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Frequency Dependence of Transformer Losses

Master of Science Thesis in the Programme Electric Power Engineering

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Abstract

In this thesis, theory concerning skin effect, eddy current and magnetic hysteresis in transformers has been investigated. An electric equivalent model of a transformer based on this theory is proposed. The validity of this theory and the usefulness of the model have been tested through laboratory measurements. In these measurements three scenarios where studied: a no load test of a transformer with varying frequency, a transformer loaded with a diode rectifier and last, the transformer was loaded with a pulse generating load. The model has been used to perform a prediction of the energy savings available through the use of amorphous transformer cores, and active filters. Finally a comparison of the two most widespread down rating standards for transformers has been made.

Analytical calculations suggest that the core impedance of a transformer increases as the square root of the frequency. A no load test with varying frequency supports this. The analysis also shows that at high frequencies the resistance of a copper winding increases as the square root of the frequency. The proposed model proved more accurate at higher frequencies compared to lower frequencies. A comparison of the European down rating standard with the standard proposed in this report shows that the two standards behaves similarly for lower frequencies, but that the European standard is stricter for the higher frequencies. The model suggests that the use of amorphous steel in the transformer core decreases the core losses by 48% using assumptions stated in the report; the model also shows that the use of amorphous steel decreases the losses that are generated by load harmonics by 8%.

Content

List of symbols

h	Harmonic order
h_{max}	Highest significant harmonic order
I_h	rms current at harmonic 'h'
I	rms load current
I_1	rms fundamental current under rated frequency and amplitude
P_{EC}	Winding eddy current loss
P_{OSL}	Other stray loss
P_f	Eddy current loss at fundamental frequency
e	The eddy current loss at the fundamental frequency divided by the loss generated by a DC current of an amplitude corresponding to the RMS value of the sinusoidal current
q	Exponential constant dependant on type of winding
$\mathfrak{I}_0$	Bessel function of first type and the order of zero
K_0	Modified Bessel function of second type and the order of zero
P_{EC-R}	Winding eddy current loss under rated conditions
P_{OSL-R}	Stray loss under rated conditions

1. Aims and Objectives

Many of the transformers in our network were designed in a time when most loads were passive. Consequently they were designed to work with mostly sinusoidal currents and voltages. Modern equipment on the other hand often uses semiconductors to adjust load voltages, as well as to compensate for supply voltage variations. These semiconductors might use a switching frequency that is more than thousand times as high as the network fundamental frequency. As the use of higher technology spreads in the society, demands on the power delivering network changes. In the future, rectifiers and converters might constitute the main part of the loads in a network. However due to the strong amplitude dependency of the power losses, the lower harmonics are still the most important.

Much effort has been made to increase the understanding of the effects of harmonic currents on transformers [3], [4] and [5]. Generally the behavior of the magnetic flux inside the transformer core is hard to foresee, since the complexity of the core geometry makes a purely analytical model impossible.

The objective of this work is to use mathematical models to get a picture of how the harmonics of the load affects the losses in the transformer, split into winding losses and core losses. Moreover, to compare these findings to contemporary methods of loss estimation.

1.1 Limitations

During data analysis it proved impossible to get an accurate enough measurement of the phase angle. Therefore the apparent power could not be divided into reactive power and real power when the measurements where too complicated to allow a visual control of the correctness of the values. Instead the phase angle derived from the theoretical model, which is presented in this work, was used to determine the amount of real and reactive power in the apparent power.

The amplifier which was used to inject harmonics into the transformer from the load side does in itself introduce some unintended distortion since it was not able to produce a perfect sine wave under the given conditions. The effect of this unintended distortion was not included in the calculations.

The lab transformer used as a test object has no documentation. This means that the exact thickness of the laminations in the core is unknown due to the difficulty of measuring the thickness of the insulation between the laminations. The material in the core was also unknown which means that the relative permeability and conductivity had to be estimated. Although the transforming ratio was known, the exact number of winding was not known. Therefore this had to be estimated.

2. Theory

In this part the physical grounds necessary to understand the losses in a transformer will be presented. And a model for the power losses of the transformer will be suggested. This will be used as a reference for comparison of the real-world data and the theory.

2.1 Magnetic and electric fundament

2.1.1 Magnetic material

A magnetic material has the property that the arbitrarily oriented atomic currents can align to an external magnetic field. The result is that magnetic fields in the vicinity of a core will concentrate its flow through the core. This effect is used to control the magnetic fields around the coils of transformers

2.1.2 Electromagnetism

Amperes law states that the line integral of the magnetic field around any closed path is proportional to the current which is bounded by the closed path,

$$\oint H \cdot ds = I \qquad (1)$$

The relationship of the magnetic flux and the field is often modeled as being linear as

$$H = \frac{B}{\mu} \qquad (2)$$

However, their relationship is in reality more complicated,

$$B = f(H) \qquad (3)$$

Figure 2.1 shows a loosely wound coil. Note that the field lines are more closely spaced and almost parallel the closer one comes to the center of the coil, indicating that the field in the center is strong and almost homogenous.

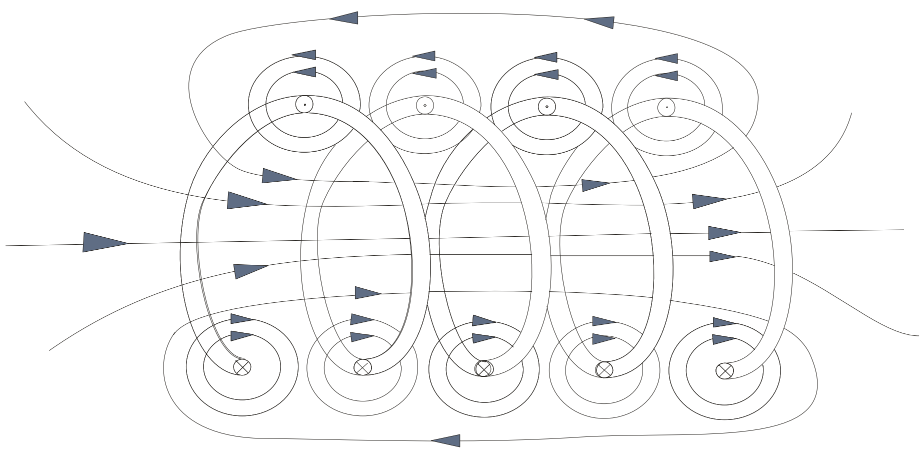

Figure 2.1 Magnetic field lines around an energized coil

2.1.3 Induction

Faradays law of induction states that the voltage in a coil is proportional to the change of the external magnetic field

$$\varepsilon = -N \frac{d\Phi}{dt} \tag{4}$$

$$\Phi = \int_S B \cdot dS \tag{5}$$

Lenz's law follows out of the law of induction and states that the induced current in a coil creates a magnetic field which counteracts the external field.

It can be useful to make an analogy between magnetic flux and electric current. In the same way as the material in a conductor offers electrical resistance to the current, the magnetic conductor offers resistance to the flux. This resistance is called reluctance.

$$\Re = \frac{l}{\mu \cdot A} \tag{6}$$

The inductance of a coil can be calculated through

$$L = \frac{N^2}{\Re} \tag{7}$$

2.1.4 Hysteresis

For each cycle of the external magnetic field the magnetization of the core material must be overcome in order to change the flux in the material. The dependency between the magnetic field intensity and the magnetic flux density is shown in Figure 2.2.The magnetization left behind in a magnetic material after the external magnetic field is removed is called remanence. The area encircled by the B-H curve represents the resulting energy loss when a material is subject to a changing magnetic field.

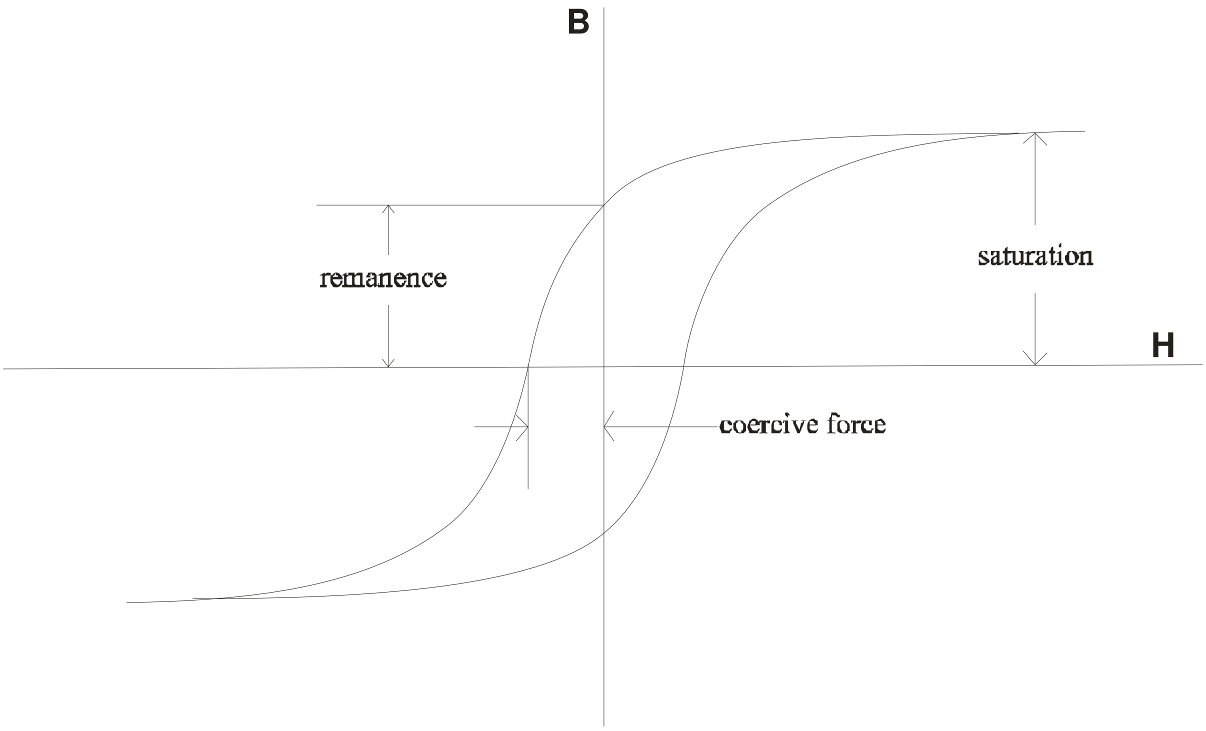

Figure 2.2 Typical B-H curve for magnetic material

2.1.5 Skin effect

The current density in a conductor will increase closer to the perimeter of the conductor at higher frequencies. The skin effect results in a higher resistance in the conductor than the dc-resistance. Also the magnetic flux in conducting magnetic material is subject to a skin effect at higher frequencies. The skin effect can only be active if the skin depth is small compared to the thickness of the conductor [7].

$$\delta = \sqrt{\frac{2}{\mu \varepsilon \omega}} \qquad (8)$$

Generally the skin dept is much greater for the copper windings compared to that of the iron core, due to the low relative permeability of the copper. For a round, straight conductor with the radius r the resistance due to skin effect is approximated as follows

$$R = \frac{l}{2r\pi\delta\sigma} \qquad (9)$$

Which is accurate if r ≫ δ.

2.1.6 Eddy currents

Eddy currents are small circulating flows of electrons that are generated when a conducting magnetic material is subjected to a varying magnetic field. Lenz's law shows that the circulating currents will generate a magnetic field that counteracts the external field. Eddy currents contribute to heat losses in core material.

2.1.7 The solenoid

From the definition of inductance, the flux can be found

$$N\Phi = L \cdot I \Rightarrow \Phi = \frac{L \cdot I}{N} \tag{10}$$

Using (10) for the approximation of the magnetic flux and combining it with the formula for inductance in a long straight solenoid of length l, the magnetic field intensity is as follows

$$H_0 = \frac{N \cdot I_{RMS}\sqrt{2}}{l} \qquad \text{For sinusoidal current} \tag{11}$$

2.2 Theoretical model

The losses in the transformer are often represented by an equivalent circuit consisting of resistors and inductors. In order to fully represent the transformer, the resistors and inductors must take account of all the different physical phenomena inside the core and the windings. In addition to purely resistive losses, hysteresis, and eddy currents are also present. Therefore the parameters of the transformer model must vary relative to the applied frequency.

2.2.1 Current distribution in a conductor wound around a cylindrical core

Analysis is performed on one turn. That turn is approximated with a cylindrical shell with inner diameter a, and outer diameter b = a + d, and with the length Δl, as shown in Figure 2.3 and Figure 2.4. The current distribution gives a magnetic field intensity $H = H_0$ for $r \leq a$, and $H = 0$ for $r > a$.

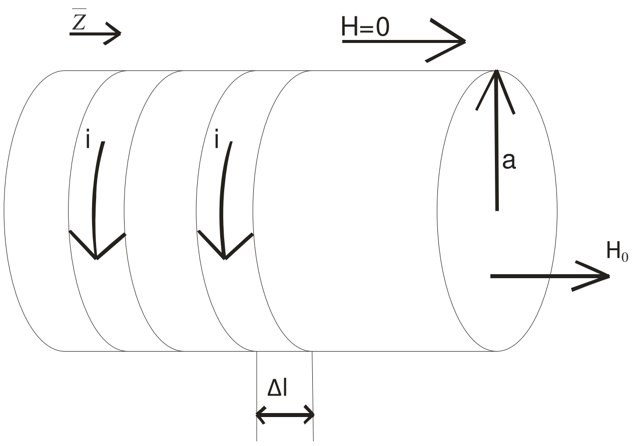

Figure 2.3 geometry for analyze of copper losses

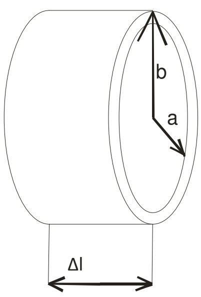

Figure 2.4 geometry for analyze of copper losses

Maxwell's equation leads to the following differential equation for the complex magnetic field intensity.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \overline{H}_z}{\partial r}\right) - j\omega\sigma\mu_0\overline{H}_z = 0 \tag{12}$$

By solving eq. (12) an expression for the current density as a function of the radius can be found.

Because of the symmetry of the cylinder the magnetic field can be said to consist solely of a coaxial component

$$\overline{H}_z = H_z(r) \tag{13}$$

Proposing the following solution

$$\overline{H}_z = A_1\Im_0(k_1 r) + B_1\mathrm{K}_0(k_2 r) \tag{14}$$

Where $\Im_0$ and K_0 are Bessel functions of order 0, and A_1 and B_1 are integration constants. The variables k_1 and k_2 are given as follows

$$k_1 = \sqrt{2}j^{3/2}\big/\delta \qquad\qquad k_2 = \sqrt{2}j^{1/2}\big/\delta \tag{15}$$

By setting up the boundary values for (14), as follows:

$H = H_0$ for r = a, and H = 0 for r = b. The expressions for the coefficients A_1 and B_1 can be found.

$$A_1 = H_0\frac{\mathrm{K}_0(k_2 b)}{\Im_0(k_1 a)\cdot\mathrm{K}_0(k_2 b) - \Im_0(k_1 b)\cdot\mathrm{K}_0(k_2 a)} \tag{16}$$

$$B_1 = -H_0\frac{\Im_0(k_1 b)}{\Im_0(k_1 a)\cdot\mathrm{K}_0(k_2 b) - \Im_0(k_1 b)\cdot\mathrm{K}_0(k_2 a)} \tag{17}$$

Current density can be found by use of Maxwell's equation

$$\nabla\times\overline{H} = J \tag{18}$$

$$J_\varphi = -\frac{\partial}{\partial r}\overline{H}_z \tag{19}$$

By combining the formulas (14), (16), (17) and (19) the current density can be expressed as

$$\overline{J}_{\varphi} = A_1 k_1 \Im_1(k_1 r) + B_1 k_2 \mathrm{K}_1(k_2 r)$$

(20)

Where $\Im_1$ and K_1 are Bessel functions of order 1

By plotting eq. (20) the effect of high frequency harmonics on current distribution in a conductor that is wound into a coil can be seen, shown in Figure 2.5 and Figure 2.6. The plot indicates that the current density increases dramatically in the side of the conductor that is closer to the core, and decreases on the outer side. The uneven distribution of current in the conductor will manifest itself as an increase in the total resistance of the winding. Thus two currents of the same amplitude but different frequency passing through the transformer winding will experience different resistance and therefore generate different levels of loss. The heat developed in the conductor due to the current will be highest closer to the core due to the higher current density there.

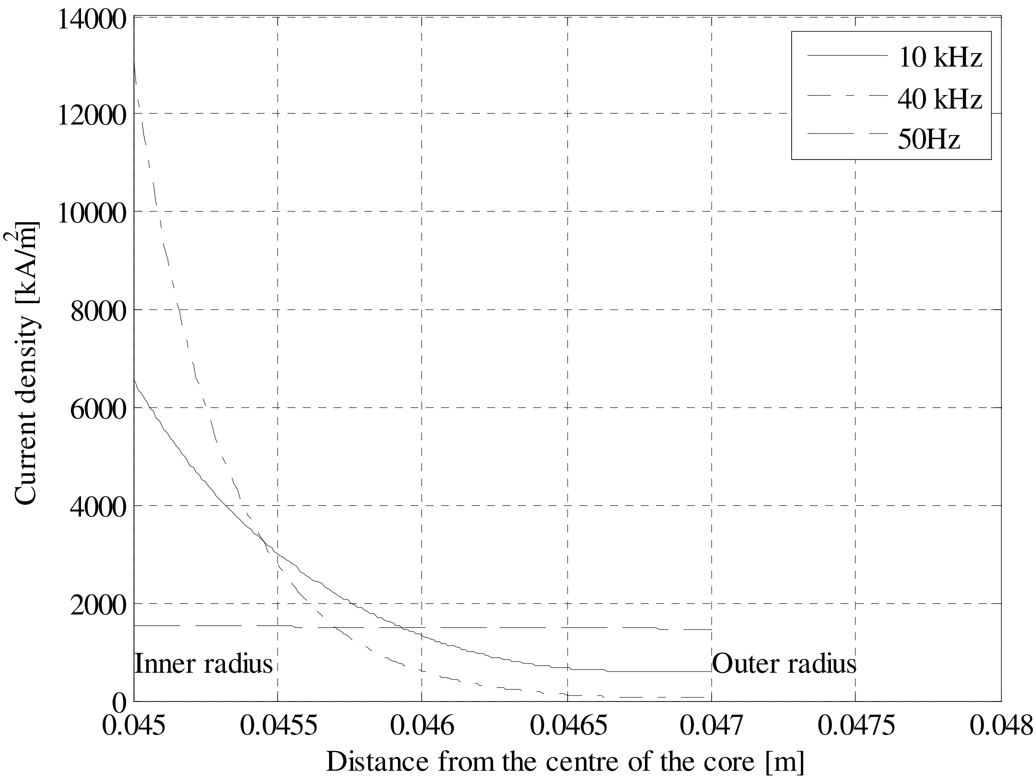

Figure 2.5 Current distribution at three frequencies and 9Amp inside one turn of a coil around circular core

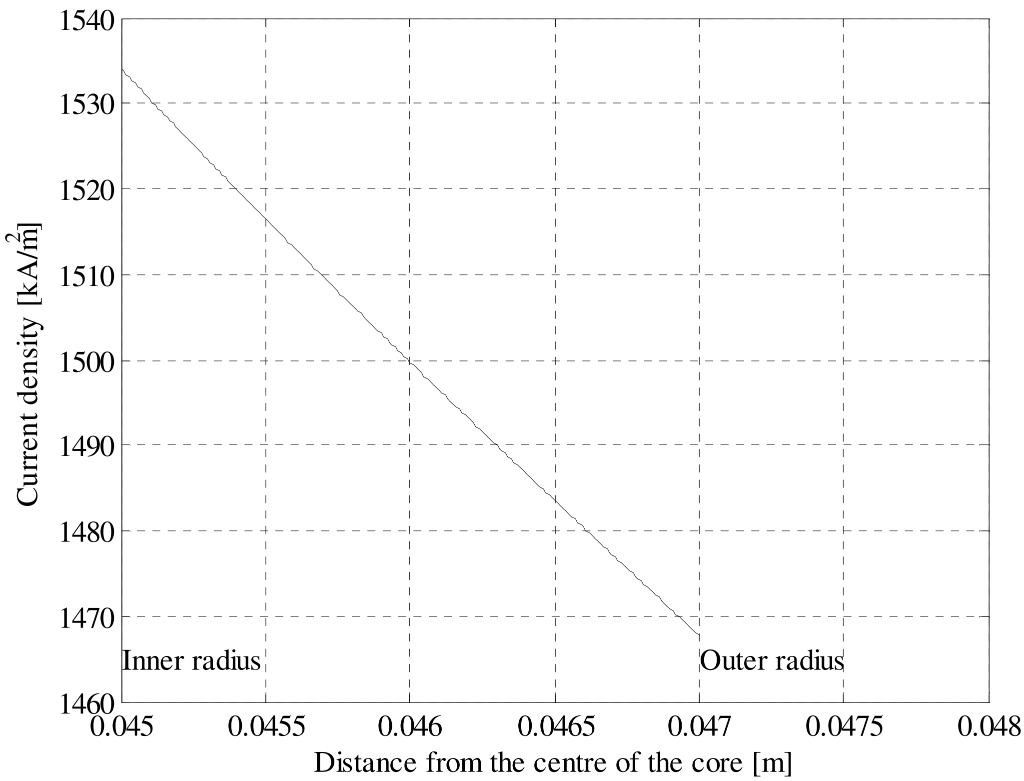

Figure 2.6 Current distribution at 50Hz inside one turn of a coil around circular core

The density of heat loss is found by use of Joule's law

$$P_\sigma = \sigma\left|E_\varphi\right|^2 = \sigma E_\varphi \cdot E_\varphi^* = \frac{1}{\sigma}\left|\bar{J}_\varphi\right|^2 \tag{21}$$

The effect loss in the volume is given by

$$P = \int P_\sigma dv = \frac{2\pi\Delta l}{\sigma}\int_{r=a}^{b}\left|\bar{J}_\varphi\right|^2 rdr \tag{22}$$

The resistance R_1 of the shell is then given by

$$P = R_1\left|\bar{i}\right|^2 = R_1\left|H_0\Delta l\right|^2 \tag{23}$$

By breaking out R_1 in eq.(22) and inserting P from eq.(23) an expression for the winding resistance can be found

$$R_1 = \frac{\frac{2\pi\Delta l}{\sigma}\int_{r=a}^{b}\left|\bar{J}_\varphi\right|^2 rdr}{\left|\bar{i}\right|^2} \tag{24}$$

By plotting eq. (24), the resistance in the conductor relative to the applied frequency can be obtained. Figure 2.7 and Figure 2.8 shows how the resistance in the coil increases as the frequency increases. In Figure 2.8 an approximation of the resistance of a straight conductor, from eq. (9), is also included. This approximation is accurate when the radius of the conductor is much greater than the skin depth. In the region of lower frequencies the resistance of the coil has a steeply increasing curve. In the region of the higher frequencies the curve appears to flatten out and increase approximately at the rate of the square root of the frequency. The resistance of the coil is much greater than that of a straight conductor, given that the length and area of the conductors are equal.

