Electrical limiting factors for wind energy installations

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Abstract

In this thesis the electrical limiting factors for installation of wind turbines stated by the Swedish connecting requirements, AMP, have been used to determine which types of power quality problems that will dominate when fix-speed wind turbines are installed. The main limiting factors are static voltage level influence and the flicker emissions. The investigation is based on field measurements on a stall-regulated and on a pitch-regulated wind turbine.

It is found that the limiting factor from the power quality point of view is the flicker emissions if one turbine is installed. If three or more turbines are installed it is the static voltage variations that sets the limit, if the summation formula from AMP is used. It is found that the XR-ratio of the grid plays a very large influence on the installation possibility, and a ratio of 1.3-2.8 is the most favourable, depending on the short circuit ratio.

The results shown in this thesis can be used to determine if there will be any problem with the static voltage level or the flicker emissions when stall-regulated wind turbines are connected to the network.
Acknowledgements

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<td>Primary current of the distribution transformer</td>
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<td>$i_{dv}$</td>
<td>Current in d-direction from the distribution transformer</td>
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<tr>
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<td>Current in q-direction from the distribution transformer</td>
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<td>$i_k$</td>
<td>Short circuit current</td>
<td>[A]</td>
</tr>
<tr>
<td>$j$</td>
<td>Imaginary unit $= \sqrt{-1}$</td>
<td></td>
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<tr>
<td>$K_{Sk}$</td>
<td>Short circuit ratio $= S_k / P_{max}$</td>
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<tr>
<td>$k$</td>
<td>Constant</td>
<td></td>
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<td>$L$</td>
<td>Inductance</td>
<td>[H]</td>
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<td>$L_{tot}$</td>
<td>Equivalent inductance of the network</td>
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<td>$N$</td>
<td>Number of turbines</td>
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<td>$n_{best}$</td>
<td>Position to the XR-ratio in the XR-vector that gives the smallest voltage variations</td>
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<td>$P_{st}$</td>
<td>Short-term flicker index</td>
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<td>Sorted IFL value that is exceeded for 0.1% of the time</td>
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<td>Sorted IFL value that is exceeded for 1% of the time</td>
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<td>$P_3$</td>
<td>Sorted IFL value that is exceeded for 3% of the time</td>
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<td>$P_{10}$</td>
<td>Sorted IFL value that is exceeded for 10% of the time</td>
<td></td>
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<td>$P_{50}$</td>
<td>Sorted IFL value that is exceeded for 50% of the time</td>
<td></td>
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<td>$P_1$</td>
<td>Load losses in the distribution transformer</td>
<td>[W]</td>
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<tr>
<td>$P_{max}$</td>
<td>Rated power of the wind turbine</td>
<td>[W]</td>
</tr>
</tbody>
</table>
\( P \)  
Active power from the wind turbine  \([\text{W}]\n\)

\( P_{\text{pu}} \)  
Active power from the wind turbine in per unit, \(1 \text{ pu}=P_{\text{max}}\)

\( P_{0,\text{pu}} \)  
Mean active power from the wind turbine in per unit

\( \Delta P_{\text{pu}} \)  
Variations in the active power from the wind turbine in per unit

\( Q \)  
Reactive power from the wind turbine  \([\text{VA}\text{r}]\n\)

\( Q_{\text{pu}} \)  
Reactive power from the wind turbine in per unit, \(1 \text{ pu}=P_{\text{max}}\)

\( Q_{0,\text{pu}} \)  
Mean reactive power from the wind turbine in per unit

\( \Delta Q_{\text{pu}} \)  
Variations in the reactive power from the wind turbine in per unit

\( R_k \)  
Short circuit resistance  \([\Omega]\n\)

\( R_{kTr} \)  
Series resistance of the distribution transformer  \([\Omega]\n\)

\( R_{Fe} \)  
Core loss resistance in the distribution transformer  \([\Omega]\n\)

\( R_{s1} \)  
Primary resistance of the distribution transformer  \([\Omega]\n\)

