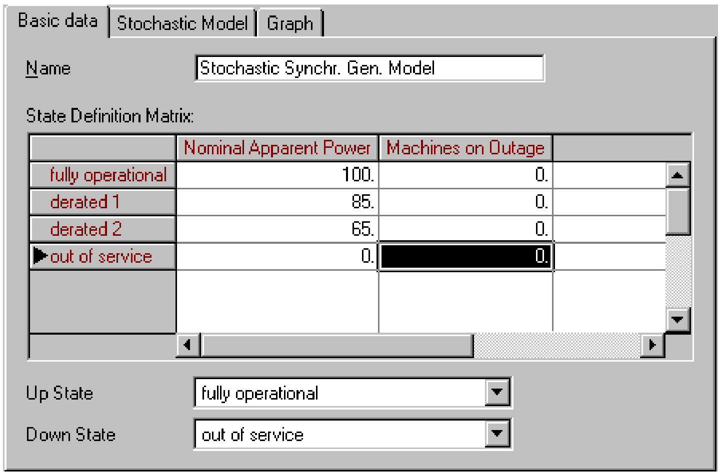

Figure 2.16: Electrical parameters for the stochastic generator

than 10% of time (942.1851 h/a), from which he may decide to check the stochastic data used.

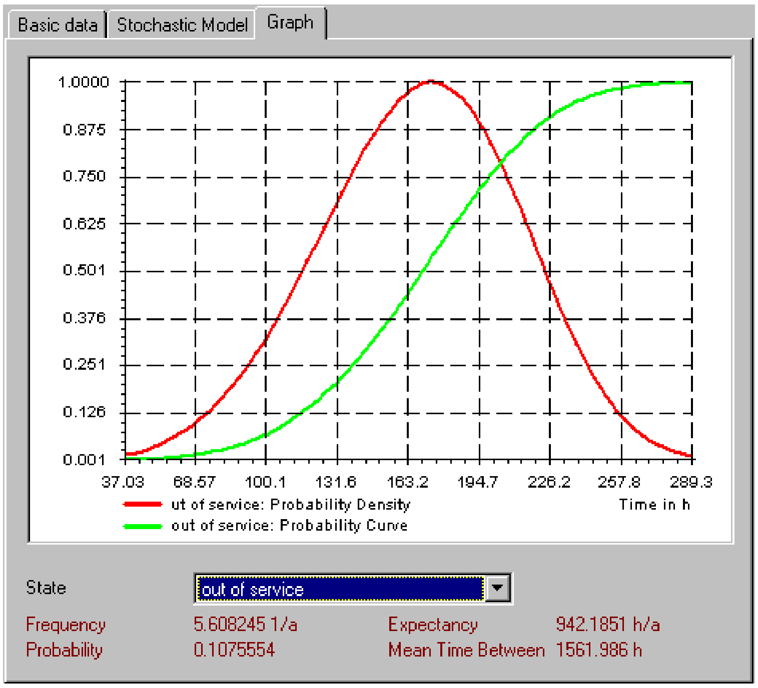

Figure 2.17: State duration graph and state parameters

Chapter 3

Applications / Examples

3.1 Comparison with Monte Carlo Simulation

The methods presented in this thesis for calculating the system state duration distribution by evaluating (2.94) and (2.95) were tested for a small system of 20 two-state components. As the test system was strictly ‘(n-1)’, two or more overlapping outages were taken to result in an interruption.

Two cases were analyzed: one with $\beta = 1.0$ for the repair duration distribution, which thus resulted in a homogenous Markov system, and one with $\beta = 3.0$, thus modeling a bell-shaped repair duration distribution. In both cases, the lifetime distribution was modeled with $\beta = 1$ and the mean repair duration was set to 5. All components used the same stochastic data in order to make hand-calculated checks possible.

Both a Monte Carlo simulation and a state enumeration method were used to find the interruption duration distribution for both cases. The four resulting distribution functions ($\mathrm{Pr}(D < d)$) are shown in Fig. 3.1.

This result shows a close resemblance between the Monte-Carlo result and the analytical result. The distribution for $\beta = 1$ is, as can be shown easily, close to the exponential distribution with a scale factor equal to half the mean repair duration. Differences between the two curves are not due to an error in the analytic (Weibull-Markov) model. In stead, they are caused by the inaccuracy in the Monte-Carlo simulation.