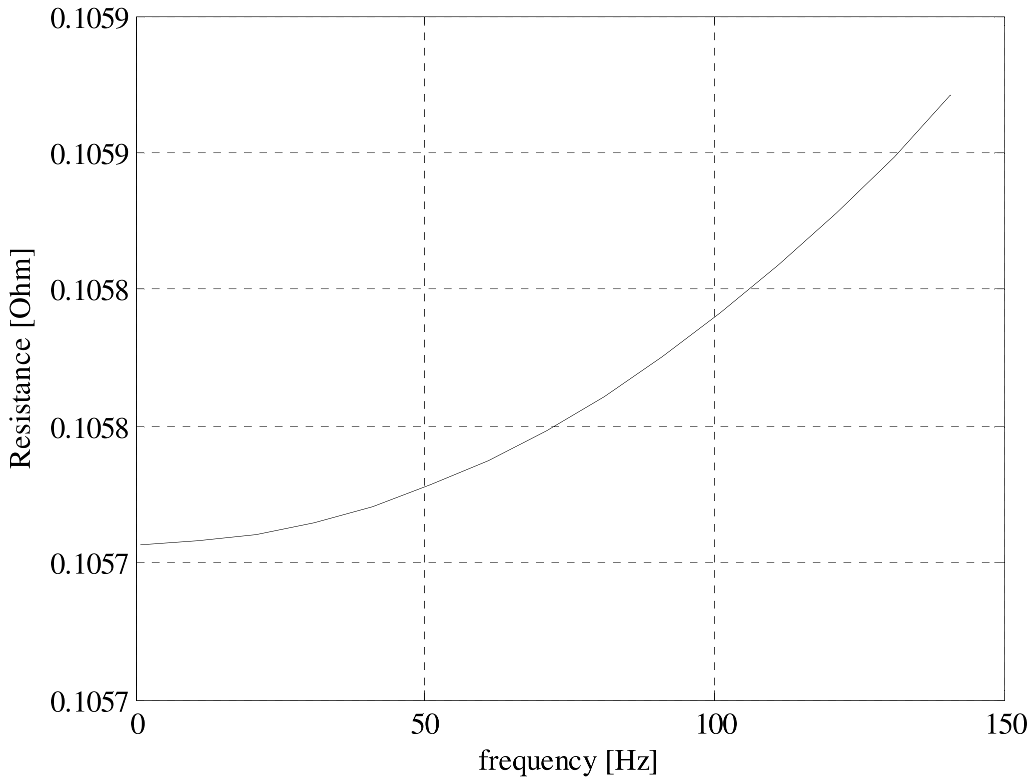

Figure 2.7 Increase of resistance of conductor due to skin effect, shown for lower frequencies

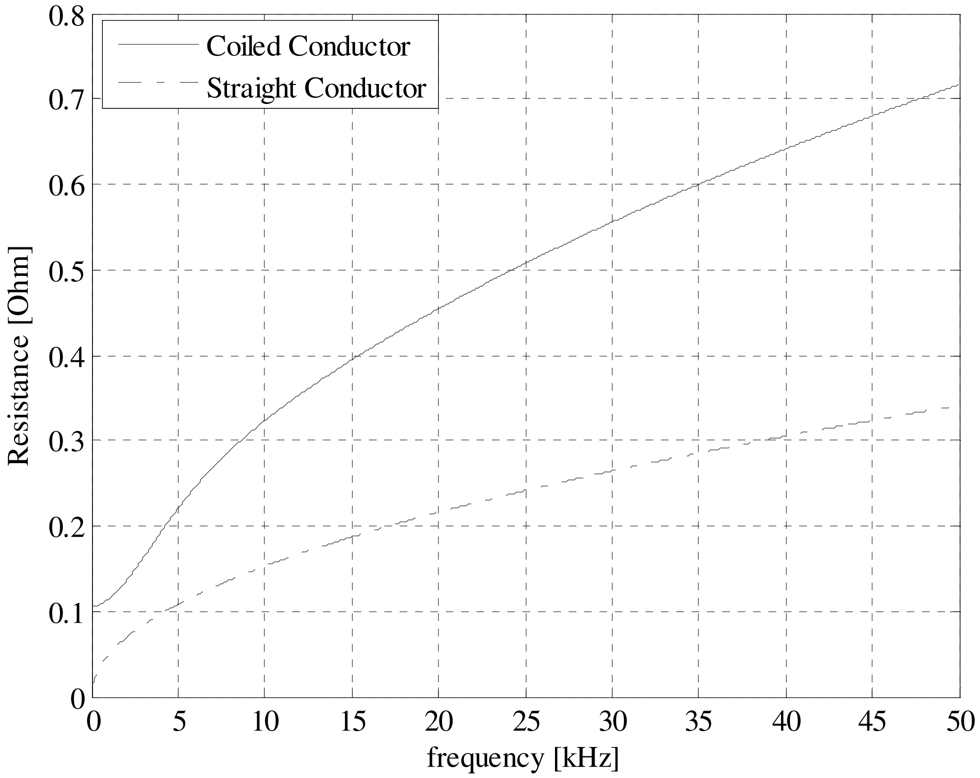

Figure 2.8 Increase of resistance of conductor due to skin effect, shown for higher frequencies

The calculated DC-resistance is lower than the measured resistance that is presented in chap. 3.3.1 (0.15ohm). This can have four reasons. First the exact properties of the conductor material are not known. Secondly the lab transformer has a square core. Which means the winding is not a perfect cylinder. Since the radius of each turn is not constant, the winding will have varying resistance along its length. The resistance will be higher where the radius is lower. This means the exact resistance is not possible to predict since the theory assumes a perfectly cylindrical winding. Thirdly, the winding conductor is not cylindrical but rather a rounded rectangle. Thus the conductor radius used in the equation is an approximation. This will decrease the accuracy further. Lastly the exact number of windings is not known.

The wound conductor shows a considerably higher resistance compared to a straight conductor. This is due to the tendency of the current to seek the shorter path. The current density will therefore be much higher close to the inner side of the conductor. This tendency will increase with frequency.

2.2.2 Magnetic hysteresis and skin effect in the transformer core

The Hysteresis curve can be approximated by an elliptical loop [4]. The magnetic intensity and the magnetic field density are represented by cosine functions, and plotted against each other, Figure 2.9. Measurements on a magnetic circuit with hysteresis yields a phase difference and B-H curve of elliptic shape.

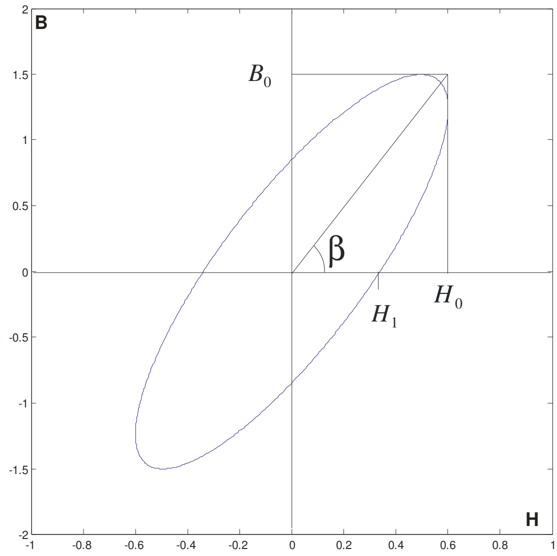

Figure 2.9 Approximation of the hysteresis curve

$$H(t) = H_0 \cos(\omega t) \tag{25}$$

$$B(t) = \mu H_0 \cos(\omega t - \alpha) \tag{26}$$

$$\mu = \frac{B_0}{H_0} = \tan\beta \tag{27}$$

$$\sin\alpha = \frac{H_1}{H_0} \tag{28}$$

The area of the ellipse represents the energy loss from the hysteresis in watt per cubic meter; the hysteresis loss can be calculated as follows.

$$P_{hyst} = W_m f Volume \tag{29}$$

The area of the ellipse is:

$$W_m = \pi B_0 H_0 \sin\alpha \tag{30}$$

And the volume of the magnetic material:

$$Volume = A_{core} l_{core} \tag{31}$$

The power loss due to the hysteresis will then be approximately

$$P_{hyst} = f \cdot \pi B_0 H_0 \sin\alpha \cdot A_{core} \cdot l_{core} \tag{32}$$

Combining (2), (11) and (32) gives

$$P_{hyst} = f \cdot \mu \frac{N^2 I^2}{l_{core}} \sin\alpha \cdot A_{core} \tag{33}$$

$$P_{hyst} = \frac{\omega N^2 I_{RMS}^2}{\Re_0} \sin\alpha \tag{34}$$

It is shown that when the harmonic frequency is high enough a magnetic skin effect becomes active [4]. This results in a broader hysteresis loop with an accordingly higher loss.

By use of complex permeability Maxwell's equations can be solved analytically for the core. The result will be a complex reluctance, which is the product of the dc-reluctance and a complex function f(jω) which takes account for the hysteresis and the eddy currents.

$$Z(\omega) = \frac{j\omega N^2}{\Re_0 f(j\omega)} \tag{35}$$

For a thin sheet of thickness d the result is

$$f(j\omega) = \left(\frac{\sqrt{2}}{2} \left(\frac{d}{\delta} \right) e^{j\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)} \right) e^{j\alpha} \coth\left(\frac{\sqrt{2}}{2} \left(\frac{d}{\delta} \right) e^{j\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)} \right) \tag{36}$$

$$\alpha = \frac{\pi}{2} - \phi \tag{37}$$

Where ∅ is the phase shift of the equivalent core impedance.

At low frequencies the reactance is

$$Z(\omega) = \frac{j\omega N^2}{\Re_0} \tag{38}$$

The serial representation can be transformed to an equivalent parallel configuration by the following procedure. As shown in Figure 2.10 and Figure 2.11.

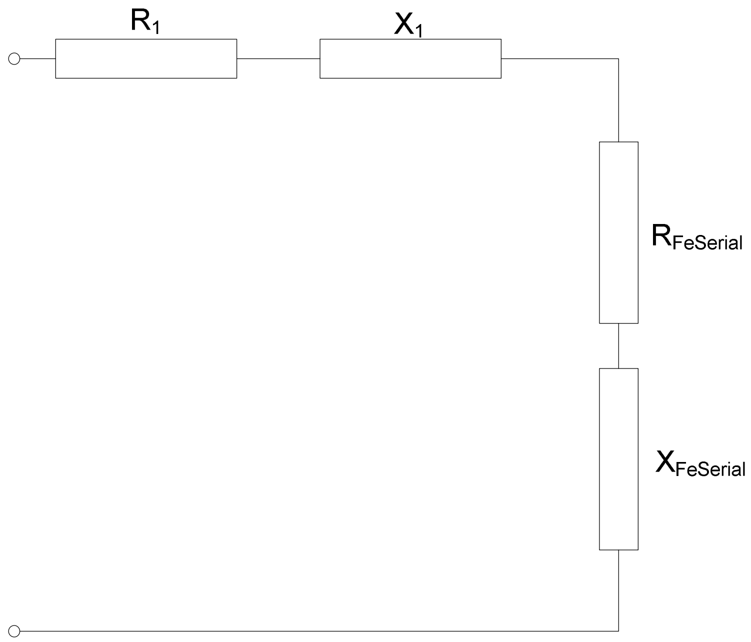

Figure 2.10 Serial representation of the transformer core

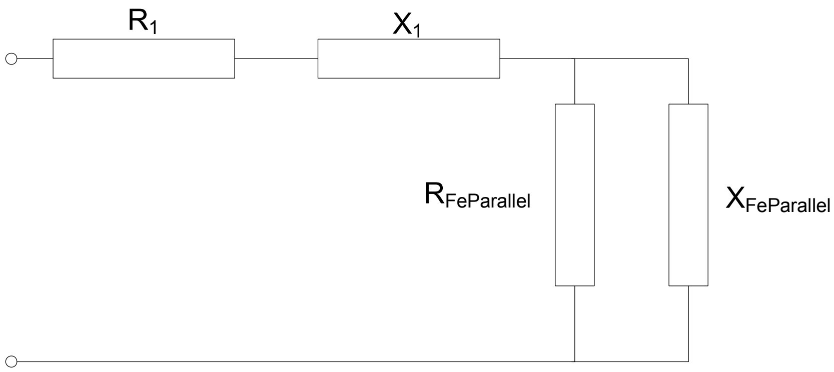

Figure 2.11 Parallel representation of the transformer core

First, the angle between the two parallel components must be found

$$\phi = \frac{\pi}{2} - \arctan\left({X_{serial}}/{R_{serial}} \right) \tag{39}$$

Then the angle is used to find the parallel resistance

$$R_{parallel} = R_{serial} \frac{1 + \tan^2(\phi)}{\tan^2(\phi)} \tag{40}$$

Since the angle between the components is known, it can be used to find the parallel reactance as follows

$$X_{parallel} = R_{parallel} \tan(\phi) \tag{41}$$

Using the above presented procedure, the following values were found for the parallel representation of the core impedance:

$R_{core\_parallel}$ = 1.5kOhm

$X_{core\_parallel}$ = 0.95kOhm

This gives the core inductance

$L_{core\_parallel}$ = 3H.

By plotting the real part and the imaginary part of eq.(35), the frequncy dependence of the resistance and reactance that represent the core losses in the transformer can be estimated. In Figure 2.12 and Figure 2.13 the real part and the imaginary part are plotted against the frequency. Both the resistance and the reactance appear to increase as the square root of the frequency for higher frequencies.

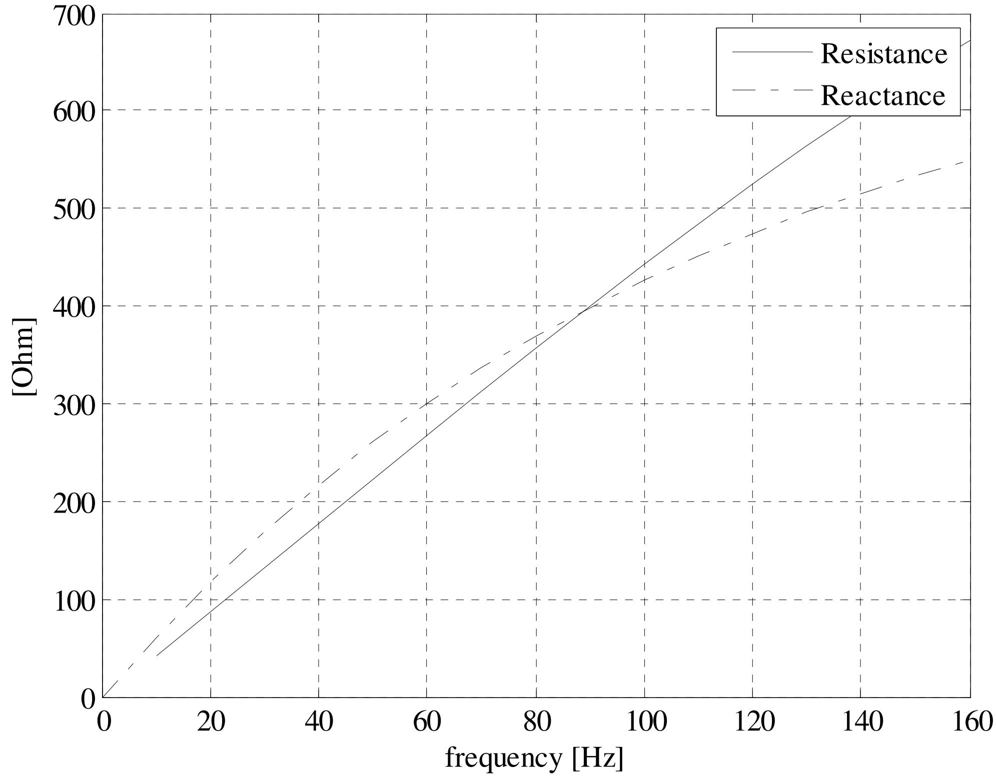

Figure 2.12 Equivalent impedance of the core due to hysteresis and magnetic skin effect, based on eq.(35)

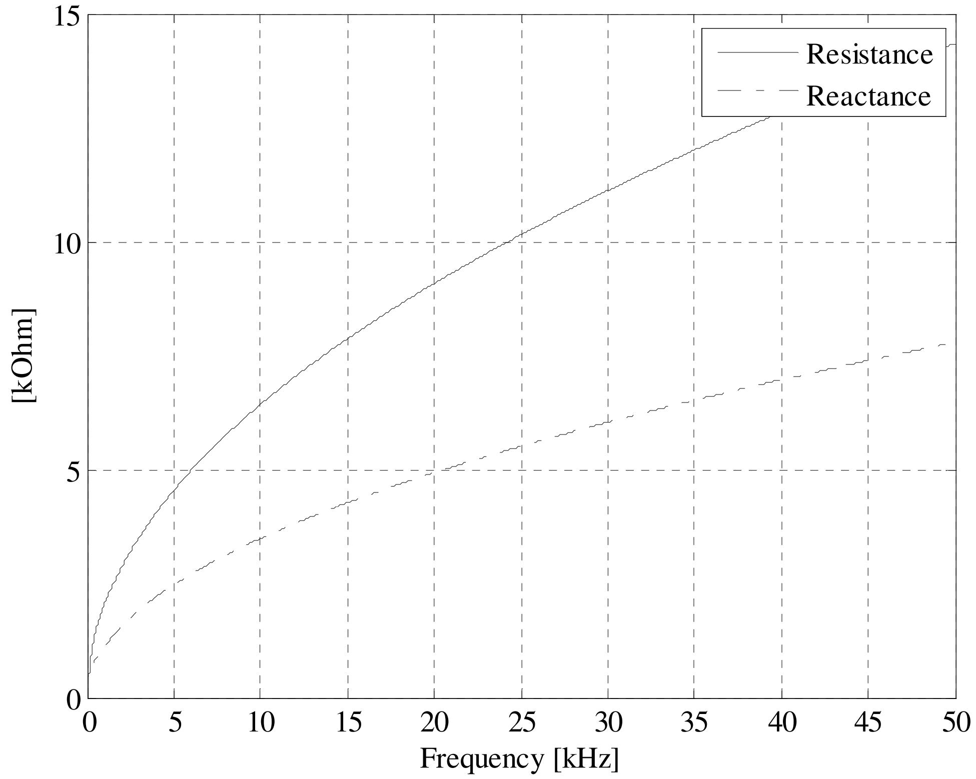

Figure 2.13 Equivalent impedance of the core due to hysteresis and magnetic skin effect, based on eq.(35)

2.3 The equivalent electric circuit of the transformer

The focus of this report is on transformers that are loaded by non sinusoidal currents. The transformers will draw 50 Hz current from the network, and the load will inject high frequency current into the transformer. Both currents will generate heat losses in the transformer. The losses generated by the 50 Hz current are often analyzed using an equivalent circuit similar to the one shown in Figure 2.14. Where Z_1 and Z_2, respectively, represent the losses in the primary winding and the secondary winding, Z_{Fe} represents the core losses, Z_L represents the load and V_1 represents the network voltage.