\( R_{s2} \)  
Secondary resistance of the distribution transformer  \([\Omega]\n\)

\( R_{\text{line}} \)  
Series resistance of the distribution line  \([\Omega]\n\)

\( R_{\text{tot}} \)  
Equivalent resistance of the network  \([\Omega]\n\)

\( S \)  
Apparent power from the wind turbine  \([\text{VA}]\n\)

\( S_k \)  
Short circuit power  \([\text{VA}]\n\)

\( S_n \)  
Rated power of the distribution transformer  \([\text{VA}]\n\)

\( t \)  
Time  \([\text{s}]\n\)

\( U_{r}, U_{s}, U_{i} \)  
Phase voltage  \([\text{V}]\n\)

\( U_1 \)  
Primary voltage of the distribution transformer  \([\text{V}]\n\)

\( U_2 \)  
Secondary voltage of the distribution transformer  \([\text{V}]\n\)

\( U_n \)  
Rated voltage of the distribution transformer  \([\text{V}]\n\)

\( U_e \)  
Phase to phase voltage of infinity strong grid  \([\text{V}]\n\)

\( U_0 \)  
Main frequency component of the voltage  \([\text{V}]\n\)

\( U_{\text{ref}} \)  
Reference voltage  \([\text{V}]\n\)

\( U_{v0} \)  
Voltage reference vector  \([\text{V}]\n\)

\( u(t) \)  
Time varying voltage  \([\text{V}]\n\)

\( u_v \)  
Phase to phase voltage on the high voltage side of the local wind turbine transformer  \([\text{V}]\n\)

\( u_{dv} \)  
Phase to phase voltage in d-direction on the high voltage side of the local wind turbine transformer  \([\text{V}]\n\)

\( u_{qv} \)  
Phase to phase voltage in q-direction on the high voltage side of the local wind turbine transformer  \([\text{V}]\n\)
\( \Delta u_v \) Variations in the phase to phase voltage on the high voltage side of the local wind turbine transformer [V]

\( v(t) \) Input voltage to the flicker meter [V]

\( Var \) Variable

\( Z_k \) Short circuit impedance [\( \Omega \)]

\( Z_{kTr} \) Series impedance of the distribution transformer [\( \Omega \)]

\( Z_b \) Base impedance of the distribution transformer [\( \Omega \)]

\( Z_{line} \) Series impedance of the distribution line [\( \Omega \)]

\( Z_{tot} \) Equivalent impedance of the network [\( \Omega \)]

\( Z_{bel} \) Magnitude of the equivalent impedance of the network = |\( Z_{tot} \)| [\( \Omega \)]

\( XR_{ratio} \) XR-ratio = \( X_{tot} / R_{tot} \)

\( XR \) Vector with values of XR-ratios

\( X_k \) Short circuit reactance [\( \Omega \)]

\( X_{kTr} \) Series reactance of the distribution transformer [\( \Omega \)]

\( X_m \) Magnetising reactance of the distribution transformer [\( \Omega \)]

\( X_{s1} \) Primary reactance of the distribution transformer [\( \Omega \)]

\( X_{s2} \) Secondary reactance of the distribution transformer [\( \Omega \)]

\( X_{line} \) Series reactance of the distribution line [\( \Omega \)]

\( X_{tot} \) Equivalent reactance of the network [\( \Omega \)]

\( \psi_k \) Short circuit angle [\( \text{rad} \)]

\( \varphi(t) \) Time varying phase angle [\( \text{rad} \)]

\( \omega \) Angular frequency [\( \text{rad/s} \)]

\( \omega_1, \omega_2, \omega_3, \omega_4 \) Constants

\( \lambda \) Constant
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1 Introduction

When installing a wind turbine to the network in Sweden the connection fee can be up to 20% of the total investment cost. This is due to the fact that in Sweden it is the utilities that have to reinforce the grid if problems with the power quality occur once a wind turbine has been connected. In order to be on the safe side, the fee taken by the utility is as mentioned rather high. To reduce the connection fee, it must be shown that the power quality impact will be acceptable. In order to do this more studies and more knowledge of which power quality problems that might be a problem are needed.