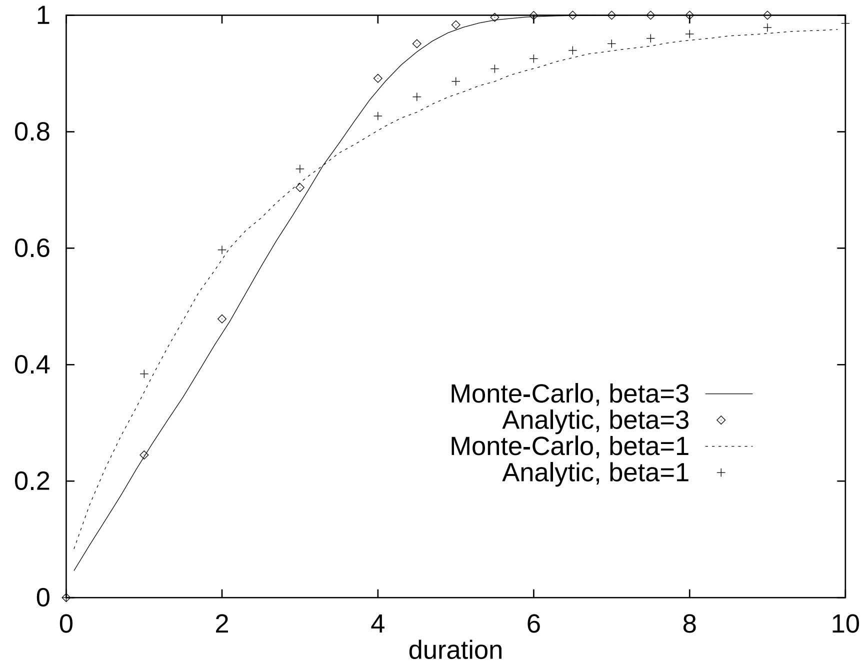

Figure 3.1: Calculated Duration Distribution Functions

3.2 Small System Example

To demonstrate both the use and the importance of the Weibull-Markov model, a small example system has been analyzed. A single line diagram of this example system is depicted in Fig. 3.2. The boxed data shows the line failure frequency and expected down time per year. This data was entered as per length values. The complete set of reliability data is listed in Table 3.1. Both the lifetime and the repair duration are negative exponentially distributed, as $\beta_0 = 1$ and $\beta_1 = 1$.

line	length km	failure rate 1/a$\cdot 100km$	β_0	repair duration h	β_1
L1	10	1	1	2	1
L2ab	30	2	1	2.5	1
L3	3	3	1	1.5	1

Table 3.1: Line data used

The load point data used is listed in Table 3.2. All three loads use the same

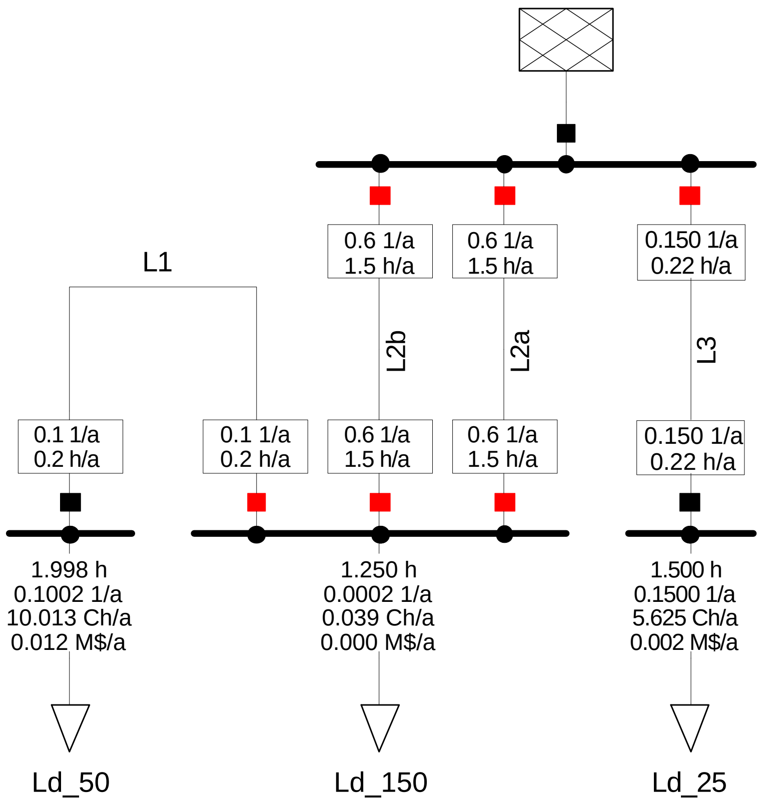

Figure 3.2: Example System with Homogenous Result

interruption costs function, which is depicted in Fig. 3.3. This function is a combination of measured data and a penalty for interruptions longer than 3 hours.