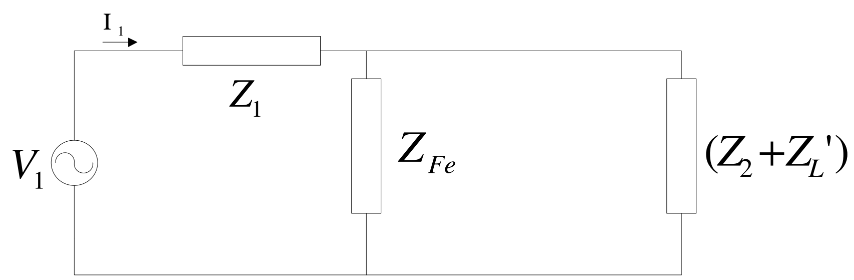

Figure 2.14 Equivalent circuit for losses related to the base frequency

The impedance seen by the grid is given as

$$Z_{tot}^{grid}(\omega_h) = \frac{\left(Z_2(\omega_h) + Z_L^{'}\right) \cdot Z_{Fe}(\omega_h)}{\left(Z_2(\omega_h) + Z_L^{'}\right) + Z_{Fe}(\omega_h)} + Z_1(\omega_h) \tag{42}$$

The losses related to the load generated harmonics will be analyzed using the circuit shown in Figure 2.15. Since the physical phenomenon of hysteresis, eddy currents and skin effect is frequency dependant. All the parameters in the circuit must also be so.

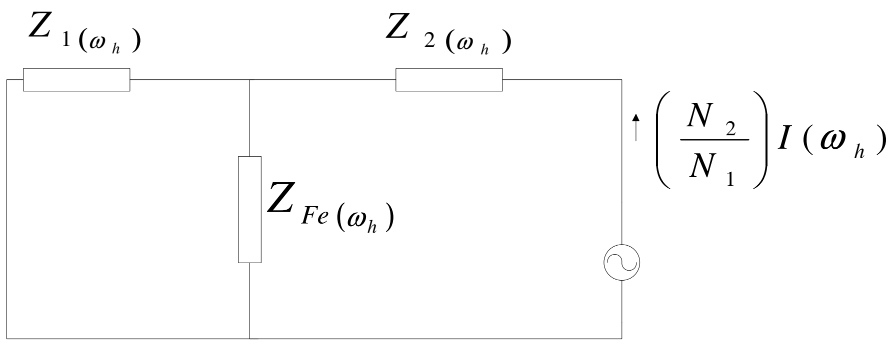

Figure 2.15 Equivalent circuit of real transformer for losses related to harmonics generated by the load

Looking at Figure 2.15 it is clear that the impedance seen by the load can be given by the following formulae

$$Z_{tot}^{load}(\omega_h) = \frac{Z_1(\omega_h) \cdot Z_{Fe}(\omega_h)}{Z_1(\omega_h) + Z_{Fe}(\omega_h)} + Z_2(\omega_h) \quad (43)$$

$Z_{Fe}(\omega_h)$ will be calculated by eq. (35). The winding resistance will be calculated according to eq.(24). However, the theory presented in this report does not treat leakage reactance. Therefore the assumption will be made that it increases proportionally to frequency. The winding impedance is

$$Z_1(\omega_h) = R_1(\omega_h) + jX_1(\omega_h) \quad (44)$$

And for the low voltage side

$$Z_2(\omega_h) = \left(\frac{N_1}{N_2}\right)^2 [R_2(\omega_h) + jX_2(\omega_h)] \quad (45)$$

The load impedance Z_L' is given by

$$Z_L^{'} = \left(\frac{N_1}{N_2}\right)^2 Z_L \quad (46)$$

2.4 Location of losses in transformers

An overview of where in the transformer losses occur, and the cause of these losses

2.4.1 Core losses

Core losses will have different magnitude on different parts of the core. This is due to the shape of the core which will increase the flux density in certain areas. This is clearly visible in Appendix-A.

The hysteresis losses due to the work required to turn the direction of the magnetic flux. These losses depend on the type of material used in the core.

The eddy currents, generated by the magnetic field, create a loss. These currents are reduced by the use of laminated cores, which reduces the length of the current path an eddy current can have in the core. The eddy currents created in the core will give rise to a magnetic skin effect. This skin effect will increase with frequency. Its consequence is a reduction of the flux density in the core, thereby reducing the power flow through the transformer.

2.4.2 Winding losses

Resistive losses will occur in the windings. The magnitude of the loss is proportional to the winding resistance, and to the square of the current.

The magnetic field around the windings will create an electric skin effect, which will increase the effective resistance of the windings, by restricting the current flow to the outer parts of the conductor. The skin effect increases with frequency. To reduce the influence of the skin effect the conductors within transformers are stranded in order to avoid that the skin depth becomes less than the radius of the strand.

To avoid the generation of circulating currents within the individual conductor due to the difference in flux experienced by each individual strand in one conductor, the strands are transposed. This technique is called Continuously Transposed Conductors (CTC).

The skin dept δ is much lower in iron due to its high permeability. This is shown in Figure 2.16. Therefore the effects of eddy currents will be visible in the core material at relatively low frequencies.

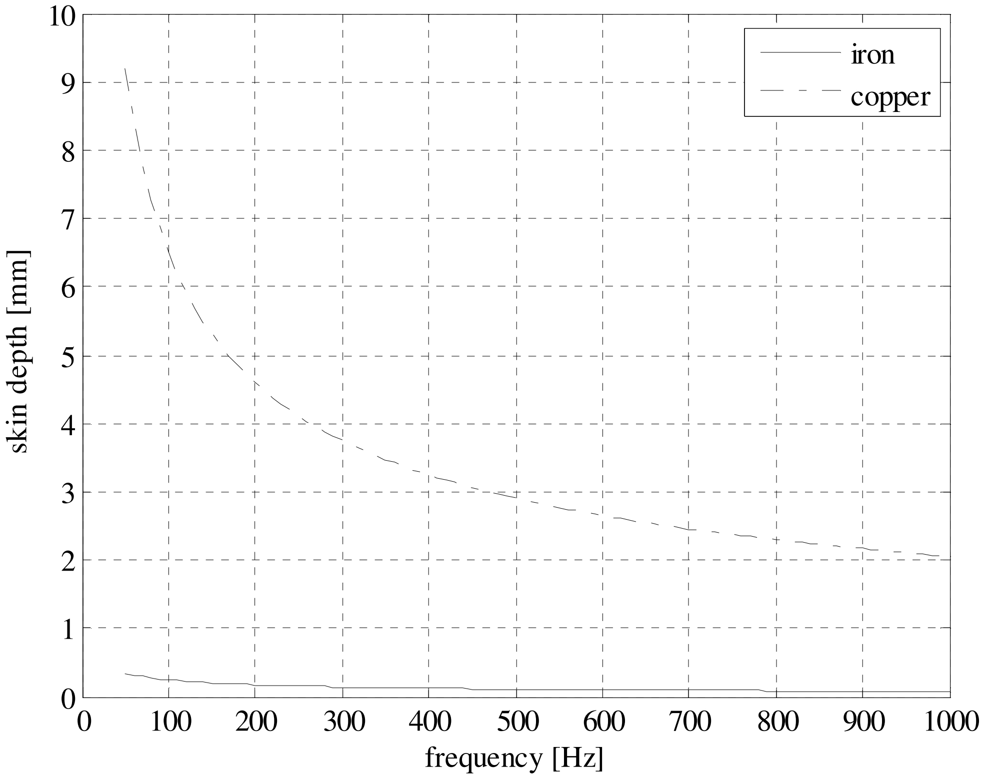

Figure 2.16 Skin dept in iron and copper relative to frequency

3. Case study

3.1 Case definition and experimental setup

A 220/110 volt transformer was used as a study object. It was subjected to different loads and supply situations in order to model the working conditions of a distribution generator. In order to map the core losses, an infrared camera was used to detect the areas where high heat losses occur.

The core is built of 170 layers of 0.4mm thick steel. The shape of the transformer core can be seen in Figure 3.1

The high voltage winding is made of round copper wire with the radius 1mm. And the low voltage winding is made of a copper conductor with a rectangular cross section. The cross section has the side dimensions 3mm and 2mm.

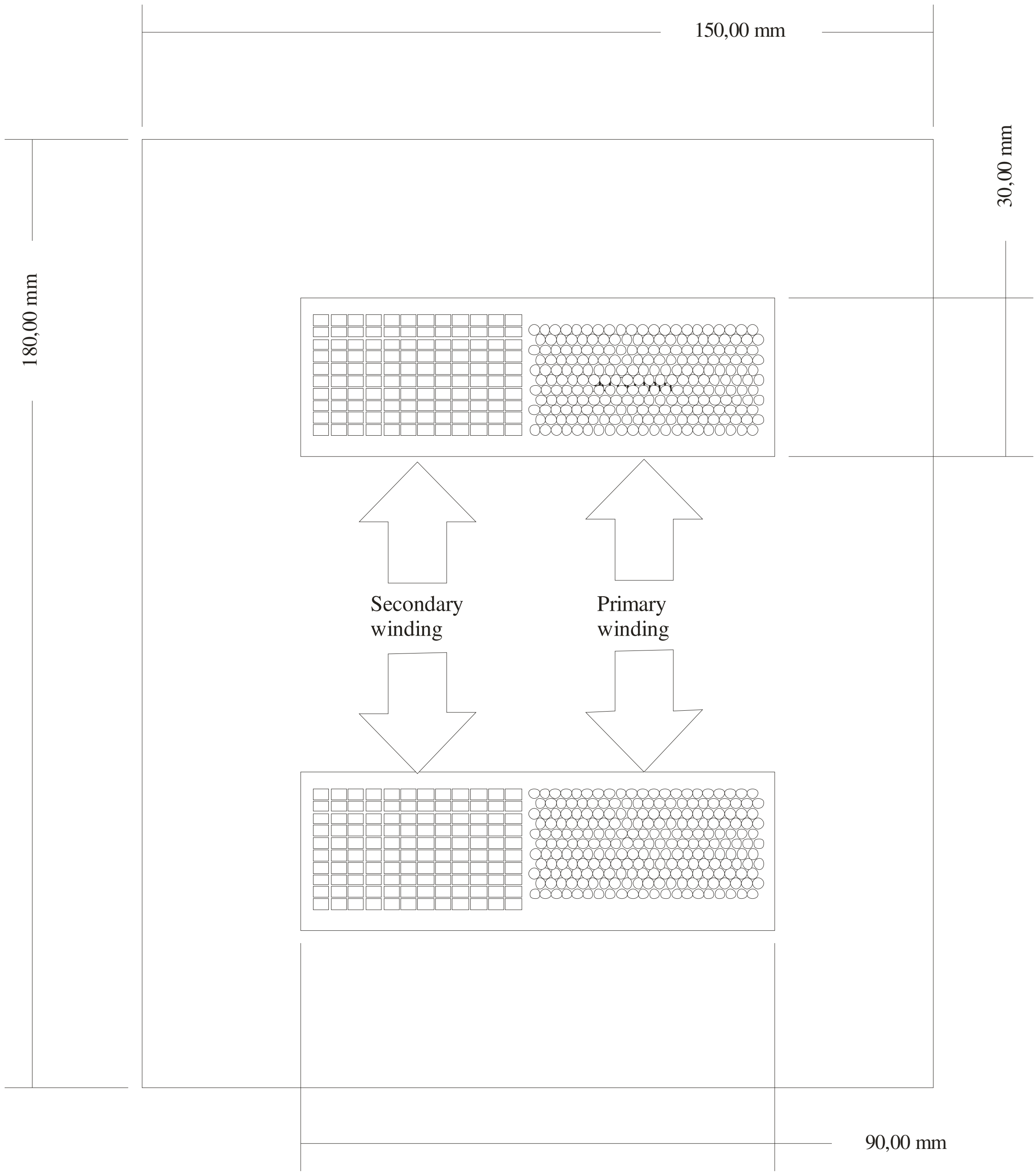

Figure 3.1 Transformer

A LeCroy oscilloscope of the model 9304CM was used for the measurements. This oscilloscope has a bandwidth of 175MHz, and the maximum sampling rate of 100MS/s. The oscilloscope is shown in Figure 3.2.

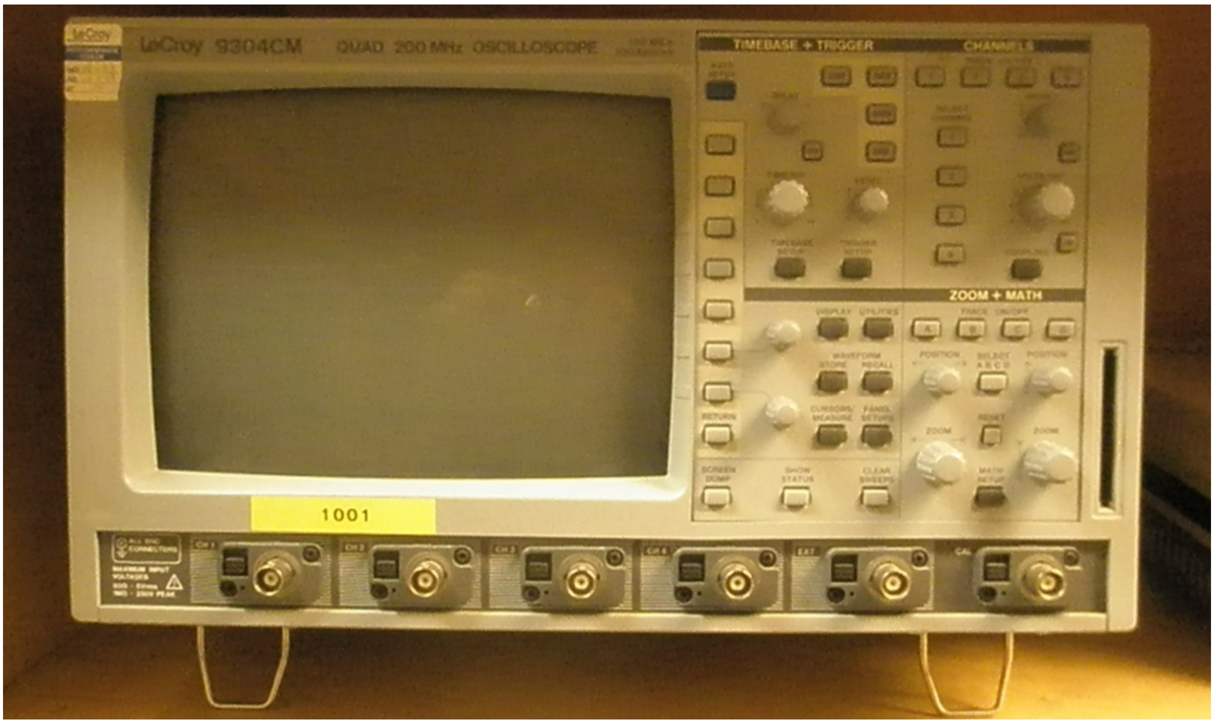

Figure 3.2 LeCroy oscilloscope

The current measurements where made with the current probe AP015, shown in Figure 3.3. This probe has bandwidth of 50MHz, a DC accuracy of ± 1% at 15A and ± 2% at 30A.

Figure 3.3 current probe

The voltage measurements were made with the AP032 voltage probe which is shown in Figure 3.4. This probe has a bandwidth of 25MHz.

Figure 3.4voltage probe

In order to measure the losses in the transformer two voltage probes and two current probes where used simultaneously. So that both the network side voltage and the network side current where recorded at the same instant that the load side voltage and the load side current were recorded. During the no load test only one of each probe was necessary.

3.1.1 Procedure for measurements on a small single phase lab transformer

DC, short circuit and no-load test were made in order to decide the parameters of the transformer

The DC test was made for both the high and low voltage windings in order to determine the resistance of these. The tests were made with currents of 1 to 4 ampere on the high voltage side, and 1 to 6 ampere on the low voltage side. Increments of 1 ampere where used in both cases. The tests showed the following DC-resistances for the high and the low voltage winding:

R_{h_DC} = 0.58Ohm

R_{l_DC} = 0.15 Ohm

The short circuit test was performed with a variable frequency source connected to the high voltage side. The results are shown in Figure 3.5.

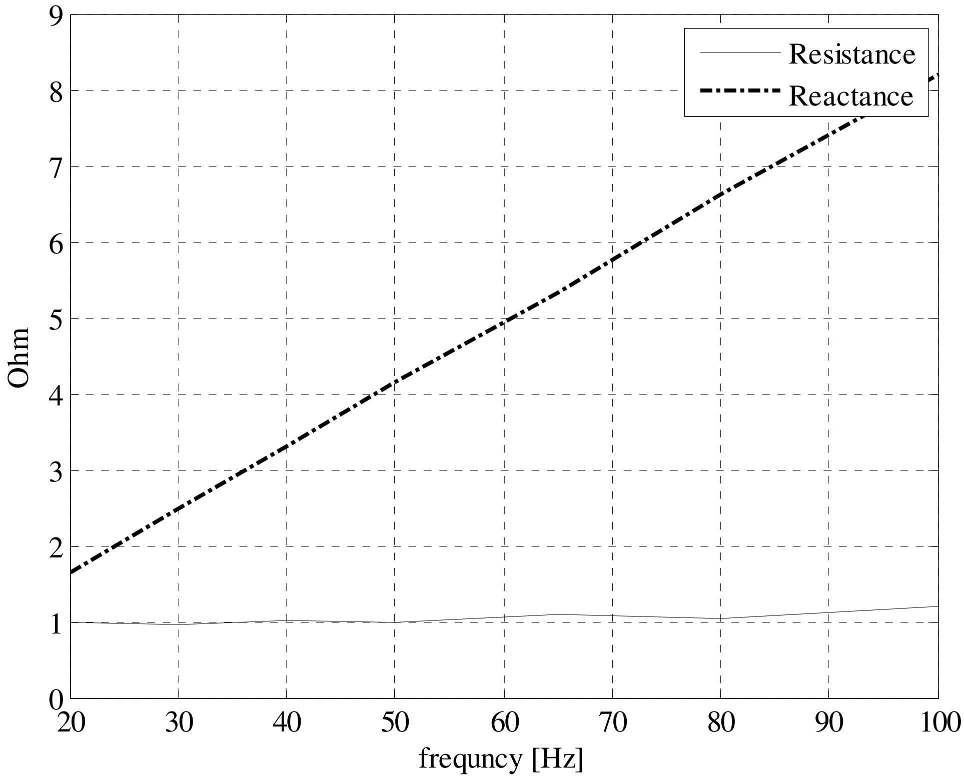

Figure 3.5 short circuit resistance and reactance

For 50 Hz the short circuit resistance is R_{SC} = 1.0Ohm and the short circuit reactance is 4.1Ohm. Which gives a short circuit inductance of L_{SC} = 13mH.

In order to isolate the core losses and test the equivalent model of the core, the transformer was operated in no-load. The tests were made with the source connected to the low voltage side. The frequency of the source was varied. The voltage was varied relative to the frequency so as to keep a constant flux level. This allows the mapping of core losses in relation to the frequency. At 50 Hz the skin dept exceeds the lamination thickness of the core and the radius of the transformer windings; therefore there will be only low eddy currents or skin effect. The only losses in the transformer will be resistive losses in the windings and hysteresis losses in the core. Since the winding resistance is very small compared to the core impedance the winding losses can be neglected in the non load measurements. The measured power loss was plotted and is shown in Figure 3.6. This figure indicates that the loss increases by the square root of the frequency in the higher register.