It has been noted that the grid short circuit power and the XR-ratio of the grid have a great influence on the power quality impact from wind turbines [3]. It has been shown that the power quality impact from wind turbines is inversely proportional towards the short circuit ratio [4, 5, 6]. A minimum for the power quality impact occurs for XR-ratios between 1.7 and 2.7 [5, 6].

The purpose with this work is to investigate which kind of power quality problems that limits a wind turbine installation. Two types of turbines are investigated: A stall-regulated and a pitch-regulated turbine with induction generators directly connected to the grid. Moreover, a goal is to determine which types of networks, such as overhead-lines, ground cables, strong and weak networks that cases which types of power quality problems.

The work has been divided into five parts. In chapter 2 the necessary theory for this work is presented. In chapter 3 a short presentation of the two measurement sites used in this work are presented. In chapter 4 the analysis is performed and in chapter 5 the conclusions are drawn.
2 Theory
In order to evaluate the impact from wind turbines on the power quality, models of the network and knowledge of which network parameters that can be expected in the point of common connection (PCC), the point in the network where the local wind turbine transformer is connected to the network, are needed. The limits for what is good power quality varies from country to country and a situation that might be acceptable at one place may not be acceptable at another place.

2.1 Limiting factors for wind turbine installations
In this report the focus is to study impact of smaller installations where it is necessary to utilise the existing lines. Further, the first customer is considered to be located on the high-voltage side of the local wind turbine transformer, so the power quality regulations must be complied at this point. To determine whether the power quality impact is acceptable the Swedish connection requirements AMP (Anslutning av mindre produktions anlägningar till elnätet) [1] is used. The limits used are for the steady-state voltage level and for rapid RMS voltage variations (flicker).

2.1.1 Steady state voltage level
Slow variations in the voltage level caused by wind turbines are more or less regular and depend on variations in the power production. All types of wind turbines produce these variations although they can be smaller from those with power electronic converters. According to AMP [1] wind turbines connected by a local transformer to a 10 kV grid should not cause slow voltage variations of the RMS voltage that exceed 2.5% in the PCC.

2.1.2 Flicker
Voltage fluctuations are fast variations in the RMS voltage. These can be sufficient in duration and frequency to allow visual observations of a change in the intensity of electric light source. These light intensity variations are called flicker. Humans are especially sensitive to luminance fluctuations around 8,8 Hz. Due to the human factor it is not easy to calculate a flicker value. The IEC (International Electromechanical Commission) has published a norm for a flicker meter, IEC 61000-4-15 [2]. This norm is the base for the flicker calculation routine that is used in this report. The flicker meter measures and calculates the flicker level based on the human annoyance. This is done by simulate how the eyes and brain respond to the luminance fluctuations of a coiled filament gas-filled lamp (230 V 60 W), this result in instantaneous flicker level, IFL, which varies in general unpredictably. Therefore a statistically processing of the IFL values are used to calculate the short-term flicker severity index $P_{st}$. The $P_{st}$ value is normally between 0 and 1. If the $P_{st}$ value exceeds 0,7 some experience luminance fluctuations and if it exceeds 1,0 most people experience luminance fluctuations.
Figure 2.1 shows the general working principal of the flicker meter.

![Diagram showing the working principle of the flicker meter.](image)

*Figure 2.1. Working principle of the IEC flicker meter.*

The variable gain amplifier at the input to the flicker meter is used to normalise the RMS value of the input voltage to a reference value. By doing this the flicker meter removes dependence of the actual voltage level. This means that it is only the relative voltage change that is used to calculate the $P_{st}$ value. In appendix A the flicker meter is described more in detail. According to AMP [1] the flicker emission from a single source connected to a 10 kV grid should not exceed $P_{st}=0.35$ or $P_{lt}=0.25$ as a weighted average over 2 hours. Where $P_{lt}$ is the long-term flicker severity index. The long-term flicker index is expressed as:

$$P_{lt} = \sqrt[3]{\frac{\sum_{i=1}^{N} P_{st_i}^3}{N}}$$

(2.1)

$P_{st_i}$ is the $i$:th reading of the short-term flicker severity index $P_{st}$.