Name	Total Active Power MW	Connected Customers
Ld_25	0.64	25
Ld_50	1.26	50
Ld_150	2.90	150

Table 3.2: Load data used

No failure data was used other than for the line failures. The load point reliability indices were calculated for the homogenous system by using a state enumerated assessment ([16]) for all system states. A larger system would only be analyzed up to the third or fourth contingency level, as deeper

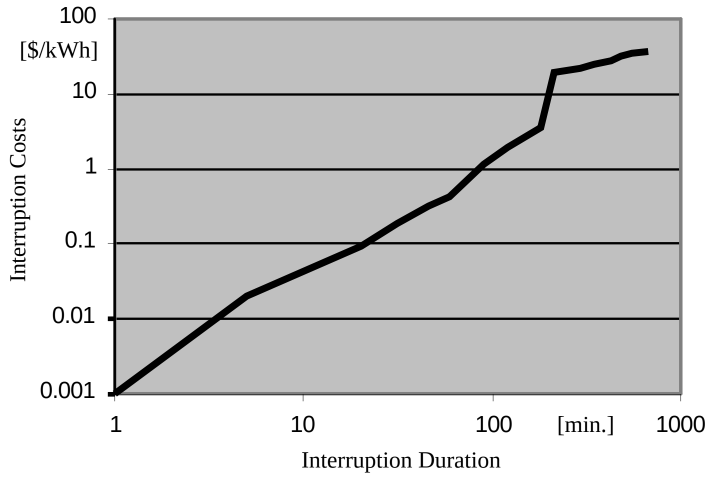

Figure 3.3: Load Point Interruption Cost Function

levels would enormously increase the computing time. The results for the homogenous system are depicted in Fig. 3.2. Each load point shows the

- avg. interruption duration in h
- avg. customer interruption frequency in 1/a
- load point interruption time in customers·h/a
- load point expected interruption costs in M$/a

The reliability assessment was repeated for a bell-shaped repair duration distribution by setting $\beta_1 = 3$ for all lines. The system-overall results for some reliability indices are listed in Table 3.3. The steady state non-monetary indices are not influenced by the change of the interruption distribution, as expected. The values for the expected interruption costs (EIC) and the interrupted energy assessment rate (IEAR), however, are about one-third of the values assessed by using homogenous models. Using interruption cost indices obtained by homogenous assessment methods may thus lead to unneeded investments or uneconomical contracts.

		Homogenous	Weibull	diff.
CAIFI	1/Ca	0.039	0.039	-
CAIDI	h	1.78	1.78	-
ENS	MWh/a	0.397	0.397	-
ACCI	kWh/Ca	1.76	1.76	-
EIC	k$/a	36.7	14.5	253%
IEAR	$/kWh	92.5	36.6	253%

Table 3.3: Results

3.3 Larger System Example

An example is shown here for a 11kV distribution system. This system is based on the test system described in [6]. Although hypothetical, it is based on a real UK system. It is chosen here because of its size and complexity.

The system has 13 busbars, 72 terminals and 56 load points. A reliability analysis was made for a single load level according to table 3.4. Each load point has a fixed number of customers,which is also listed in table 3.4. All loads have the same interruption cost function, which is listed in table 3.5. This cost function is a variant of the one given in [17], extended for longer interruption durations.

The failure data used for lines, cables, transformers and busbars is listed in tables 3.7 and 3.8. The cables encircled in the single line diagram have a common mode failure mode defined, which is given in table 3.6.

A reliability assessment was made for the whole system in the single line diagram. A maximum number of overlapping failures of two was used. The reliability assessment calculated overall system indices as well as individual load point indices for all load points in the system. Reliability worth indices were calculated too. These results are listed in table 3.9 and 3.10. The less frequently interrupted loads are not listed.