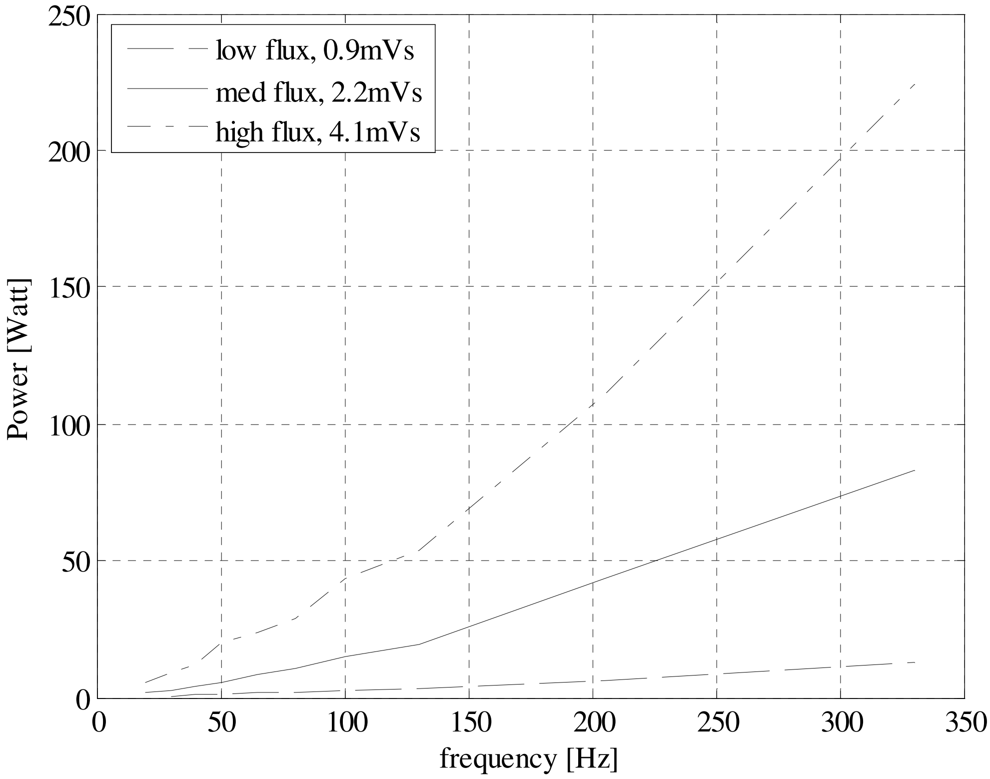

Figure 3.6 Power loss in transformer core relative to frequency, measured by no load test

The measurements were compared to the electrical equivalent of the core by plotting the measured power loss and the theoretical power loss, Figure 3.7 and Figure 3.8. The theoretical power losses were adjusted to fit the measured values by changing the permeability of the transformer steel, and by changing the variable phi. Thus the impedance of the model and the relative size of the resistance and the reactance were changed. The theoretical power loss is obtained by multiplying the square of the current with the equivalent core impedance found in eq.(35).

The results used throughout this report are $\mu_r = 5500\text{Vs/Am}$, and $\emptyset = 57°$.

Example

Calculation of effect loss at 100Hz. The first step is to find the skin dept by eq. (8)

$$\delta = \sqrt{\frac{2}{5.5\cdot 10^3 \cdot 4\pi \cdot 10^{-7} \cdot 6.25\cdot 10^6 \cdot 2\pi \cdot 100}} = 0.27\cdot 10^{-3}$$

Now f(jω) was calculated by inserting skin depth, alpha and laminate thickness into eq. (36)

$$f(j\omega) = \left(\frac{\sqrt{2}}{2}\left(\frac{0.4\cdot 10^{-3}}{0.27\cdot 10^{-3}}\right) e^{j\left(\frac{\pi}{4}-\frac{0.576}{2}\right)}\right) e^{j0.576} \coth\left(\frac{\sqrt{2}}{2}\left(\frac{0.4\cdot 10^{-3}}{0.27\cdot 10^{-3}}\right) e^{j\left(\frac{\pi}{4}-\frac{0.576}{2}\right)}\right)$$

$$= 0.858 + j0.891$$

In order to find the impedance of the core, f(jω) is inserted into eq. (35)

$$Z(2\pi 100) = \frac{j2\pi 100 \cdot 110^2}{10 \cdot 10^3 (0.858 + j0.891)} = (395 + \mathrm{j}380)\Omega$$

The impedance is now multiplied by the square of the measured current RMS value in order to get the effect loss at 100 Hz

$$S_{Core}^{100Hz} = (395 + \mathrm{j}380)\Omega \cdot 0.17^2 A = \underline{\underline{(12 + \mathrm{j}11)VA}}$$

Repeating the calculation for 50Hz gives the following result

$$S_{Core}^{50Hz} = \underline{\underline{(3.9 + \mathrm{j}4.5)VA}}$$

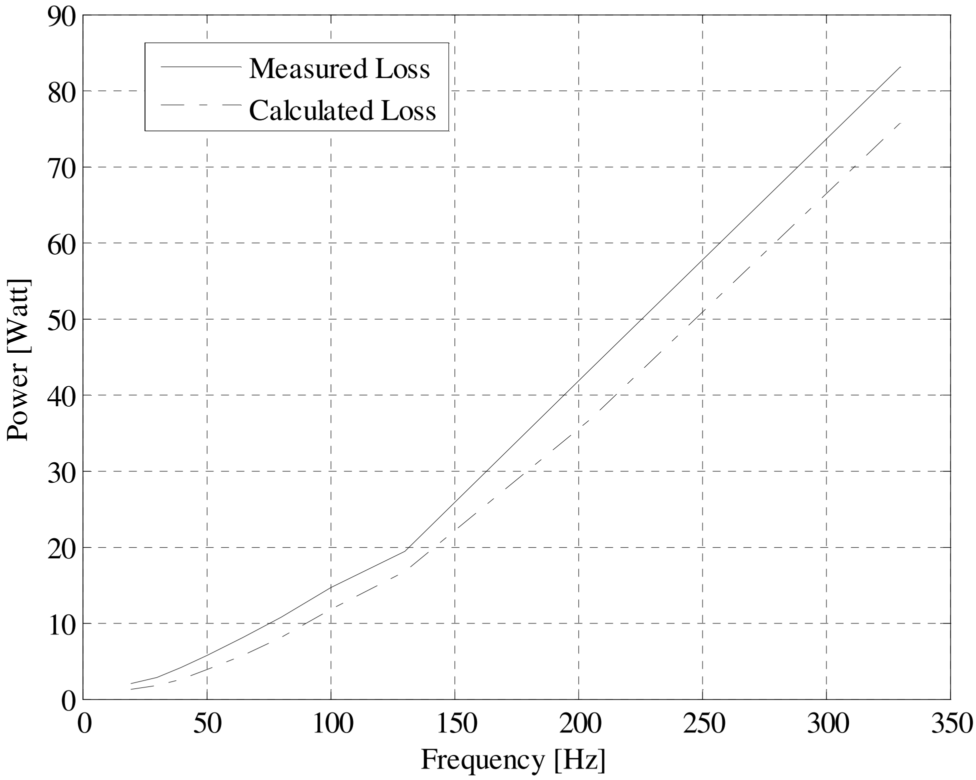

Figure 3.7 Measured power loss compared to calculated power loss

The Measured losses are higher than the calculated losses, ca 13%. The difference appears to be constant throughout the frequency specter.

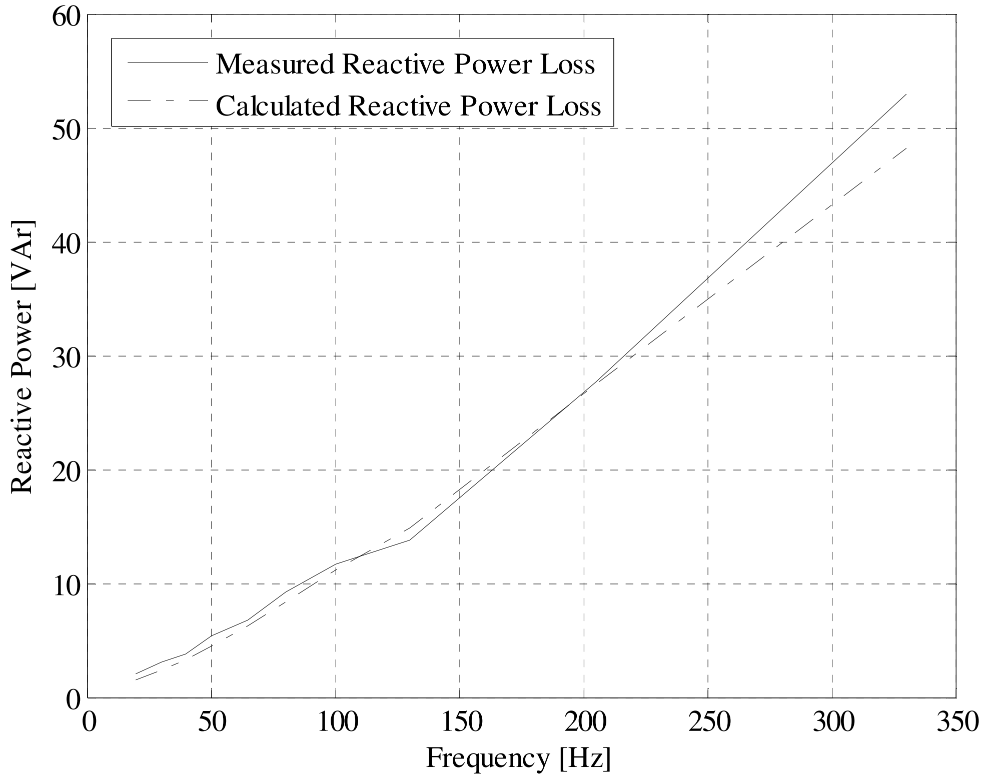

Figure 3.8 Measured reactive power compared to calculated reactive power

The reactive power gives a good fit in the lower part of the spectrum. In the higher part the measured power loss rises faster than the calculated power loss.

Figure 3.9 shows the value of the equivalent resistance of the core. The values are derived by dividing the voltage by the current measured in the no-load test. Both the resistance and reactance are increasing proportionally to the square root of the frequency. This rate of increase corresponds with the resistance and reactance increase of the core found in eq. (35), and illustrated in Figure 2.13.

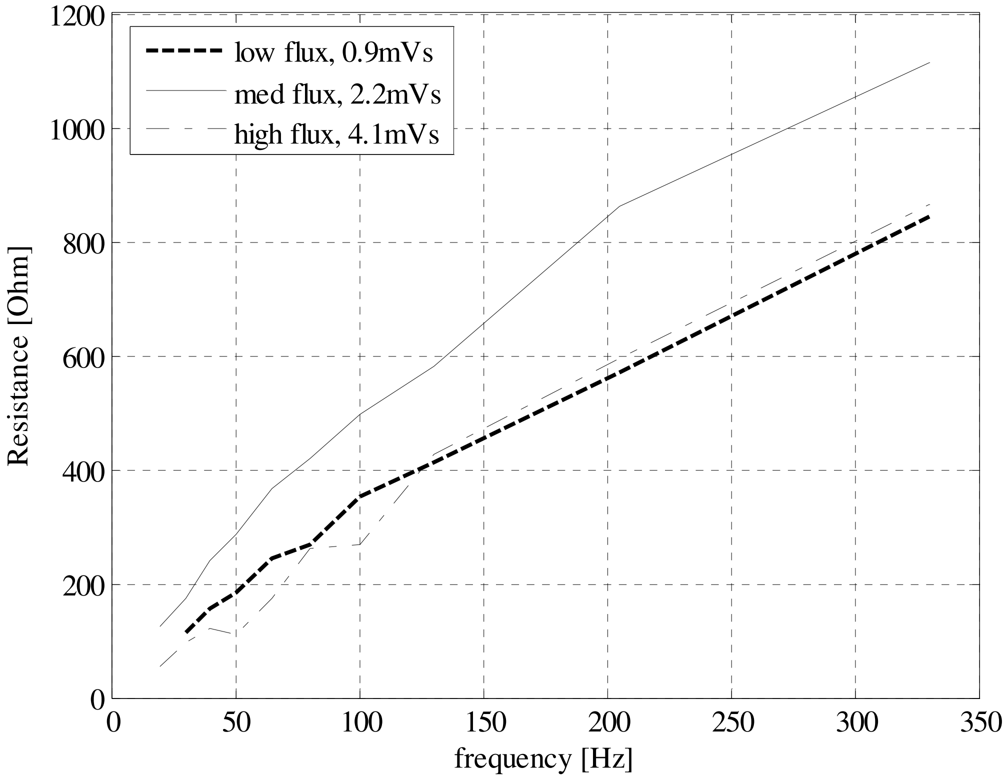

Figure 3.9 equivalent resistance of the core for different frequencies and different flux levels

As the figure above shows the resistance is higher at medium flux levels. Since the impedance is proportional to the permeability, it can be observed that the permeability is at its highest at medium flux levels.

Figure 3.10 and Figure 3.11 shows the resistance and the reactance, respectively. The measured values are compared to theoretical values of the impedance. The theoretical impedance was adjusted to fit the measured values.

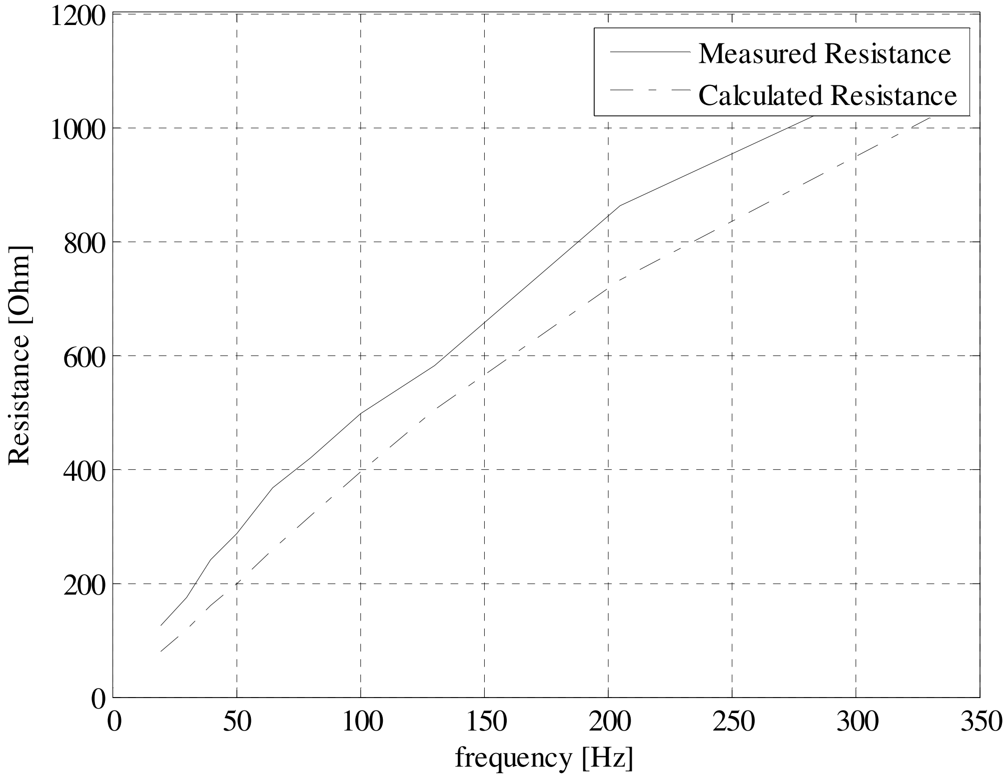

Figure 3.10 Resistance of the core found by no-load test

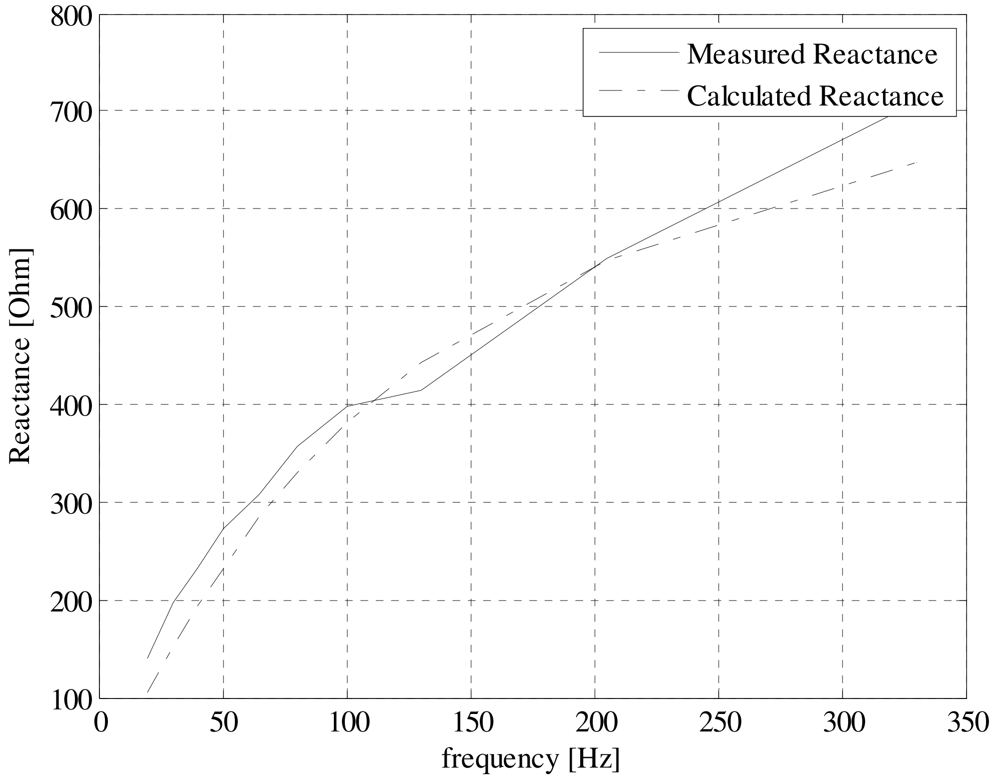

Figure 3.11 Reactance of the core found by no-load test

Example

Converting from serial representation to parallel representation of the core impedance at 100 Hz. First the angle between the parallel components are found by using eq. (39)

$$\phi = \frac{\pi}{2} - \arctan\left(380 / 395\right) = 0.805$$

Now the parallel resistance can be found by inserting the angle into eq. (40)

$$R_{parallel}^{100\,Hz} = 395 \frac{1 + \tan^2(0.805)}{\tan^2(0.805)} = \underline{\underline{760\Omega}}$$

Finally the reactance can be found by using eq. (41)

$$X_{parallel}^{100\,Hz} = 760 \tan(0.805) = \underline{\underline{790\Omega}}$$

Repeating the calculations for 50Hz yields

$$R_{parallel}^{50\,Hz} = \underline{\underline{470\Omega}}$$

and

$$X_{parallel}^{50\,Hz} = \underline{\underline{402\Omega}}$$

3.1.3 Measurements with rectified load

The transformer feeds an impedance load through a rectifier with a smoothing conductor on the DC side, as shown in Figure 3.12. This will generate harmonics. These harmonics will each have a different hysteresis curve and level of skin effect, which will show up as different levels of loss for each harmonic frequency

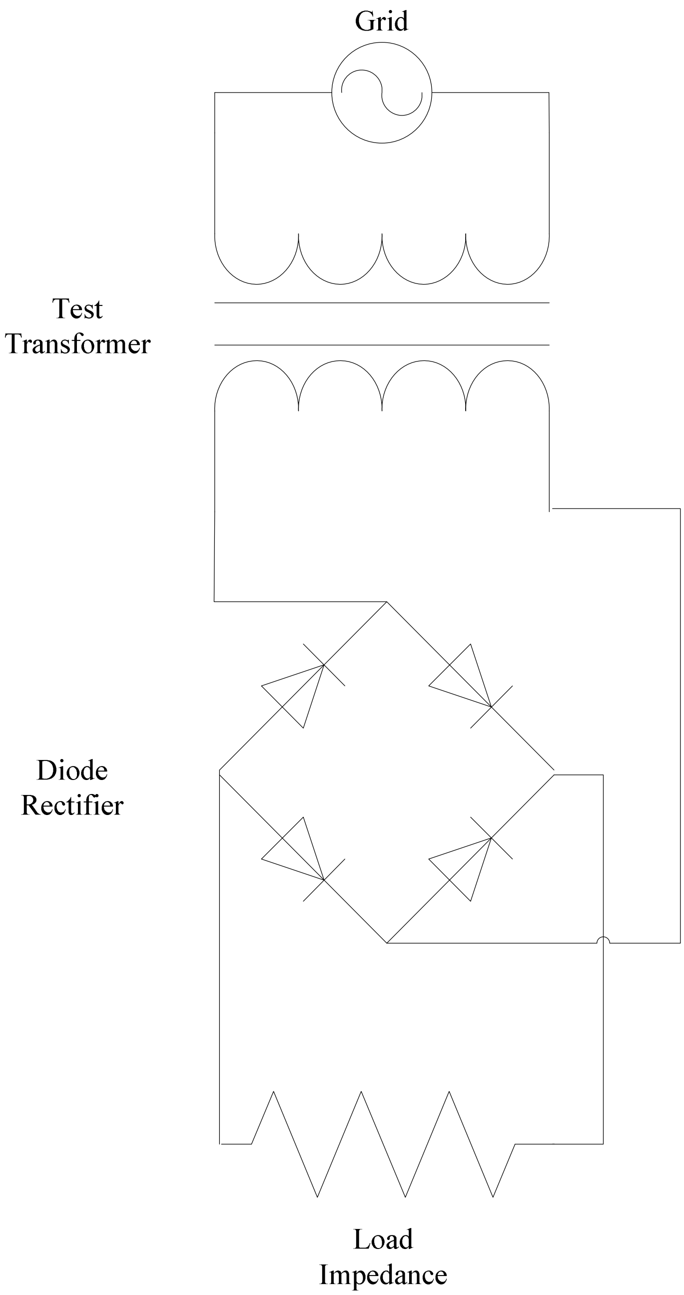

Figure 3.12 test circuit for diode rectified load

In Figure 3.14 the losses are plotted relative to the frequency. The measured power losses are compared to the values which are obtained by multiplying the square of the current with the calculated impedance of the transformer.