From equation 2.1 it is seen that if the wind turbine produces almost the same $P_{st}$ value for two hours the $P_{lt}$ value will be almost the same as the $P_{st}$ value. Therefore the limit for the $P_{st}$ value in this report is set to 0.25.

2.1.3 Line transmission capacity

The transmission capacity of the network must be investigated before the wind turbine can be connected to the network, this to avoid overloading the network. Which type of line used and the rated power of the distribution transformer settle the limit on the line transmission capacity. The limit for the rated power of the wind turbine is settled by the transmission capacity of the network and by the power flow along the network, how much active and reactive power that is consumed/produced along the line. This dependence on the local conditions makes it difficult to set a general limit on the rated power of the wind turbine installation, it must be calculated from case to case.
2.2 Network model

Two different network models has been designed, one general and one simplified model. The simplified model is the one used in this report.

2.2.1 General network model

In order to investigate the impact on the power quality that wind turbines has, representative calculation models of the energy transmission system are needed. In figure 2.2 a typical transmission system of a constant speed wind turbine installation is presented, in this case consisting of only one wind turbine. It consists of an equivalent circuit of the supplying grid, distribution transformer, transmission lines, local wind turbine transformer, capacitor and the wind turbine. The capacitor provides reactive power to compensate for the no-load reactive power consumption of the wind turbine.

\[ i_c = i_{dc} + j i_{qc} \]

\[ Z_k \]

\[ +jU_c > 10 \text{ kV} \]

\[ \text{Consumer} \]

\[ 10 \text{ kV} \]

\[ 690 \text{ V or 400 V} \]

\[ \text{AG} \]

\[ \text{Gear} \]

\[ \text{Wind turbine} \]

\[ \text{Local wind turbine transformer} \]

\[ \text{Distribution line} \]

\[ \text{Distribution transformer} \]

\[ \text{Supplying grid} \]

\[ i_k = \sqrt{3} i_{\text{phase}} \]

\[ S_k \]

\[ \psi_k \]

\[ Z_k \]

\[ \text{Phase to phase voltage of infinity strong grid.} \]

\[ \text{Short circuit current} \]

\[ \text{Short circuit power} = |i_k| \| U_c \| = |U_c|^2 / |Z_k| \]

\[ \text{Short circuit angle} = \arg(Z_k) \]

\[ \text{Short circuit impedance} = R_k + jX_k \]

Figure 2.2. Typical energy transmission system of a fix speed wind turbine installation.

To represent the supplying grid, the Thevenin equivalent circuit presented in figure 2.3 is used. To determinate the grid parameters \( R_k \) and \( X_k \) information of the supplying network short circuit power and short circuit angle is used.

\[ i_k \]

\[ R_k \]

\[ jX_k \]

\[ +jU_c \]

\[ \text{Consumer} \]

\[ \text{Supplying grid} \]

\[ \text{Distribution transformer} \]

Where: \( U_c \) Phase to phase voltage of infinity strong grid.
The parameters $U_e$ and $Z_k$ are used in the Thevenin equivalent circuit for the grid. One simplification with this representation of the supplying grid is that the voltage $U_e$ is approximated to be constant. In figure 2.4 the base and the simplified models of a transformer and a transmission line are presented. For the purpose of determining the voltage impact of wind turbines the simplified models provide quite satisfactory results.

In order to simplify the calculations all components on the high voltage side of the distribution transformer has been transformed to the low voltage side. The model of the energy transmission system becomes as in figure 2.5 with these simplifications.

To determine the voltage impact by a wind turbine measured active ($P$) and reactive ($Q$) power from a reference wind turbine are feed into the network on the high voltage side of the system.
the local wind turbine transformer, the point of common connection, as shown in figure 2.5.