Name	MW	MVAr	Cust.	Name	MW	MVAr	Cust.
C2a	0.50	0.02	100	C11a	0.40	0.08	80
C2b	0.52	0.02	104	C11b	0.30	0.06	60
C2c	0.40	0.02	80	C11c	0.40	0.08	80
C2d	0.90	0.18	180	C12a	1.50	0.30	300
C3a	0.54	0.02	108	C12b	1.20	0.24	240
C3b	0.62	0.02	124	C12c	1.20	0.24	240
C3c	1.10	0.22	220	C13a	2.20	0.45	440
C4a	0.90	0.18	180	C13b	2.20	0.45	440
C4b	0.40	0.08	80	C14a	0.10	0.02	20
C4c	1.60	0.32	320	C14b	0.10	0.02	20
C5a	1.00	0.20	200	C14c	1.70	0.35	340
C5b	1.40	0.28	280	C15a	1.80	0.37	360
C5c	2.30	0.02	460	C15b	0.70	0.14	140
C6a	1.00	0.20	200	C15c	1.20	0.24	240
C6b	1.40	0.28	280	C16a	0.10	0.02	20
C6c	1.90	0.39	380	C16b	0.10	0.02	20
C7a	1.20	0.24	240	C16c	0.80	0.16	160
C7b	0.90	0.18	180	C17a	0.10	0.02	20
C7c	2.10	0.43	420	C17b	0.10	0.02	20
C8a	0.74	0.15	148	C17c	1.40	0.28	280
C8b	0.90	0.18	180	C20a	0.10	0.02	20
C9a	0.74	0.15	148	C20b	1.40	0.28	280
C9b	0.74	0.15	148	C21a	0.10	0.02	20
C9c	0.74	0.15	148	C21b	0.30	0.06	60
C9d	0.74	0.15	148	C21c	0.50	0.10	100
C10a	0.90	0.18	180	C22a	1.40	0.28	280
C10b	0.90	0.18	180	C22b	1.70	0.35	340
C10c	0.40	0.08	8	C23a	1.50	0.30	300

Table 3.4: Load Data

duration minutes	costs $/kWh	duration minutes	costs $/kWh
1	0.001	960	26.000
20	0.093	1440	46.000
60	0.428	2880	94.000
240	4.914	4320	186.000
480	15.690		

Table 3.5: Interruption Cost Function

Name	Failure Freq. 1/a	Failure Expect. h/a	Failure β	Repair Mean h	Repair β
Cables 0113a and 13b	0.01	3	1	300	3
Cables 02a and 02b	0.03	9	1	300	3
Lines 0109 and 10	0.05	15	1	300	3
Lines 0113a and 13b	0.05	15	1	300	3

Table 3.6: Common mode cable failure data

	BusBar Failure			Connection Failure		Repair	Repair
Name	Freq. 1/a	Exp. h/a	β	Freq. 1/a	Exp. h/a	Mean h	β
11kV Bar	0.002	0.028	1	0.005	0.07	14	3
33kV Bar	0.0025	0.06	1	0.015	0.36	24	3

Table 3.7: Busbar failure data

Name	Failure Frequency 1/a	Failure Expectancy h/a	Failure β	Repair Mean h	Repair β
11kV Cable	3.2	107.2	1	33.5	3
33kV Cable	3.2	107.2	1	33.5	3
33kV OHL	2.5	530.0	1	212	3
Transformers	0.02	6.86	1	343	3

Table 3.8: Branch failure data

State Enumeration Analysis System Summary		
Customer Avg. Interruption Freq. Index	CAIFI	0.08 1/Ca
Customer Avg. Interruption Duration Index	CAIDI	7.1 h
Avg. Service Unavailability Index	ASUI	0.000068
Expected Energy Not Supplied	ENS	31 MWh/a
Avg. Customer Curtailment Index	ACCI	0.003 MWh/Ca
Expected Interruption Cost	EIC	0.036 M$/a
Interrupted Energy Assessment Rate	IEAR	1.2 $/kWh