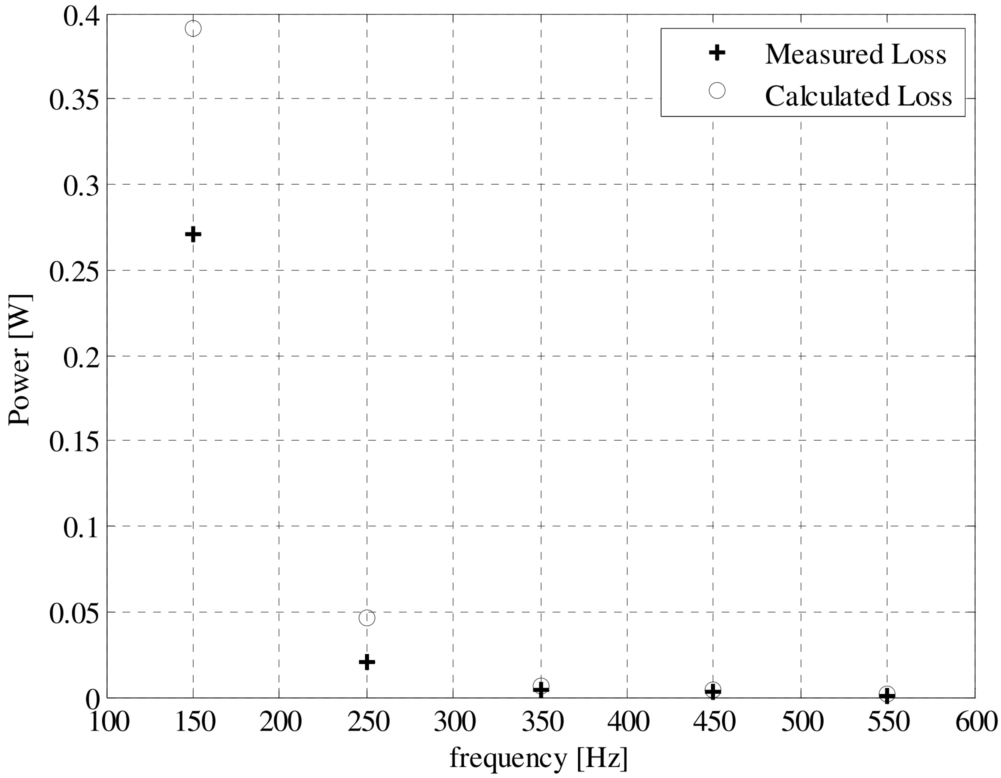

Figure 3.13 power losses generated by the harmonics

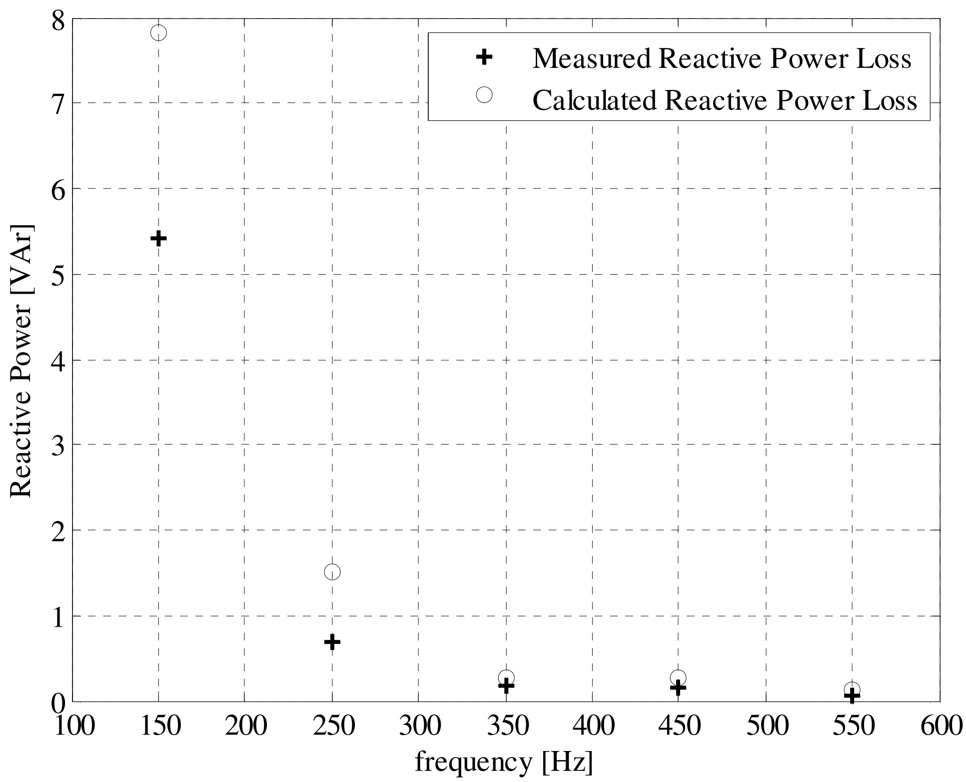

Figure 3.14 reactive power in the transformer, related to the load generated harmonics

The measured losses are lower than the calculated losses. 32% lower for the 3rd harmonic. This indicates that the proposed equivalent model gives a pessimistic estimate of the losses.

3.1.4 Measurements with pulse generating load

This lab set up is an attempt to model an active load which generates a pulsed load current of high frequency over layered on a 50Hz sinus curve. The primary side of the transformer was fed with a normal 50Hz voltage from the grid. On the load side a resistive load was used. An injection transformer was connected in series with the resistive load. The injection transformer was fed by a pulse generator and an amplifier. The setup is illustrated in Figure 3.15 and Figure 3.16.

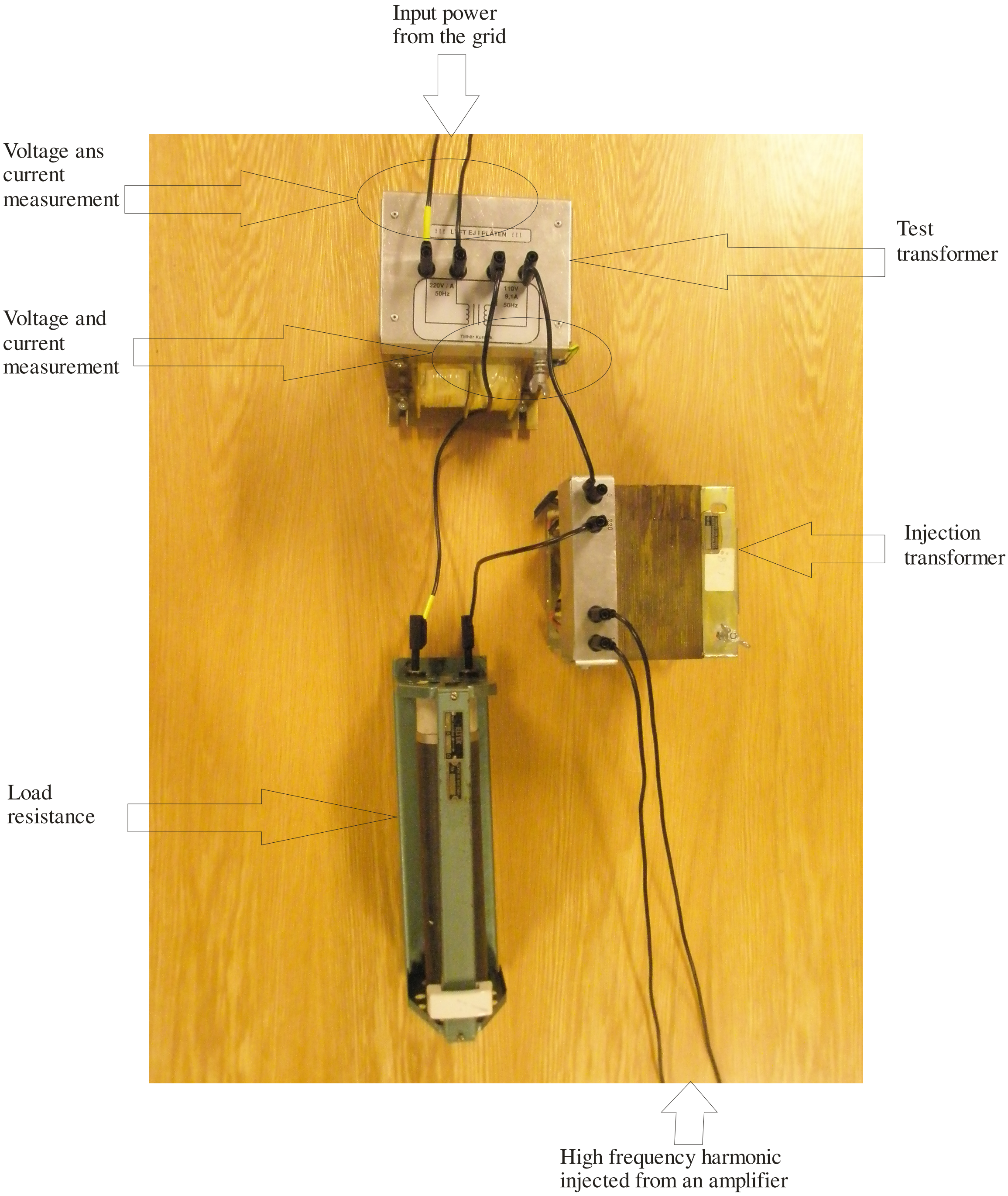

Figure 3.15 lab setup that simulates a harmonic generating load

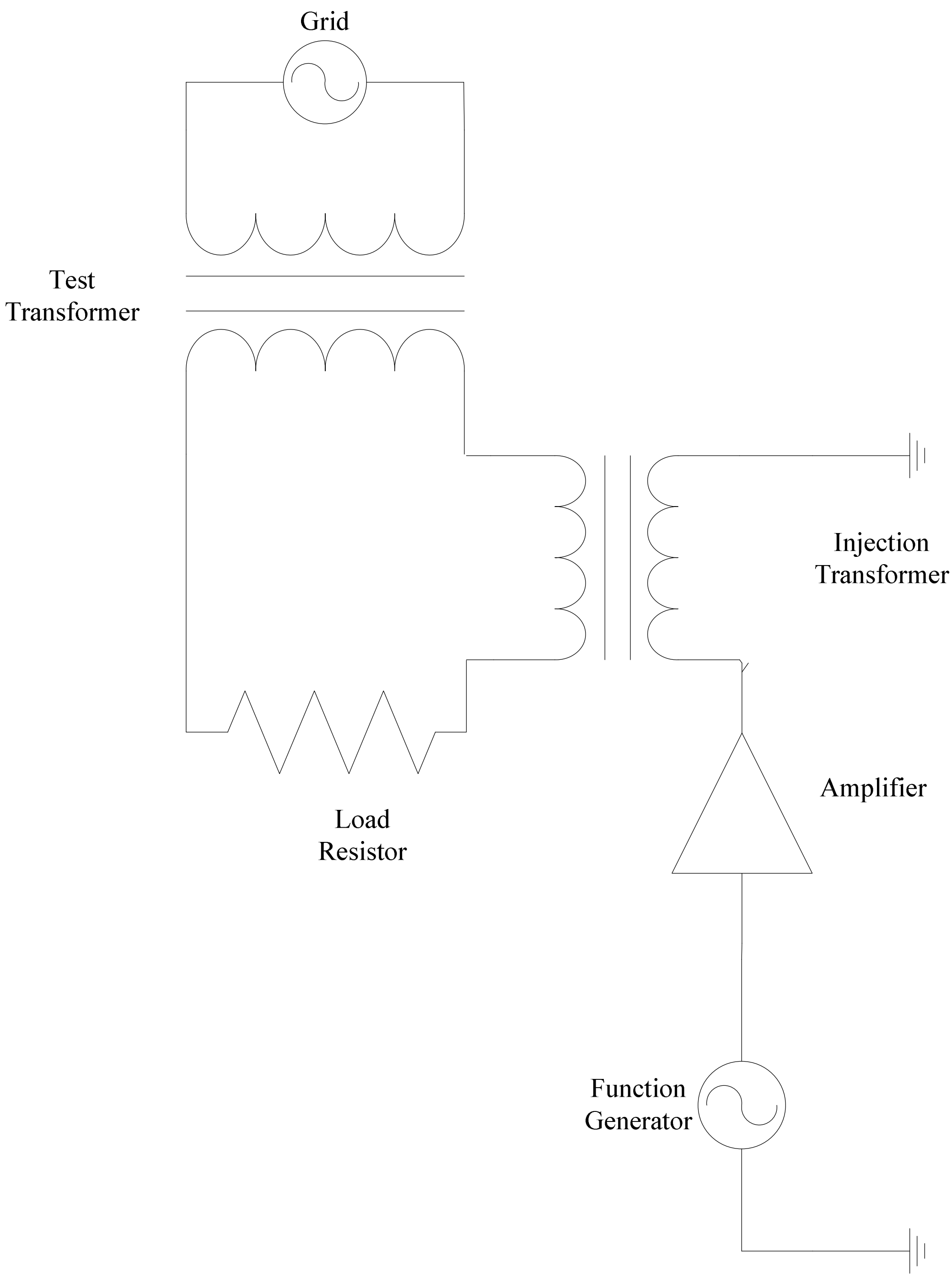

Figure 3.16 lab setup that simulates a harmonic generating load

The pulse generator was set to inject one sinus wave at the time. Starting at 0.5 kHz and ending at 3 kHz, with steps of 0.5 kHz. Two sets of tests were made. The first test was made with grid voltage of $14V_{RMS}$ and grid current of $0.4A_{RMS}$. The second test was made with grid voltage of $28V_{RMS}$ and grid current of $0.8A_{RMS}$.

In Figure 3.17 and Figure 3.19 the power loss generated by the over layered harmonic is plotted, as well as the theoretical power loss based on the equivalent electrical circuit presented in chap. 2.3. In

Figure 3.18 and Figure 3.20 the reactive power is compared in a similar way. The leakage reactance is from the short circuit test.

Example

Calculating power loss from a 1.5kHz harmonic. First the skin depth in the core for this specific frequency is determined by using eq. (8)

$$\delta = \sqrt{\frac{2}{5.5 \cdot 10^3 \cdot 4\pi \cdot 10^{-7} \cdot 6.25 \cdot 10^6 \cdot 2\pi \cdot 1500}} = 70 \cdot 10^{-6}$$

Then the equivalent electric resistance of the core is calculated using eq. (36),

$$f(j\omega) = \left(\frac{\sqrt{2}}{2} \left(\frac{0.4 \cdot 10^{-3}}{70 \cdot 10^{-6}} \right) e^{j\left(\frac{\pi}{4} - \frac{0.576}{2}\right)} \right) e^{j0.576} \coth \left(\frac{\sqrt{2}}{2} \left(\frac{0.4 \cdot 10^{-3}}{70 \cdot 10^{-6}} \right) e^{j\left(\frac{\pi}{4} - \frac{0.576}{2}\right)} \right)$$

$$= 1.91 + \mathrm{j}3.54$$

and then eq. (35)

$$Z_{Fe}(2\pi 1500) = \frac{j2\pi 1500 \cdot 220^2}{10 \cdot 10^3 (1.91 + \mathrm{j}3.54)} = (8.8 + \mathrm{j}4.8) \cdot 10^3 \,\Omega$$

The skin depth of the copper in the windings at the specific frequency is calculated by eq. (8)

$$\delta = \sqrt{\frac{2}{4\pi \cdot 10^{-7} \cdot 59 \cdot 10^6 \cdot 2\pi \cdot 1500}} = 1.7 \cdot 10^{-3}$$

Then skin depth is used to calculate k_1 and k_2 from eq. (15)

$$k_1 = \sqrt{2} j^{3/2} \Big/ 1.7 \cdot 10^{-3} = -595 + j595$$

$$k_2 = \sqrt{2} j^{1/2} \Big/ 1.7 \cdot 10^{-3} = 595 + j595$$

k_1 and k_2 are inserted into eq. (16) and (17) to obtain A_1 and B_1, H_0 can be set to any random number.

$$A_1 = H_0 \frac{\mathrm{K}_0(k_2 b)}{\Im_0(k_1 a) \cdot \mathrm{K}_0(k_2 b) - \Im_0(k_1 b) \cdot \mathrm{K}_0(k_2 a)} = (11.5 - j7.45) \cdot 10^{-9}$$

$$B_1 = -H_0 \frac{\Im_0(k_1 b)}{\Im_0(k_1 a)\cdot \mathrm{K}_0(k_2 b) - \Im_0(k_1 b)\cdot \mathrm{K}_0(k_2 a)} = (-3.75 + j8.22)\cdot 10^{15}$$

Matlab is now used to find the current density eq. (20), which will be a function of the radius. The current density is integrated in order to find the current, the result in this example is I(f=1.5kHz) = 9Amp. This can be put into eq. (24).

(Observe that the values of H_0 and I do not matter, these variables are only there to make the numerical calculations in Matlab possible)

$$R_1^{1.5kHz} = \frac{\frac{2\pi 0.002}{59\cdot 10^6}\int_{r=a}^{b}\left|\overline{J}_{\varphi}\right|^2 r dr}{|9|^2} = 0.1845\Omega$$

The winding reactance at the specific frequency is found by multiplying the 50Hz reactance that was found by a short circuit test. Which gives $\mathrm{X}_1^{1.5\mathrm{kHz}} = 2.1\frac{1500\mathrm{Hz}}{50\mathrm{Hz}} = 63\Omega$.Thus $\mathrm{Z}_1^{1.5\mathrm{kHz}} = (0.18 + \mathrm{j}63)\Omega$

The same procedure is used for the secondary winding. The only difference being that the winding impedance must be transposed over to the primary side of the transformer. The result is $\mathrm{Z}_2^{'1.5\mathrm{kHz}} = (0.94 + \mathrm{j}63)\Omega$.