2.1.2 Simplified network model
In the simplified network model, presented in figure 2.6, all impedances between the first customer and the strong grid is added to one equivalent. This is, of course, a simplification, but quite relevant for many situations. Further an advantage is that it facilitates the analysis and presentation of the power quality impact by a wind turbine.

Figure 2.6. The simple network model.

In this simplified model there is one customer connected in parallel with the wind turbine. This means the power quality at this customer must be satisfactorily.

2.3 Voltage impact calculation methods
To calculate the impact that wind turbines have on the voltage from measured active and reactive power, equations that describes the voltage as function of active and reactive power are needed. In the first part of this chapter these equations are derived and in the second part the accuracy of the network model is discussed. In the last part a Matlab function that calculates the XR-ratio that minimises the voltage variations caused by the wind turbine is presented.

2.3.1 Calculation of voltage at the wind turbine from P and Q
The inputs to the network model are measured active, P, and reactive, Q, power from the investigated wind turbines. In order to calculate the power quality impact and the currents, the voltage $u_v$ at the wind turbine must be determined. In this chapter the necessary equations to calculate the voltage from P and Q for the simplified model in chapter 2.1.2 are presented. The apparent power from the wind turbine in the dq-coordinate system is:
\[ S = P + jQ = u_v \bar{i}_v = u_v \frac{u_v - jU_e}{R_{tot} - jX_{tot}} \]  \hspace{1cm} (2.2)

Where \( \bar{x} \) is the complex conjugate.

\[ u_v = u_{dv} + ju_{qv} \]
\[ i_v = i_{dv} + ji_{qv} \]

If the complex and real parts of this equation are separated and solved for \( u_{dv} \) and \( u_{qv} \) the following expressions are obtained:

\[ u_{dv} = \frac{QR_{tot} - PX_{tot}}{U_e} \]
\[ u_{qv} = \frac{U_e \pm \sqrt{U_e^2 + PR_{tot} + QX_{tot} - \left( \frac{QR_{tot} - PX_{tot}}{U_e} \right)^2}}{2} \]

\[ = \frac{U_e \pm \sqrt{U_e^2 - u_{dv}^2 + PR_{tot} + QX_{tot}}}{2} \]

In the expression for \( u_{qv} \) the positive solution is valid, because when the power is zero the voltage \( u_{qv} \) must be equal to \( U_e \) otherwise the current would not be zero in equation 2.2. The voltages \( u_{dv} \) and \( u_{qv} \) are used for calculation of the current, but to calculate the power quality, the magnitude of the voltage vector is needed. The magnitude of the voltage vector is determined by:

\[ |u_v|^2 = u_{dv}^2 + u_{qv}^2 = \frac{U_e^2}{2} + PR_{tot} + QX_{tot} + \sqrt{\left( \frac{U_e^2}{2} + PR_{tot} + QX_{tot} \right)^2 - (P^2 + Q^2) \left( R_{tot}^2 + X_{tot}^2 \right)} \]
\[ = \frac{U_e^2}{2} + PR_{tot} + QX_{tot} + \left( \left( \frac{U_e^2}{2} + PR_{tot} + QX_{tot} \right)^2 - |S|^2 |Z_{tot}|^2 \right) \]  \hspace{1cm} (2.4)

In order to get a feeling of the size of the variables in equation 2.4 the variables in it is substituted against:

\[ |Z_{tot}| = \sqrt{R_{tot}^2 + X_{tot}^2} = \frac{U_e^2}{S_k} = \frac{U_e^2}{K_{SK} P_{max}} \]
\[ R_{tot} = \frac{|Z_{tot}|}{\sqrt{1 + XR_{ratio}^2}} \]
\[ X_{tot} = \frac{|Z_{tot}| XR_{ratio}}{\sqrt{1 + XR_{ratio}^2}} \]

Where \( K_{SK} \) is the ratio between the short circuit power, \( S_k \), and the rated power of the generator, \( P_{max} \), called short circuit ratio and \( XR_{ratio} = \frac{X_{tot}}{R_{tot}} \) = XR-ratio is the ratio.
between the grid reactance and resistance, in other words an alternative way to express the short circuit angle of the grid.