Table 3.9: Overall system reliability indices

State Enumeration Analysis Load Point Interruptions							
Name	LPIT Ch/a	LPIF C/a	AID h	LPENS MWh/a	LPEIC k$/a	ACIF 1/a	ACIT h/a
C13a	440.67	60.58	7.27	2.20	2.54	0.14	1.00
C23a	356.70	32.15	11.09	1.78	2.24	0.11	1.19
C14c	331.71	43.51	7.62	1.66	1.83	0.13	0.98
C10a	284.19	15.97	17.79	1.42	1.87	0.09	1.58
C5c	261.28	22.19	11.77	1.31	1.70	0.05	0.57
C15a	256.51	21.58	11.89	1.28	1.70	0.06	0.71
C7c	256.00	36.60	7.00	1.28	1.59	0.09	0.61
C22a	231.10	18.89	12.23	1.16	1.55	0.07	0.83
C6c	217.13	33.11	6.56	1.09	1.41	0.09	0.57
C4c	185.22	27.88	6.64	0.93	1.19	0.09	0.58
C10b	180.85	19.37	9.34	0.90	1.09	0.11	1.00
C9b	179.72	18.63	9.65	0.90	1.11	0.13	1.21
c9c	179.72	18.63	9.65	0.90	1.11	0.13	1.21
c9d	179.72	18.63	9.65	0.90	1.11	0.13	1.21
C12a	178.32	35.28	5.05	0.89	0.90	0.12	0.59
C7a	166.75	29.18	5.71	0.83	0.94	0.12	0.69
C3c	151.54	12.26	12.36	0.76	0.83	0.06	0.69
C13b	146.30	19.07	7.67	0.73	0.84	0.04	0.33
C17c	134.96	15.89	8.49	0.67	0.85	0.06	0.48
C15c	133.56	29.21	4.57	0.67	0.74	0.12	0.56

State Enumeration Analysis Load Point Interruptions, continued							
Name	LPIT Ch/a	LPIF C/a	AID h	LPENS MWh/a	LPEIC k$/a	ACIF 1/a	ACIT h/a
C20b	129.12	38.63	3.34	0.65	0.58	0.14	0.46
C10c	122.72	8.59	14.29	0.61	0.78	0.11	1.53
C9a	120.36	18.66	6.45	0.60	0.68	0.13	0.81
C2b	113.12	7.96	14.22	0.57	0.68	0.08	1.09
C5a	112.59	8.64	13.03	0.56	0.74	0.04	0.56
C6a	112.49	13.52	8.32	0.56	0.74	0.07	0.56
C2a	108.76	7.65	14.22	0.54	0.66	0.08	1.09
C4a	100.58	8.68	11.58	0.50	0.67	0.05	0.56
C8b	92.05	9.46	9.73	0.46	0.55	0.05	0.51
C3a	80.96	8.29	9.77	0.40	0.41	0.08	0.75
C16c	73.74	9.08	8.12	0.37	0.48	0.06	0.46
C12c	61.88	25.85	2.39	0.31	0.15	0.11	0.26
C12b	53.36	28.27	1.89	0.27	0.06	0.12	0.22
C2d	45.63	7.87	5.80	0.23	0.18	0.04	0.25
C15b	45.11	17.06	2.64	0.23	0.20	0.12	0.32
C11a	43.05	4.80	8.97	0.22	0.23	0.06	0.54
C21c	34.68	5.59	6.21	0.17	0.19	0.06	0.35
C3b	34.60	9.55	3.62	0.17	0.02	0.08	0.28
C7b	33.12	21.93	1.51	0.17	0.05	0.12	0.18
C11b	32.28	3.60	8.97	0.16	0.17	0.06	0.54
C8a	31.27	18.71	1.67	0.16	0.03	0.13	0.21
C11c	24.51	4.81	5.09	0.12	0.08	0.06	0.31
C22b	23.37	13.73	1.70	0.12	0.03	0.04	0.07
C21b	21.49	3.35	6.41	0.11	0.12	0.06	0.36
C16a	21.43	0.64	33.49	0.11	0.15	0.03	1.07
C17a	21.43	0.64	33.49	0.11	0.15	0.03	1.07
C2c	19.08	6.16	3.10	0.10	0.01	0.08	0.24
C5b	12.20	12.18	1.00	0.06	0.01	0.04	0.04
C21a	10.71	0.32	33.49	0.05	0.07	0.02	0.54
C20a	10.71	0.32	33.49	0.05	0.07	0.02	0.54