The winding resistances and the core impedance can now be put into eq. (43) in order to find the total resistance seen by the harmonic generating load at the frequency 1.5kHz

$$\mathrm{Z}_{\mathrm{tot}}^{1.5\mathrm{kHz}} = \frac{(0.18 + \mathrm{j}63)\cdot(9.6 + \mathrm{j}3)\cdot 10^3}{(0.18 + \mathrm{j}63) + (9.6 + \mathrm{j}3)\cdot 10^3} + (0.94 + \mathrm{j}63) = (1.49 + \mathrm{j}125)\Omega$$

The RMS value of the current that is emitted by the load at the frequency 1.5kHz is $\mathrm{I}_{1.5\mathrm{kHz}} = 25\mathrm{mA}$, which must be transposed to the primary side. This gives $\mathrm{I}'_{1.5\mathrm{kHz}} = 12\mathrm{mA}$ which gives a power loss of $1.49\Omega\cdot 12\mathrm{mA}^2 = 0.2\mathrm{mW}$

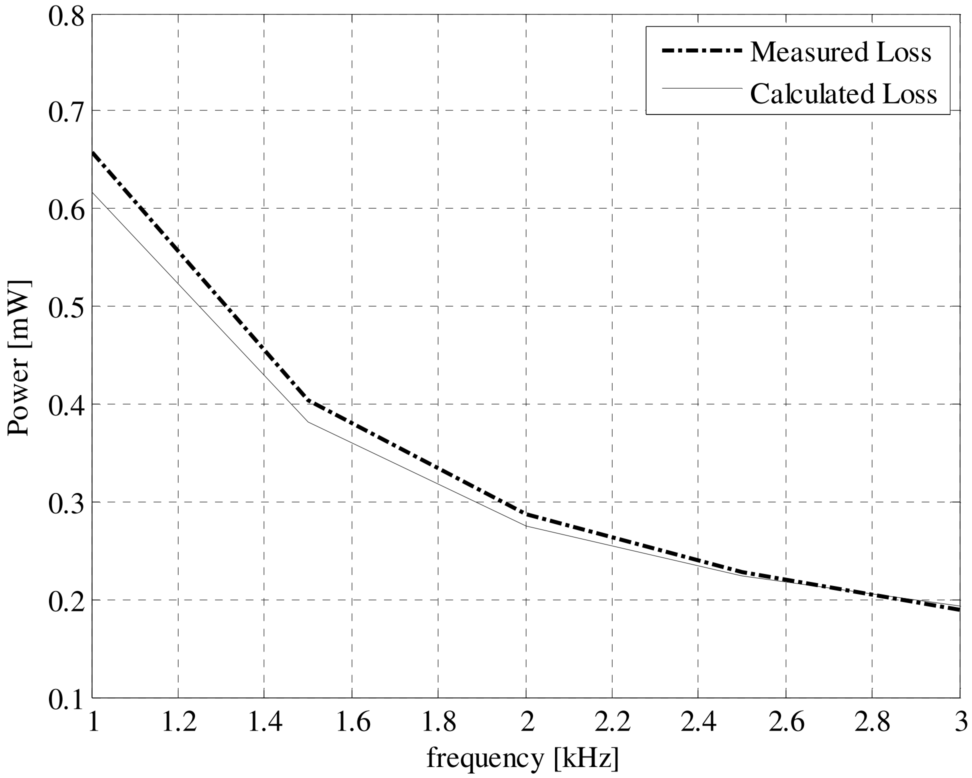

Figure 3.17 Power losses generated by the harmonic. Grid voltage is $14V_{RMS}$

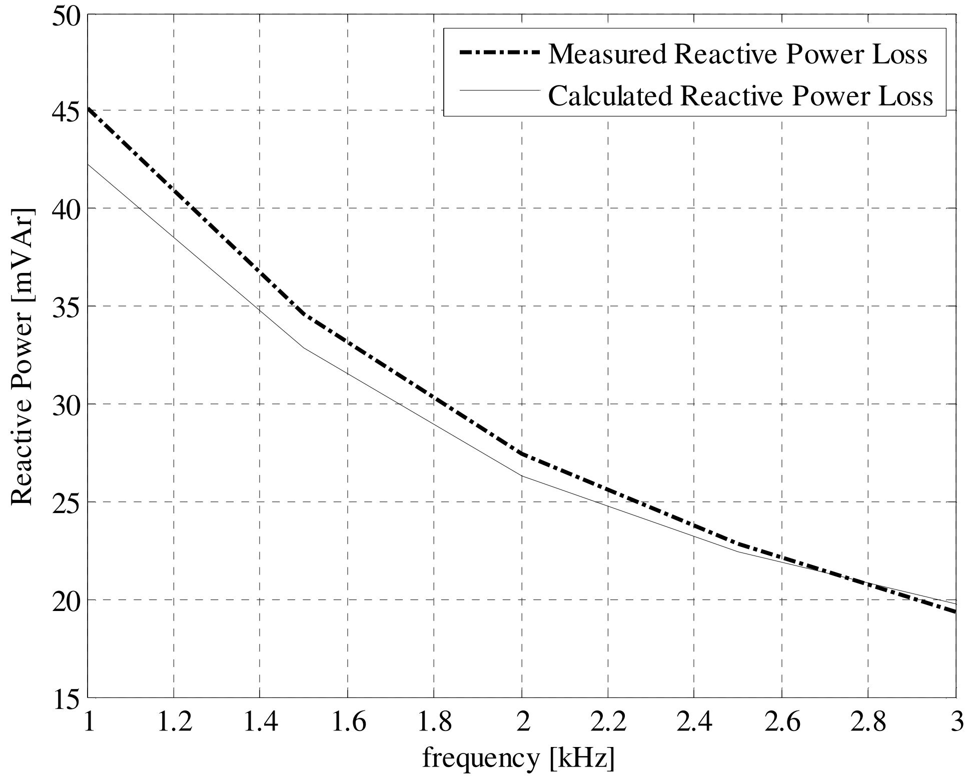

Figure 3.18 Reactive power in the transformer, related to the harmonic. Grid voltage is $14V_{RMS}$

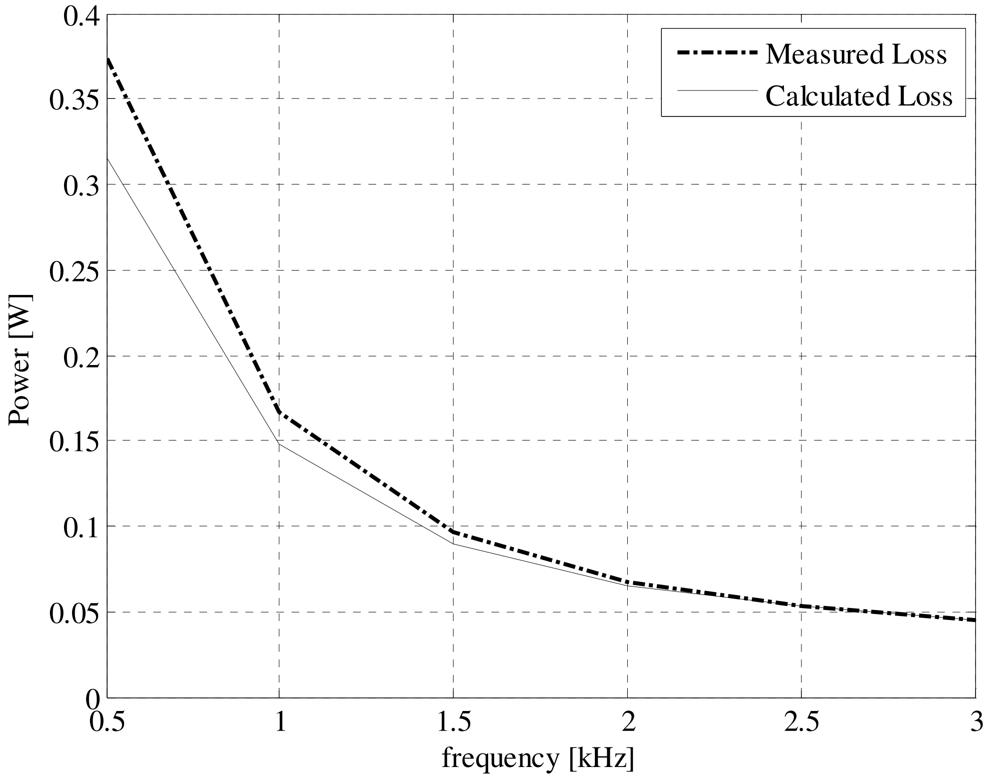

Figure 3.19 Power losses generated by the harmonic Grid voltage is 28V_{RMS}

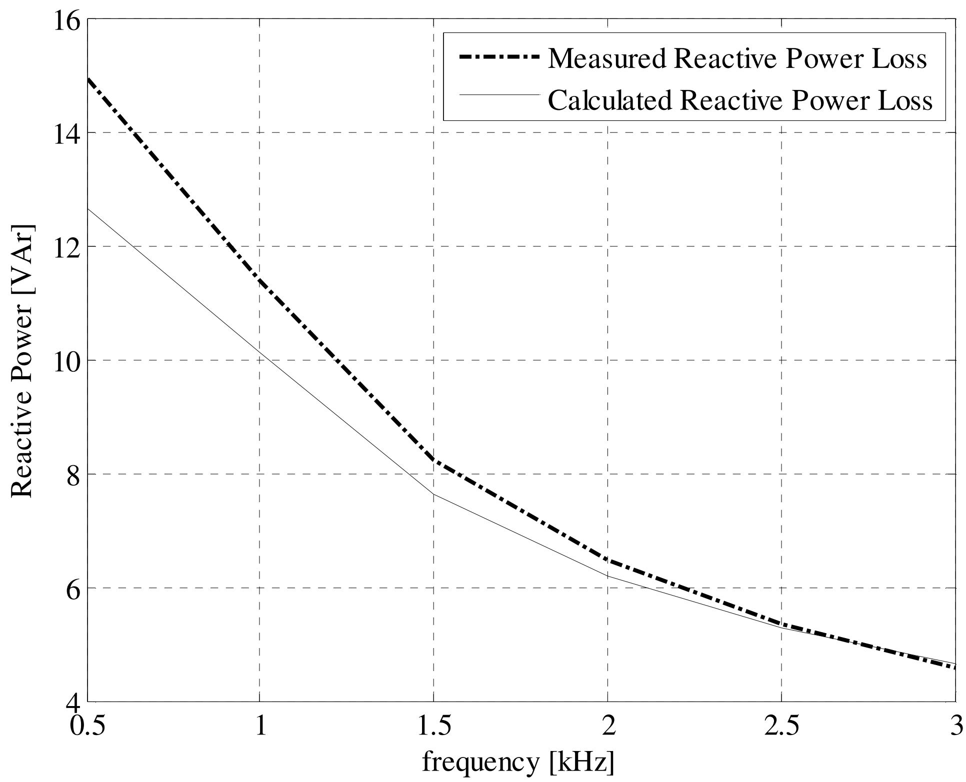

Figure 3.20 Reactive power in the transformer, related to the harmonic. Grid voltage is 28V_{RMS}

The calculated amplitude is ca 10% to low for the lowest frequencies, and appears to be crossing the curve of the measured amplitudes at about 2.7 kHz. Thus the model appears to predicting too low losses for the low frequencies too high losses in the higher frequencies. It is quite possible that this error is due to the varying amplitude of the injected harmonic. The amplitude was higher for the lower frequencies than for the higher frequencies.

Since it was not possible to keep the current amplitude constant in the during the laboratory measurements giving a better picture of the frequency dependence of the transformer impedance was attempted by calculating the impedance that is seen by the harmonic which is injected into the transformer from the load side. This can be achieved by dividing the measured power loss with the square of the measured current $z_{transformer} = \frac{P_{Loss}}{I_{Load}^2}$. The result is shown in Figure 3.21.

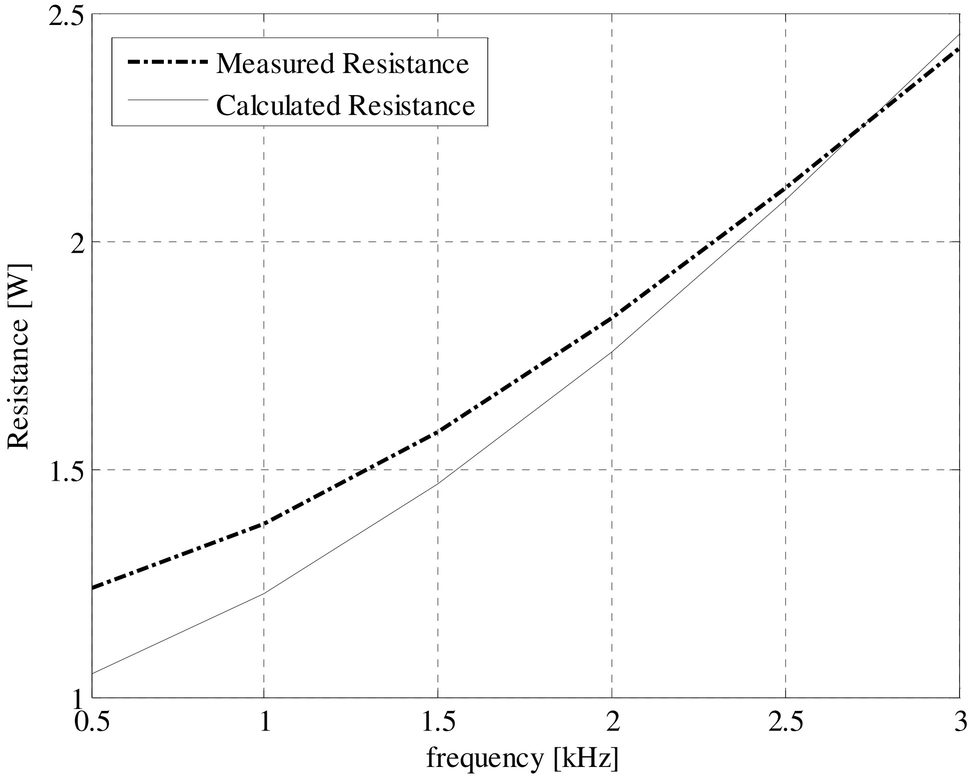

Figure 3.21 Impedance in the transformer, Grid voltage is 28V$_{RMS}$

The results shown in the figure above illustrates how each harmonic that is injected into the transformer will experience different impedance depending on which frequency it has.

3.2 Measurements from a transformer that is operating in the grid

The company Öresundskraft has provided measurements from one of their transformers. The 22kV/420V transformer feeds three 400 Volt frequency converters and they in turn feed three compressors. The converters generate harmonics which will pass through the transformer. These harmonics are in the range 100Hz to 1.25 kHz, and have amplitudes between 60 and 0 Ampere. An active filter is mounted on the load side of the transformer. This filter dampens the harmonics by injecting currents that are of the same frequency and amplitude as the harmonics, but shifted a half period compared to the harmonic. The filter has an average switching frequency of 10 kHz, and has a rated compensation current of 100Amp/phase. The filter will therefore also induce its own losses in the transformer. Since the measurement equipment mounted on the transformer is limited to frequencies below 6 kHz, the filter generated loss is not registered in the measurements.

When the system is operating with filter the loss generating currents that pass through the transformer will consist of three groups of frequencies. When the filter is off only two groups of frequencies are present in the transformer. The different load situations are listed in Table 1.

With Filter	Without Filter
50Hz required by the frequency converter and the filter	50Hz required by the frequency converter
Low order harmonics generated by the frequency converter, partially damped by the filter	Low order harmonics generated by the frequency converter
High order harmonics generated by the filter	

Table 1

3.2.1 Without filter

The spectrum of the power consumed on the loadside of the transformer when the filter is swithced off is shown in Figure 3.22. The measurements register a THD of 65% in the current.

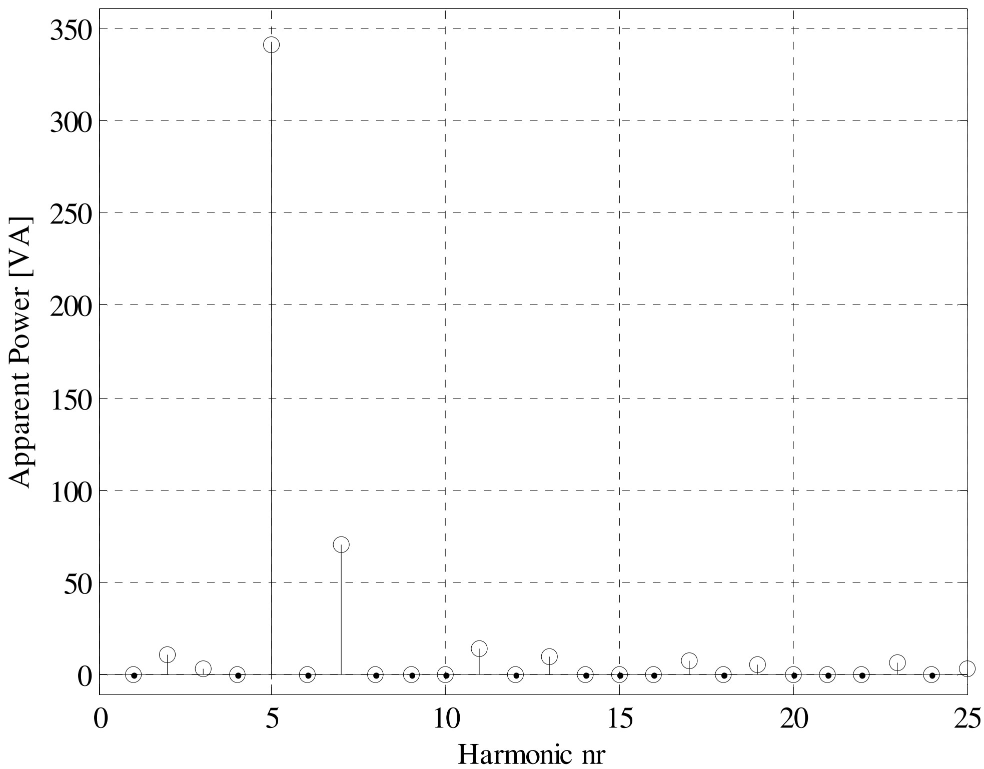

Figure 3.22 Power consumed by the load without a filter, relative to frequency

3.2.2 With filter

The spectrum of the power consumed on the loadside of the transformer when the filter is swithced on is shown in Figure 3.23. Due to the limitations of the measurement equipment the measurements do not show what happens above 2kHz. This makes it impossible to know wethever the filter injects haromics in to the transforer. Harmonics that could originate in the swithing of the filter. With the filter switched on the equipment registers a THD of 15% in the current.

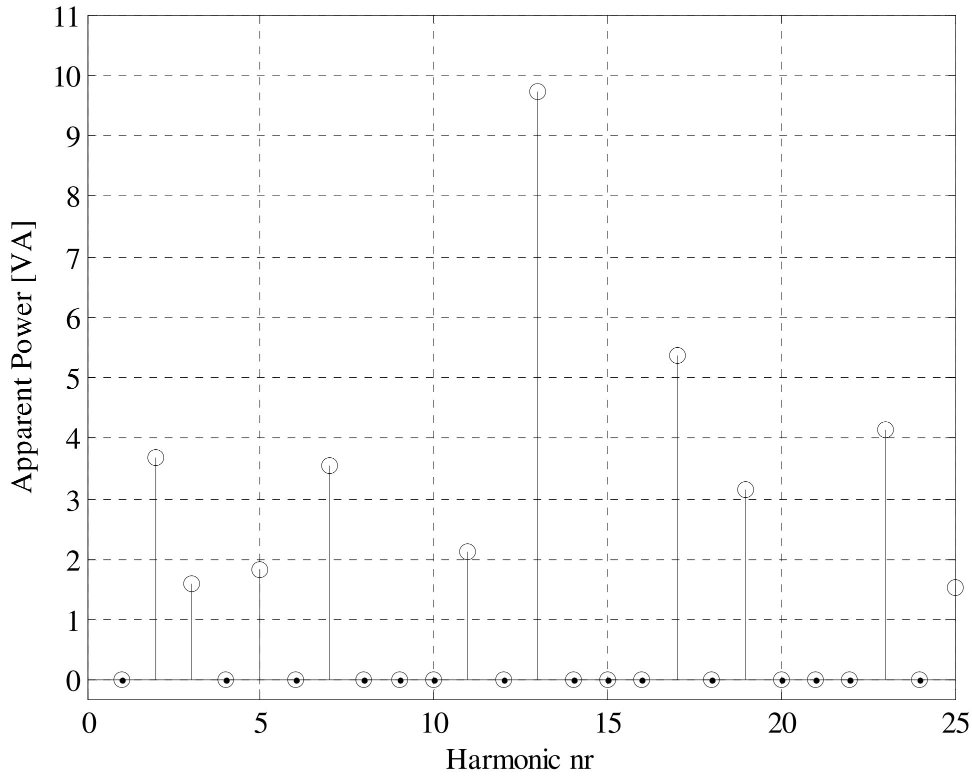

Figure 3.23 Power consumed by the load with filer, relative to frequency

3.2.3 Modeling of infield transformer measurements

In order to estimate how the filter affects the transformer, a hypothetical scenario was used as the base for calculation. The transformer is loaded under two different conditions. In scenario one, a diode rectifier is connected on the load side. The amplitude of the harmonics the rectifier generates, are adjusted so that the THD equals 65%. The same value the infield measurements show when the filter is not activated. In scenario two, the harmonics are adjusted down so that we get a THD of 15% in the current. This will emulate the situation where the filter is switched on. These two scenarios will be used to calculate the reduction in harmonic generated power loss we can expect by switching in the filter. The two load currents are shown in Figure 3.24 and Figure 3.25. The base frequency is included in the plots for comparison.

The reduction in power can be used to investigate if the power consumed by the filter is lower than the power loss induced by the harmonics.