Table 3.10: Load point reliability indices

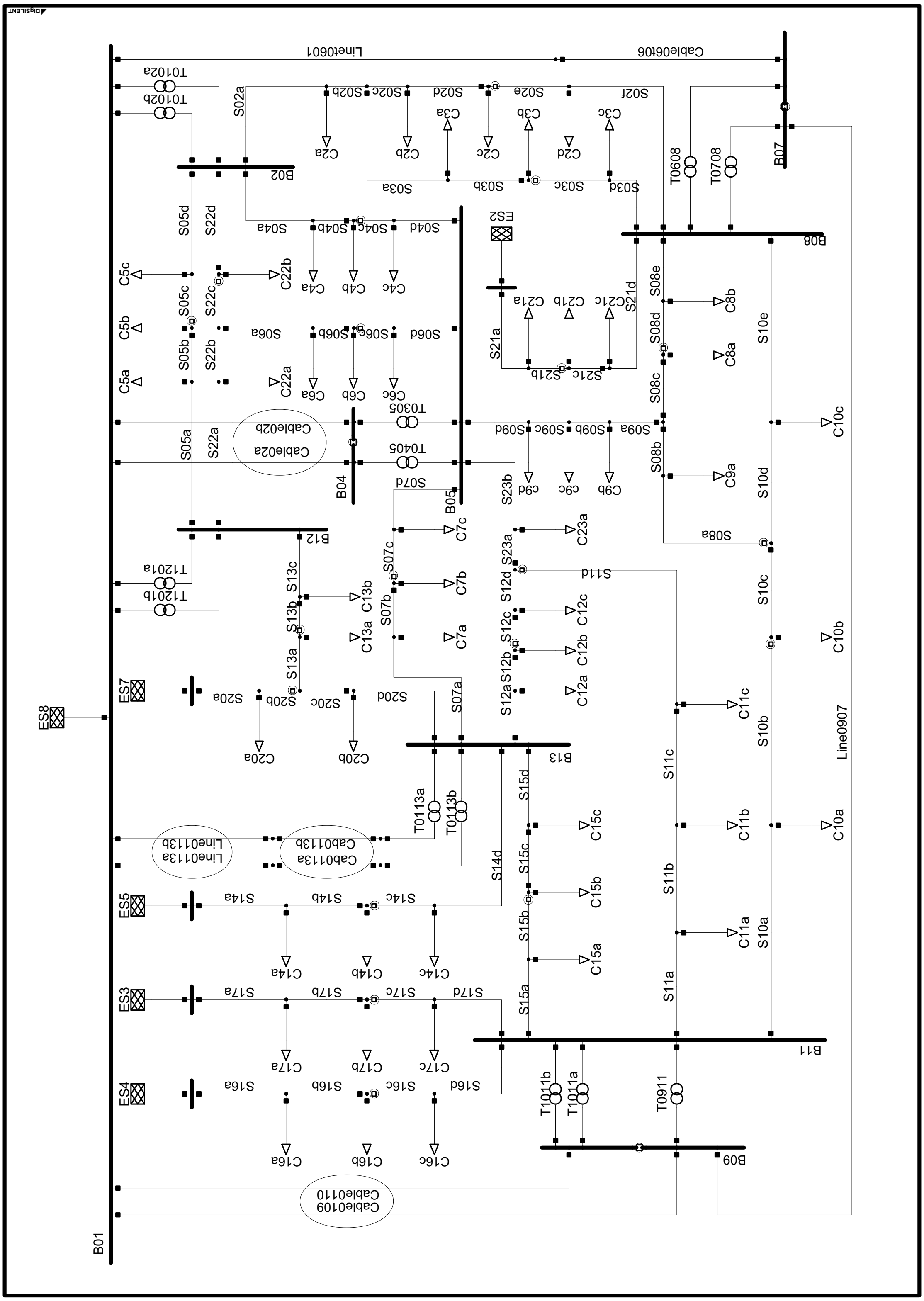

Figure 3.4: The distribution test system

Chapter 4

Conclusions

The Weibull-Markov stochastic model that is introduced in this Licenciate Thesis is 100% compatible with the widely used homogenous Markov model. It enables all necessary analytic calculations on the component level, including the calculation of the component state frequencies and probabilities.

A system that is build from Weibull-Markov components is not a Markov model but the stationary system again becomes a semi-Markov model. The calculation of system state probabilities and frequencies can also be performed when the system is not stationary, by combining the contributions of the components, and does not require the solving of large matrix equations.

An expression has been found for the system state duration distributions. This expressions allows for a fast assessment of interruption costs during a state enumerated reliability assessment. Such costs assessment are not possible with the homogenous Markov model.