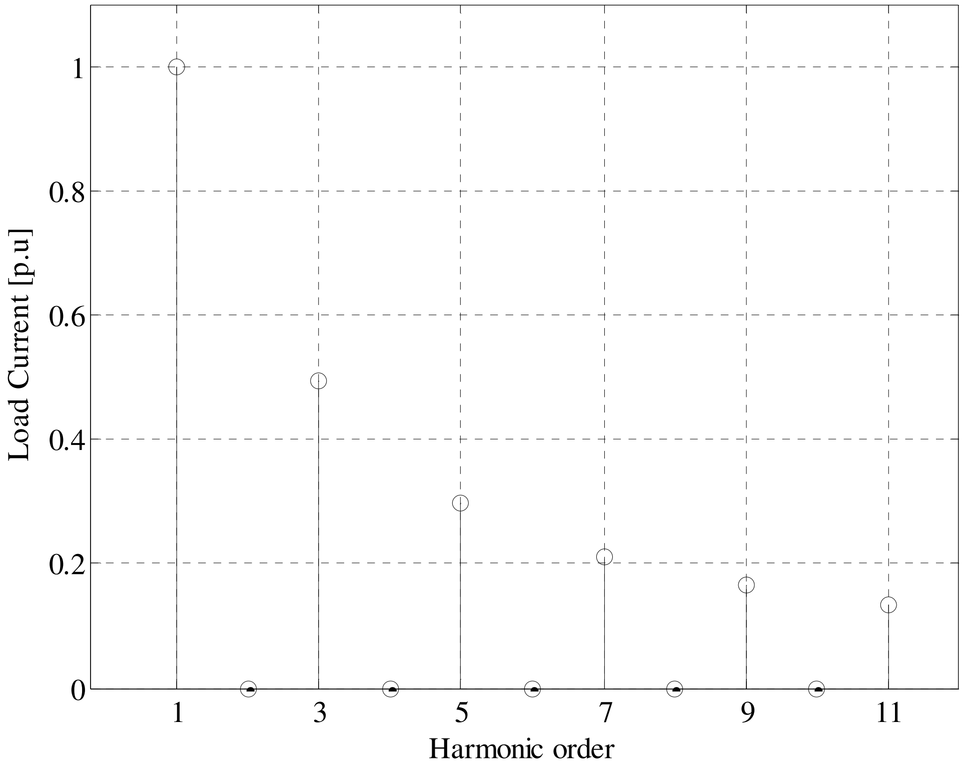

Figure 3.24 Model of currents on the load side when no filter is activated, THD = 65%

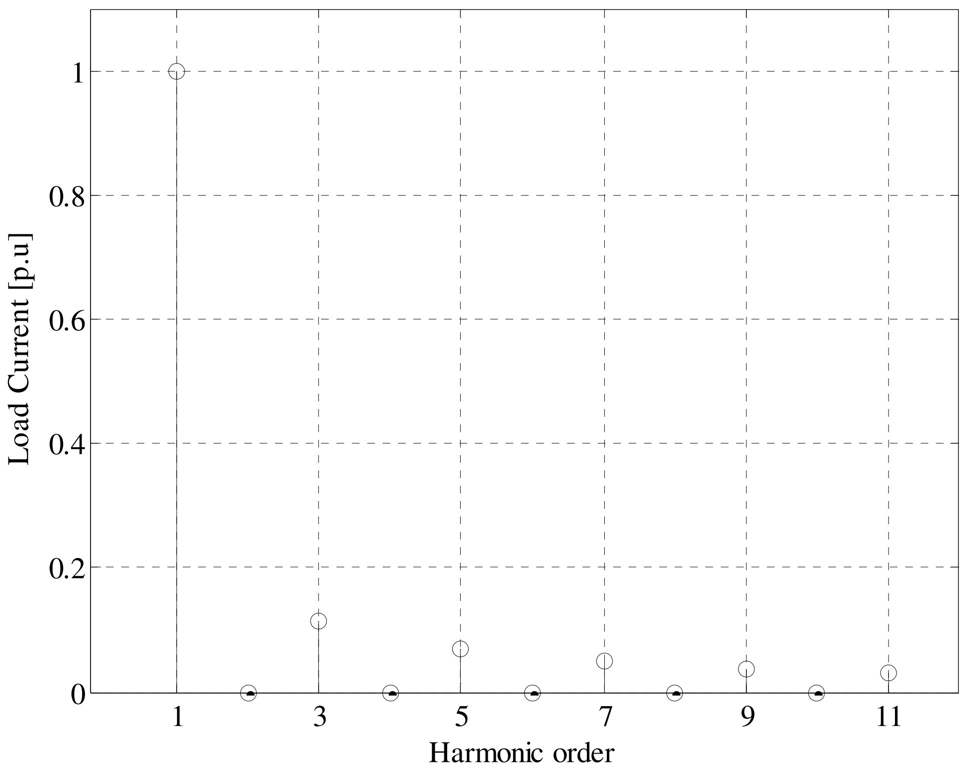

Figure 3.25 Model of the current on the load side when filter is activated, THD = 15%

The power loss generated by the base current was in both of these setups calculated to be 49W. The Power loss generated by the unfiltered load was 4.3W. The power loss generated by the filtered load was 230mW. This means that the power loss is reduced by 8% when the harmonics are damped.

4. Establishing transformer capability when supplying non-sinusoidal load currents

At the present time there are two standards in use for specifying a suitable transformer for a harmonic load. One aims at down-rating a normal transformer to suit a load with relatively large amount of harmonics. The other method aims at designing a transformer with is particularly good at handling the losses associated with high harmonic content in the load.

A third system has also been suggested. The harmonic loss factor (F_{HL}) is a function of the harmonic current distribution and does not take the relative harmonic amplitude relative to the base frequency into consideration. Two transformers of different rated secondary current might therefore have the same F_{HL}. The K_factor on the other hand takes both the magnitude and the distribution into consideration.

4.1 K_{f_USA}

In the U.S the K_{f_USA} is calculated and assigned to a transformer by the manufacturer. The K_{f_USA} serves to grade the capability of the transformer to handle load generated harmonics without exceeding its operating temperature limits. The K_{f_USA} can be used directly to specify a transformer for a given load [8].

The K_{f_USA} of the load must also be calculated in order to determine the transformer requirements.

$$P_t = P_f \sum_{h=1}^{h_{\max}} I_h{}^2 h^2 \tag{47}$$

The K_{f_USA} can be calculated as

$$K_{f\_USA} = \frac{P_t}{P_f} = \sum_{h=1}^{h_{\max}} \left(\frac{I_h}{I_1} \right)^2 h^2 \tag{48}$$

The K_{f_USA} of the load can also be used to estimate the de-rating necessary due to a harmonic load, in a simple two step analysis.

The First step is to calculate the losses at full load. Often the eddy current losses are assumed to be 10% of the total loss, and the copper losses assumed to be the balancing 90%. Total loss at 100% load is given in as

$$P_{100\%} = 0.9 + 0.1 \cdot K_{f\_USA} \tag{49}$$

The next step is to calculate the de-rating. The total loss at X% load is given by the expression

$$P_{lossX\%} = \left(\frac{I_{load}}{I_{nominal}} \right)^2 \cdot P_{100\%} \tag{50}$$

The maximum allowed load with harmonics present is calculated by setting X_{loss} =1 and solving for X_{load}

$$I_{load} = \sqrt{\frac{P_{lossX\%}}{P_{100\%}}} \tag{51}$$

Example

If the K_{f_USA} of the load is calculated to be 6, the total losses using (50) will be $P_{100\%} = 0.9 + 0.1 \cdot 6 = 1.5$. The maximum load of the transformer can now be calculated with (51)

$$\frac{I_{load}}{I_{nominal}} = \sqrt{\frac{1}{1.5}} = 0.8$$

Thus the transformer can be loaded to 80% of nominal current with the given load characteristic.

4.2 K_{f_EU}

The K_{f_EU} is taken as the square root of the total losses in the transformer, and is used to determine how much the transformer must be de-rated for a given harmonic content in the load. This method is mostly used in Europe. Calculation of the K_{f_EU} is done as follows

$$K_{f\_EU} = \left[1 + \frac{e}{1+e}\left(\frac{I_1}{I}\right)^2 \times \sum_{h=2}^{h_{\max}} h^q \times \left(\frac{I_h}{I_1}\right)^2\right]^{0.5} \tag{52}$$

And the result is used to determine the necessary de rating of the transformer

$$X_{load} = \frac{1}{K_{f\_EU}} \tag{53}$$

The amount of down rating that is necessary for a given load is shown in Figure 4.1. Two load profiles where used. One represents a group of lower harmonics (3rd, 5th, 11th, 13th order harmonics), and one represents a group of higher harmonics (38th, 40th, 42th and 44th order harmonics).The amplitude of the harmonics are gradually increased, and the recommended maximum load is plotted against the THD. As expected, the example shows that the higher the frequency of the harmonics the lower is the acceptable load. It also shows that the K_{f_USA} is a much stricter standard than the K_{f_EU}.

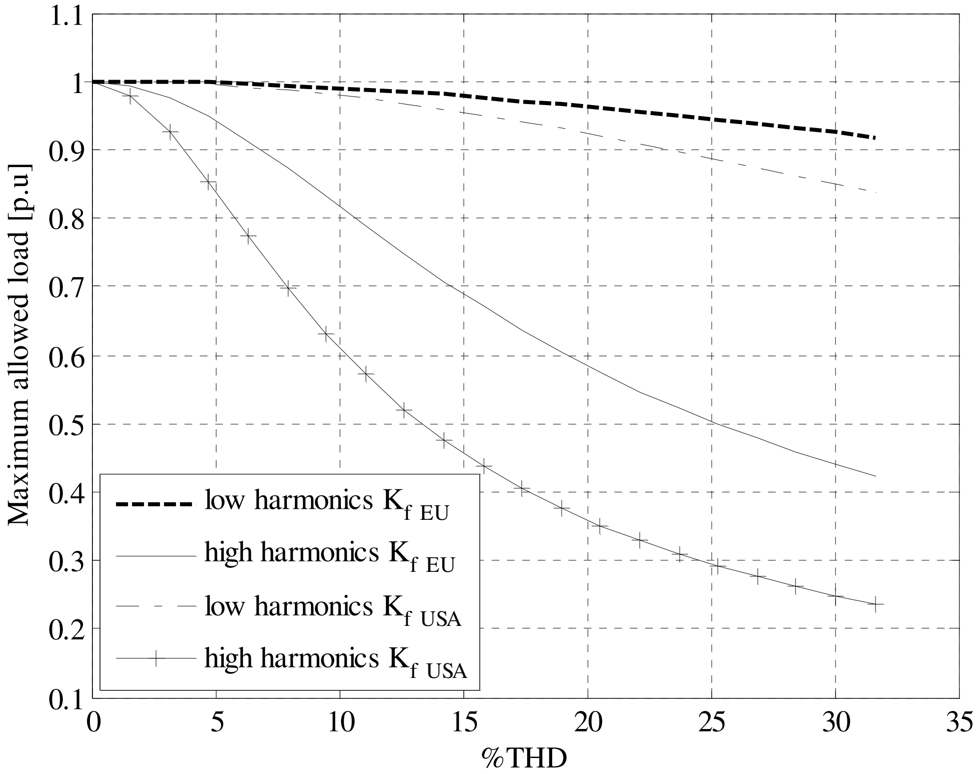

Figure 4.1 Comparison of K_{f_EU} and K_{f_USA}, using two different load profiles

4.3 Harmonic loss factor

This is a recommendation for a method of establishing capability in two winding transformers subjected to sinusoidal load currents with some harmonics present based on [6].

The losses of the transformer are subdivided into excitation loss (power needed to energize the transformer). And stray loss, which is attributed to eddy currents, induced by the stray magnetic field in the windings (P_{EC}), core and other metal parts (P_{OSL}).

The winding eddy current loss (P_{EC}) is considered to be proportional to the square of the amplitude and the square of the frequency.

The other stray losses (P_{OSL}) are also proportional to the square of the current amplitude. But they are related to the frequency by an exponential growth factor of not more than 0.8. The importance of the P_{OSL} is dependent on the type of transformer. In a dry type transformer the heat generated in the core will simply be carried away by the cool air flow. On the other hand, in a liquid filled transformer these losses contribute to the heating of the insulating liquid.

In order to perform these calculations, the characteristics of the load current must be known. The magnitude of the harmonics must be known either in terms of magnitude of the fundamental frequency or as magnitude of the total RMS-current. Additionally information on the winding eddy current loss density must be known.

The RMS load current is given by the following expression

$$I = \sqrt{\sum_{h=1}^{h_{max}} I_h^2} \tag{54}$$

The eddy current loss due to a non sinusoidal load current is estimated by the following equation

$$P_{EC} = P_{EC-R} \sum_{h=1}^{h_{max}} \left(\frac{I_h}{I_1} \right)^2 h^2 \tag{55}$$

Where P_{EC-R} is the winding eddy current loss under rated conditions

The other stray losses are given a conservative approximation as follows

$$P_{OSL} = P_{OSL-R} \sum_{h=1}^{h_{max}} \left(\frac{I_h}{I} \right)^2 h^{0.8} \tag{56}$$

Where P_{OSL-R} is the stray loss under rated conditions

Harmonic loss factor due to winding eddy current is given by the following expression

$$F_{HL} = \frac{\sum_{h=1}^{h_{max}} I_h{}^2 h^2}{\sum_{h=1}^{h_{max}} I_h{}^2} \tag{57}$$

Estimated harmonic loss factor due to other stray losses

$$F_{HL-OSL} = \frac{\sum_{h=1}^{h_{max}} I_h{}^2 h^{0.8}}{\sum_{h=1}^{h_{max}} I_h{}^2} \tag{58}$$

4.4 A Suggestion for Transformer down Rating

A new down rating system is proposed. The system is based on the European down rating standard. The difference is that the power loss calculations presented in this report are substituted for the power loss calculations that are used in the European standard. So we get the following formulae

$$X_{load} = \frac{1}{\sqrt{1 + P_{Loss}}} \tag{59}$$

P_{Loss} is the per unit power loss from the harmonics of the load.

The per unit power loss generated by the load harmonics are calculated by calculating the harmonic generated loss as suggested in cap. 2.3. And then normalizing it by dividing by the power loss that is generated by the 50Hz grid current in the transformer.

By choosing a frequency of 1kHz for the harmonic and then letting the harmonic increase from 0 to 0.4p.u in amplitude, the necessary down rating that the two systems suggests could be compared. The result is shown in Figure 4.2.

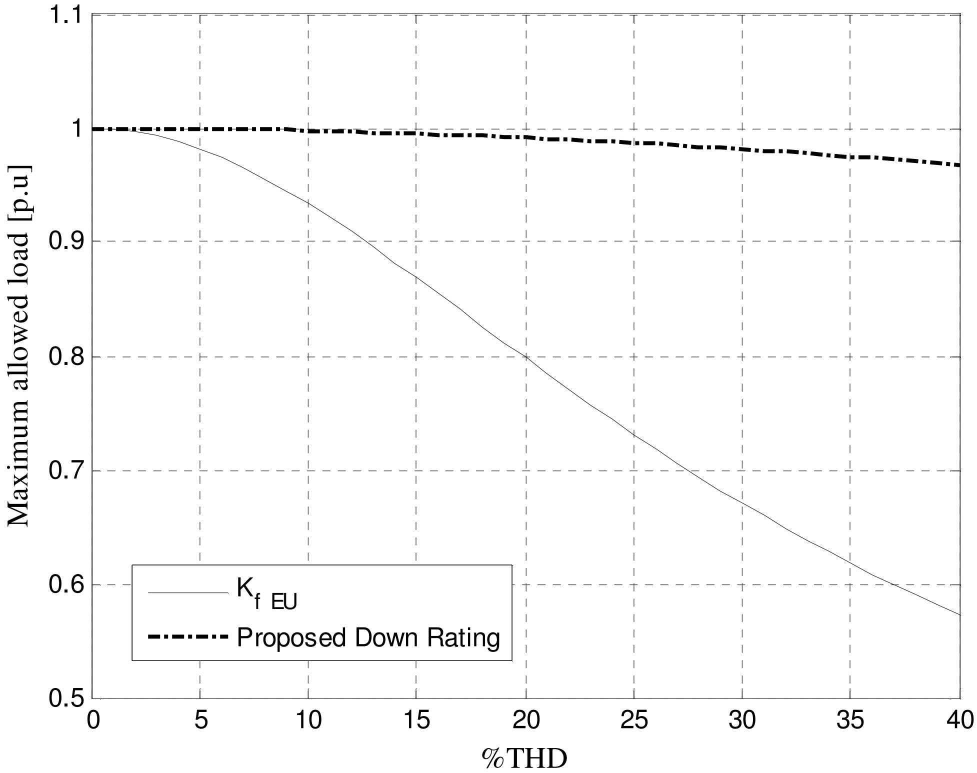

Figure 4.2 Down rating with a harmonic at 1kHz

The proposed down rating standard is much less strict compared the European standard. This is because the analysis shown in this report predicts that the winding resistance will only increase as the square of the frequency for higher frequencies. This can be seen in Figure 2.8.

In order to see how the two systems compared for lower frequencies the calculations were repeated with a harmonic frequency of 150Hz, which correspond to the third harmonic in the grid. The result is shown in Figure 4.3.

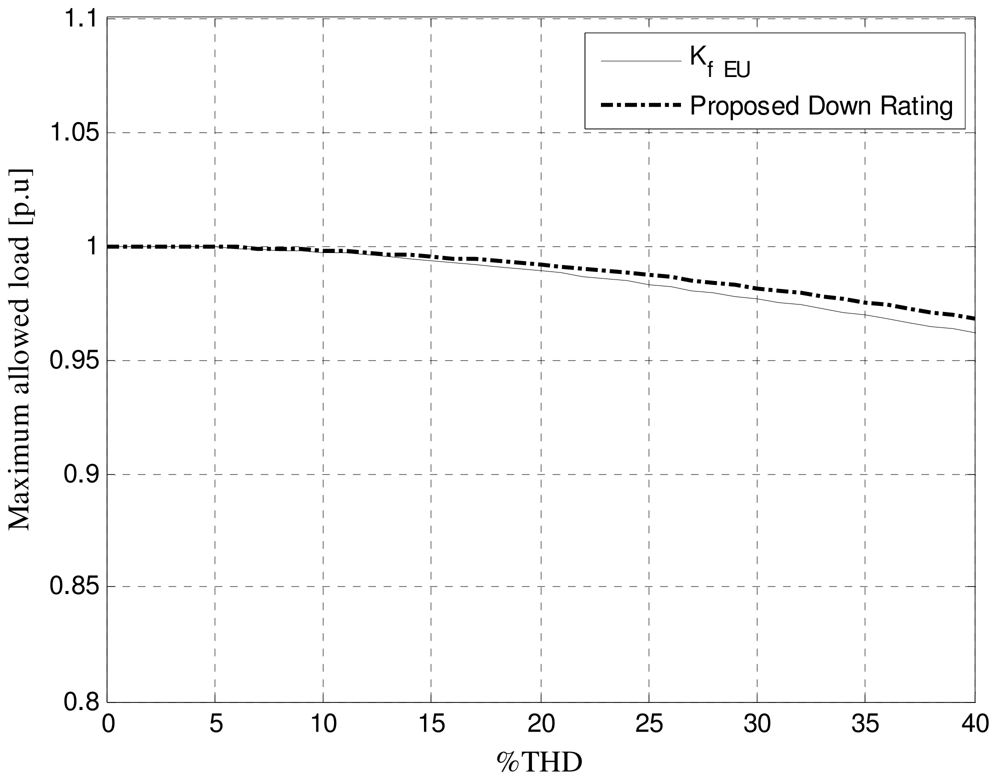

Figure 4.3 Down rating with a harmonic at 150Hz

The figure above shows that the European standard and the proposed standard behave very similarly in the lower frequency register. This is as expected since the European standard assumes that the effect loss increases as the harmonic order to the power of 1.7, and the analysis of the resistance in the copper windings show that the resistance increases as the square of the frequency for the for the lower harmonics Figure 2.7.

5. Cost

5.1 Use of amorphous material in transformer cores

A transformer built with amorphous steel in the core will have lower core losses. The copper losses of the windings will on the other hand remain unchanged. To gain a view of the savings made by using amorphous steel in the transformer core, the core losses of one conventional transformer can be compared to those of a transformer with an amorphous steel core.

To make this comparison two operating scenarios were imagined. The first scenario is when the transformer is loaded with a simple resistive load. Scenario number two is that the transformer is loaded by a diode rectifier; the diode rectifier will generate low order harmonics.