At this point in research, it is believed that all reliability assessment methods that have been developed for the homogenous Markov model can also be used in combination with the Weibull-Markov model. The Weibull-Markov models is therefore a good alternative to the homogenous Markov model.

Appendix A

Glossary of Terms

Stochastic power system component. A power system component is said to be stochastic when it has a countable set of functional states in which it may reside, and when the time spend in a state, and the (number of the) next state are both stochastic quantities.

Stochastic component behavior. The stochastic behavior of a component is the description of the way in which the component changes from one state to another. This includes

- all the possible transitions from one state to another
- a description of the stochastic durations for each possible state
- the probabilities for the transitions to other states, given the current state.

Component A component is a typical part of the electric power system which is treated as one single object in the reliability analysis. Examples are a specific switch, transformer, line, generator, etc. Component may reside in different states, such as 'being available', 'being repaired', etc.

System A 'system' is short for an electrical power system. A system is build from components and changes state when one of its components changes state. The resulting 'system state' is the combination of all component states.

Event An event is a transition of a component between its states.

Outage An component outage is defined as the situation in which the component cannot be used, either because it has been taken out of service deliberately (‘planned’ outage), either because it has failed (‘unplanned’, or ‘forced’, outage).

Healthy State A healthy system state is a situation in which no unplanned outages are present.

Contingency A contingency is a system state in which one or more unplanned outages are present.

Coherent System Also called “consistent system”. A system in which additional outages will never improve the system performance.

Active Failure A failure of a component which activates the automatic protection system. Active failures are normally associated with short-circuits.

Adequacy The ability of the electrical power system to meet the load demands under various steady state system conditions.

Availability The fraction of time a component is able to operate as intended, either expressed as real fraction, or as hours per year.

Base State A system state in which no failures or outages are present.

Contingency A system state in which one or more unplanned outages are present.

Distribution Function The distribution function for the stochastic quantity X equals the cumulative density function CDF(x).

CDF(x) = the probability of X to take a value smaller than x.

Failure A failure is an undesirable event of a component.

Failure Effect Analysis (FEA) The electrical steady state and/or dynamic analysis of the system, possibly combined with switching, generator rescheduling, or other alleviation techniques, in order to assess the number of loads which would have to be curtailed.

Hazard Rate Function The function HRF(x), describing the probability of a stochastic quantity to be larger than x+dx, given the fact that it is larger than x, divided by dx. The hazard rate may thus describe the probability of a element to fail in the next period of time, given the fact that it is still functioning properly. The hazard rate is often used to describe ageing and wear out. A famous example is the so-called "bath-tub" function which describes the probability of a device to fail in the next period of time during wear-in, normal service time and wear-out.

Hidden Failure An event of a component which will prevent it from operating as intended the next time will be called upon.

Interruption An unplanned zero-voltage situation at one or more load points due to outages in the system.

Maintenance The planned removal of one or more primary components from the system.

(n-1) system A system for which all relevant components are redundant units.

(n-k) system A system for which the outage of any k components will never lead to an interruption in the base state.

Outage The removal of a primary component from the system.

Passive Failure A failure of a component which does not activate the automatic protection system.

Probability Density Function The function PDF(x), describing the probability of the stochastic quantity to take a value from an interval around x, divided by the length of that interval. The PDF(x) is the derivative of the CDF(x).

Redundant Unit A component which outage will never lead to an interruption in the base state, but for which at least one contingency state exists for which its additional outage will lead to an interruption.

Repair The restoration of the functionality of a component, either by replacing the component or by repairing it.

Scheduled Outage The planned removal of a primary component from the system, i.e. for preventive maintenance.

Security The ability of the system to meet the loads demands during and after a transient or dynamic disturbance of the system.

Spare Unit A reserve component, not connected to the system, which may be used as a replacement for a component on outage by switching or replacing.

Statistic Statistic calculation methods are used to analyze stochastic quantities. A simple example is the method for calculating a mean repair duration by dividing the total time spend repairing by the number of repairs performed.

Information obtained by using statistic tools on measured data can be used to build stochastic models of the observed equipment.

Stochastic A quantity is said to be stochastic when its value is random and thus unknown. The range of possible values, however, may be known as well as the likelihood of these possible values. The number of eyes thrown with a dice is random, the possible outcomes are {1,2,3,4,5,6} and the likelihood is $\frac{1}{6}$ for each outcome. For a continuous range of possible outcomes, the likelihood is a continuous function, known as the Probability Density Function or “PDF”.

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