In the examples the previously researched lab transformer was used as virtual test object. For the calculations involving amorphous steel the dimensions and the laminate thickness were kept unchanged, only the permeability and conductivity are changed.

For amorphous steel the following values were used:

Relative permeability $\mu_r = 600 \cdot 10^3$

Conductivity $\sigma = 770 \cdot 10^3 \Omega^{-1} m^{-1}$

Example

The transformer is loaded with resistive load of 12ohm on the low voltage side, and the high voltage side is fed by the grid (220Volt, 50Hz).

The resistance seen by the grid is given by eq. (42)

$$Z_{tot}^{grid}(\omega_1) = \frac{(0.6 + j2.1 + 48)\cdot(795 + j929)}{(0.6 + j2.1)\cdot + 48 + (795 + j929)} + (0.58 + j2.1) = (48 + j5)\Omega$$

Now the apparent power drawn from the grid can be calculated by $S = \frac{U_{grid}^2}{Z_{tot}^{*grid}}$ which gives $P_{grid}^{st} = 999W$ and $Q_{grid}^{st} = 114VAr$

The current drawn from the grid will be $I_{grid} = \frac{U_{grid}}{Z_{tot}^{*grid}}$ which gives $I_{grid} = 4.5Amp$ with an angle of 6°.

The current flowing through the core can be calculated with a current divider $I_{core} = I_{grid} \frac{Z_2 + Z_{load}^{'}}{Z_2 + Z_{load}^{'} + Z_{Fe}}^{*}$ which gives $I_{core} = 177mA$ with an angle of 51°. This gives the core loss $P_{core}^{st} = 25W$ and $Q_{core}^{st} = 31VAr$.

The load power can be found by first finding the load current $I_{load}^{'} = I_{grid} - I_{core}$ and then the load power by $S_{load} = Z_{load}^{'*} I_{load}^{'2}$. Which gives $P_{load} = 936W$.

The winding losses can now easily be found by $P_{winding} = P_{grid} - P_{core} - P_{load}$ which equals 24W.

The calculations are now repeated but with the permeability and conductivity of amorphous steel used for the calculations of the equivalent core impedance. The equivalent core impedance will in this case be $Z_{Fe}^{am} = (48 + j27)\text{kOhm}$.

The power drawn from the grid will now be $P_{grid}^{am} = 977\text{W}$ and$Q_{grid}^{am} = 83\text{VAr}$. The losses in the amorphous core will be $P_{core}^{am} = 0.7\text{W}$ and$Q_{core}^{am} = 0.5\text{VAr}$.

The calculations above show that by going from a steel core to an amorphous core the core losses would be reduced from $\frac{25\text{W}}{24\text{W}+25\text{W}}100\%$=51% to $\frac{0.7\text{W}}{24\text{W}+0.7\text{W}}100\%$=2% of the total losses. The total losses were reduced by 48% by going from transformer steel to amorphous steel.

Example

The transformer is loaded by a diode rectifier. The load draws the same current as in the previous example; additionally, current harmonics are generated by the rectifier. The harmonics will generate losses in the transformer.

Five harmonics are used in the calculation, nr 3, 5, 7, 9 and 11. The amplitude of the harmonic is given as one through the harmonic order. For example the 3rd current harmonic has the amplitude $I_3 = \frac{1}{3} I_1$ Where I_1 is the amplitude of the fundamental current.

First the calculations are done with the steel core.

The impedance seen by the load is given by eq. (43) and gives the following values for the five different frequencies $Z_3^{st} = (0.67 + j23\,)\Omega$, $Z_5^{st} = (0.67 + j27\,)\Omega$, $Z_7^{st} = (0.67 + j32)\Omega$, $Z_9^{st} = (0.68 + j36)\Omega$ and $Z_{11}^{st} = (0.68 + j40\,)\Omega$.

The power loss generated by each current harmonic in the transformer can now be calculated by $S_h^{st} = Z_h^{st} \cdot I_h^2$. The power loss in the transformer due to the harmonics is $P_{harmonics}^{st} = 2.6\text{W}$ and $Q_{harmonics}^{st} = 104\text{VAr}$.

The core losses can be found by first determining the current through the core as follows $I_h^{core} = \frac{Z_1}{Z_1 + Z_{Fe}} I_h$. And then use the current and the impedance of the core for each harmonic to determine the power. The result is $P_{core}^{st} = 82\text{mW}$ and $Q_{core}^{st} = 56\text{mVAr}$.

The calculations were repeated for amorphous steel.

The impedances seen by the load are the following $Z_3^{am} = (0.62 + j23\,)\Omega$, $Z_5^{am} = (0.63 + j27\,)\Omega$, $Z_7^{am} = (0.63 + j31)\Omega$, $Z_9^{am} = (0.63 + j36)\Omega$ and $Z_{11}^{am} = (0.63 + j40\,)\Omega$.

The power loss due to the harmonics in the transformer with the amorphous core is $P_{harmonics}^{am} = 2.4\text{W}$ and $Q_{harmonics}^{am} = 103\text{VAr}$.

And the core losses are $P_{core}^{am} = 2.9\text{mW}$ and $Q_{core}^{am} = 1.6\text{mVAr}$.

A comparison of the losses in the transformer with the standard steel core and the one with the amorphous steel core shows that the amorphous core reduces the core losses with 96%. And that the total losses related to the harmonics decreases by 8%.

6 Conclusion

The mathematical model of the transformer appears to come close to the measured values. The measurements fit better for the resistive part compared to the reactive part. However, the no load measurements performed for this report only reach frequencies just below the fifth harmonic of the grid frequency. In order to verify the theory, more accurate measurements with a greater frequency spectrum and different levels of saturation are needed.

The theory concerning winding losses is harder to verify since the magnetic field will interact with any metal in the vicinity and therefore gives rise to extra losses. Therefore the model of the winding resistance can only be tested by testing the whole transformer and comparing the test results with the complete model of the transformer. The tests in this report show that the model predicts the loss to frequency relation quite well. But the magnitudes of the measured loss and the calculated loss do not coincide fully. Since the transformer was not operated at flux levels approaching saturation further tests are necessary in order to test the theory for operating conditions more closely resembling those a real transformer would experience.

The theory cannot be completely accurate since it does not take the shape of the transformer core into consideration. If the core has tight corners the magnetic flux will concentrate there and generate relatively high losses.

The most common down rating standards assume that power loss due to harmonic loads increase by the square of the harmonic order. However, the Harmonic loss factor suggests that power loss does not necessarily increase that rapidly. Furthermore, the analysis presented in this report suggests that an increase of power loss proportional to $f^{1/2}$ is possible for higher frequencies. This in turn suggests that the traditional standards of down rating might be too strict.

The losses generated by load generated harmonics appear to be located in the windings rather than the core. Therefore, the use of low loss cores does not appear to be the most efficient way to reduce the harmonic generated losses. The way to reduce these losses would instead be to use more finely stranded conductors in the windings.

The use of low loss cores appear to be most efficient in reducing the no load losses where almost all the losses are core losses.

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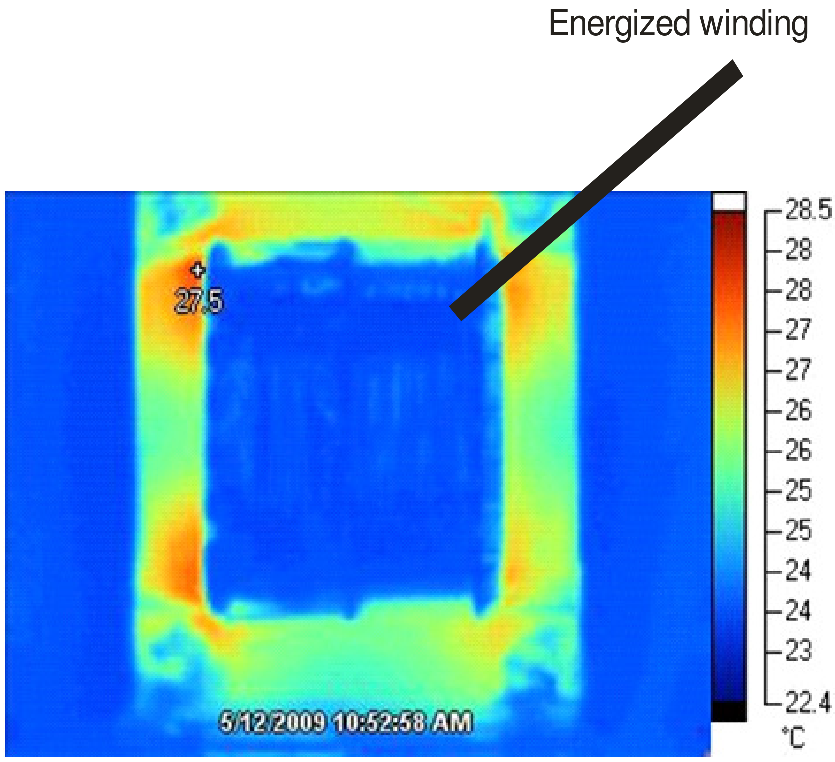
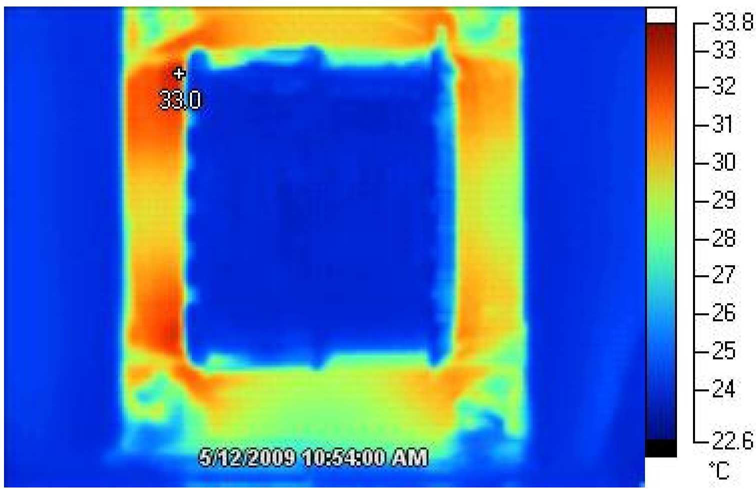
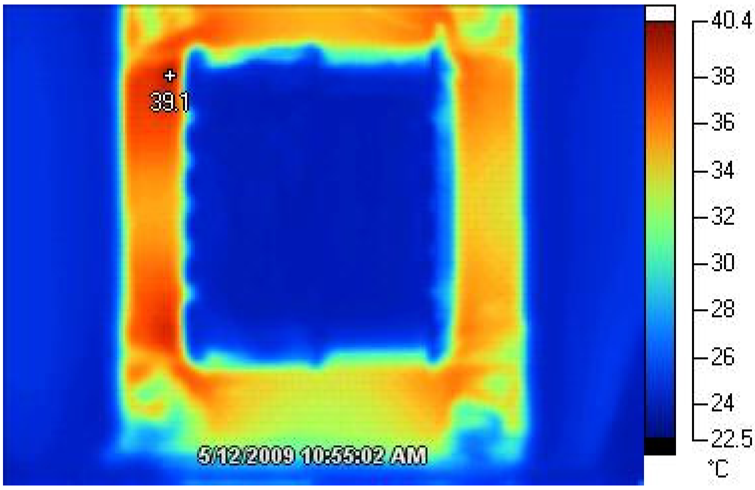
Appendix-A

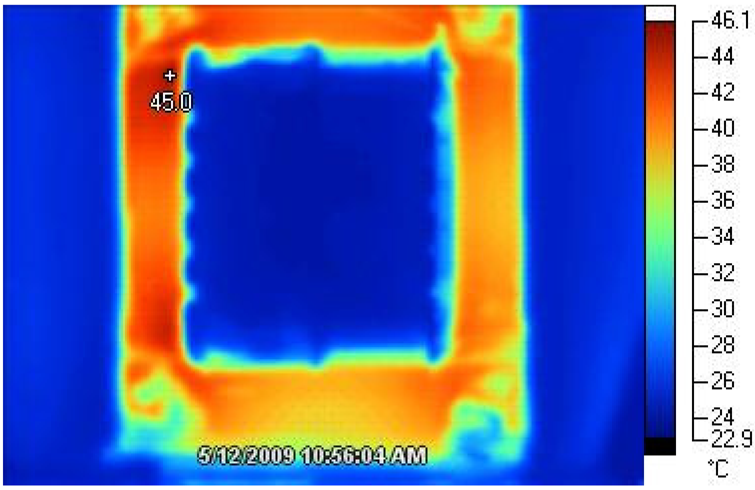
A-1 Infrared picture of heat development in small lab transformer

Transformer in no load-mode, magnetized by880Volts RMS 400Hz. Pictures taken with one minute interval, first picture taken 30sec after power switch on.



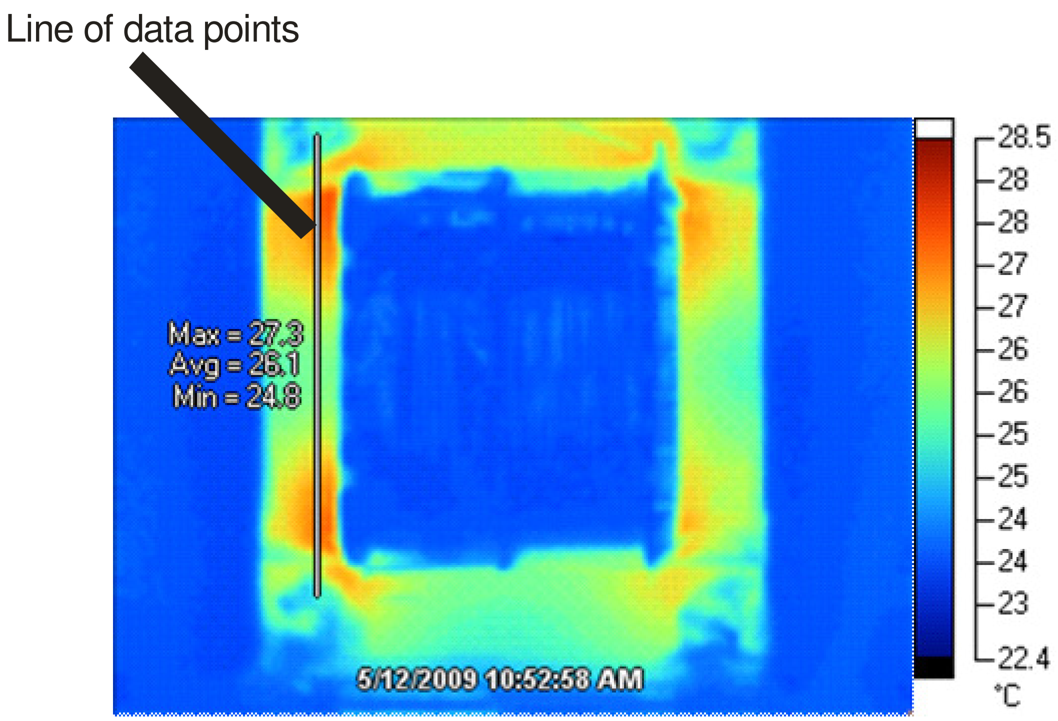


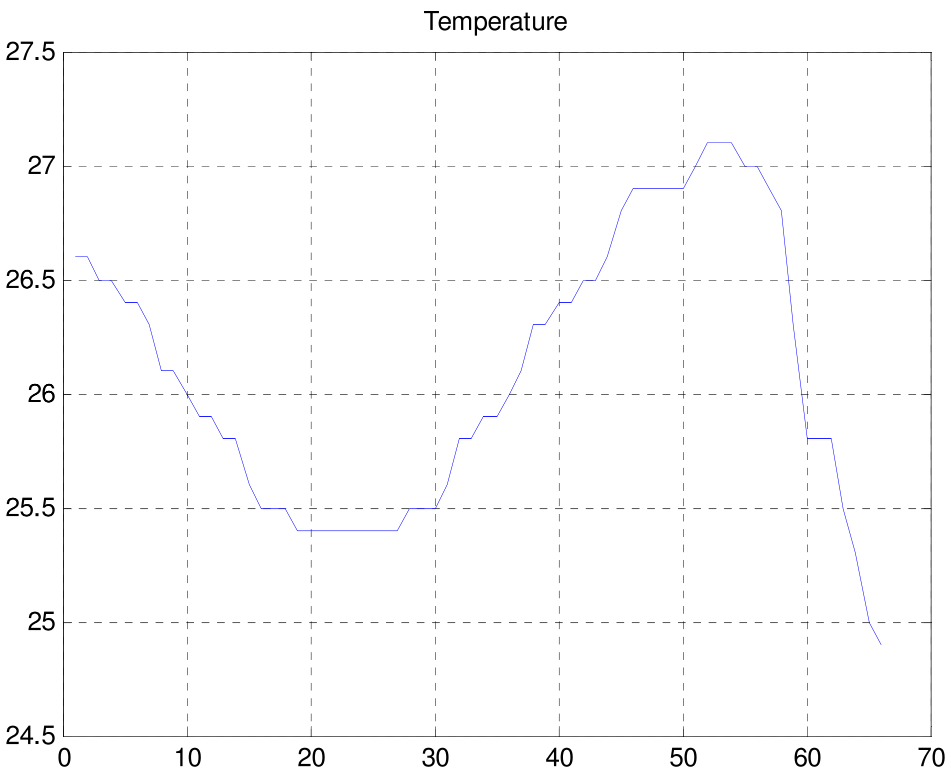

39.1
5/12/2009 10:55:02 AM
40.4
38
36
34
32
30
28
26
24
22.5
°C


45.0
5/12/2009 10:56:04 AM
46.1
44
42
40
38
36
34
32
30
28
26
24
22.9
°C

A-2 Plot of temperature along the side of a transformer core

Transformer in no load-mode, magnetized by880Volts RMS 400Hz.





Appendix-B

B-1 Matlab code, equivalent impedance of the core

```
ur = 5500;   %relativ permeablity
u0 = pi*4e-7;%vacum permeablity
mu = ur*u0;  %relativ * vacum permeablity
sigma = 6.25e6;     %conductivity
c     = 4e-4;       %laminate thickness
Aria = (63e-3)*(78e-3);
path_length = 2*(12e-2 + 7e-2);
R0 = path_length/(Aria*mu);%DC-reluctance
N_sec = 110;        %number of windings
phi = 57*(pi/180);  %phase shift
alpha = (pi/2)-phi; %angle of heel of the elipse
w     = 2*pi*freq;
delta = sqrt(2./(w.*sigma*mu));            %skin depth
u1    = 0.5*sqrt(2)*(c./delta).*exp(j*(pi/4-alpha/2));
fc    = u1.*exp(j*alpha).*coth(u1);
fcr   = real(fc); fci=imag(fc);fca=abs(fc);
ZF    = j*w*N_sec^2./(R0.*fc);             %Equivalent inductance
ZFr   = real(ZF); ZFi=imag(ZF);            %of the core
psi   = angle(ZF).*180/pi; Za=abs(ZF);
```

B-2 Matlab code, resistance of transformer winding

```
N_prim = 200;       %Number of windings
sigma  = 5.99e7;    %conductivity
mu     = 4*pi*1e-7; %permeability
a      = 0.045;     %distance from center to iner side of the iner winding
b      = 0.047;     %distance from center to outer side of the iner winding
L      = 0.002;     %Height of the winding conductor
H0     = 9.0/L;     %External field, the value of this variable is
insignificant
%
for n = 1:1:n_max;
    w     = 2*pi*freq(n);
    delta = sqrt(2/(w*sigma*mu));% Skin depth

    k1 = sqrt(2)*j^(3/2)/delta;
    k2 = sqrt(2)*j^(1/2)/delta;
    A1 =  H0*besselk(0,k2*b)/(besselj(0,k1*a)*besselk(0,k2*b)-
besselj(0,k1*b)*besselk(0,k2*a));
    B1 = -H0*besselj(0,k1*b)/(besselj(0,k1*a)*besselk(0,k2*b)-
besselj(0,k1*b)*besselk(0,k2*a));
    %
    r = a:1e-5:b;
    Jfi = A1*k1*besselj(1,k1*r) + B1*k2*besselk(1,k2*r);%Current density
    Jfia = abs(Jfi);
    Jfia2 = Jfia.^2;
    i = L*trapz(r,Jfi);       %integral of the current density
    ia = abs(i);
    pv = (2*pi*L/sigma)*r.*Jfia2; %effect los per unit of volume
    p(n) = trapz(r,pv);         %effect loss in one turn
    Z1(n) = N_prim*p(n)/(ia^2); %resistance of winding of N number of turns
end
```