This gives the following expression for the magnitude of the voltage at the wind turbine:

\[
|u_v|^2 = U_e^2 \left( \frac{1}{2} + \frac{P_{pu} + Q_{pu} X_{R \text{ ratio}}}{K_{Sk} \sqrt{1 + X_{R \text{ ratio}}^2}} + \sqrt{\left( \frac{1}{2} + \frac{P_{pu} + Q_{pu} X_{R \text{ ratio}}}{K_{Sk} \sqrt{1 + X_{R \text{ ratio}}^2}} \right)^2 - \frac{P_{pu}^2 + Q_{pu}^2}{K_{Sk}^2}} \right)
\] (2.6)

In equation 2.6 \( P_{pu} \) and \( Q_{pu} \) are active and reactive power in per unit, 1 pu is equal to the rated power of the wind turbine, \( P_{\text{max}} \). As can be noted from equation 2.6 the voltage is dependent on the relation between the active and reactive power and on the grid parameters. If the grid is mainly resistive (XR-ratio low) then the magnitude of the voltage \( |u_v|^2 \) depends mainly on the active power and if the grid is mainly inductive (XR-ratio high) it depends mainly on the reactive power. If the grid is strong (\( K_{Sk} \) large) the voltage is not affected much of the power produced by the wind turbine. From equation 2.6 it is also seen that one way to counteract the voltage rise when the wind turbine produces active power (\( P > 0 \)) is to consume reactive power (\( Q < 0 \)). An approximate relation between \( Q \) and \( P \) for keeping the voltage constant is as follows:

\[
P = -X_{R \text{ ratio}} Q = -\frac{X_{tot}}{R_{tot}} Q
\] (2.7)

This expression is valid for small \( P \) and \( Q \), which means that the last term under the square root in equation 2.6 can be neglected. From equation 2.7 it is seen that if \( R_{tot} > X_{tot} \) then much reactive power has to be consumed in order to keep the voltage down and if \( X_{tot} > R_{tot} \) less reactive power is needed to keep the voltage down.

In the left plot in figure 2.7 the reactive power as function of produced active power for two induction generators, one with high reactive consumption, dashed curve, and one with low, solid curve, are shown. As can be observed the generators consume more reactive power when the active power production is high which is typical for induction generators. In the right plot the voltage change at the point of common connection in per cent is shown. The voltage change is calculated with equation 2.6 for two different short circuit ratios and with the two PQ-curves in the left plot. The PQ-curves have been calculated from the equivalent circuit of an induction machine with no-load compensation of the reactive power.
**Figure 2.7.** In the left plot the two PQ-curves for the different induction generators are shown. The solid curve is for the generator with low reactive power consumption and the dashed is for the generator with high reactive power consumption. In the right plot the voltage change in per cent for an XR-ratio equal to 2 is shown. The solid lines is for the low reactive consumption generator, black curve is for short circuit ratio equal to 10 and the grey is for short circuit ratio equal to 20. The dashed and dash-dotted curves is for the high consumption generator, dashed curve is for short circuit ratio equal to 10 and the dash-dotted is for short circuit ratio equal to 20.

From the left plot in figure 2.7 it can be seen that the ratio P/Q is not constant, which leads to that the voltage change is not constant. For low active power production the voltage increases since the reactive consumption increases slowly. When the power increases the increase in voltage gets smaller because the consumption of reactive power increases more and at high powers the voltage even decrease. In the voltage plot it is seen that the influence on the voltage gets less when the grid is stronger, solid grey and dash-dotted curves. It is also noticed that the different between the voltage curves for the strong and weak grid is less than the difference between the short circuit ratios, less than 2 times. For the voltage curve caused by the low Q generator the voltage change gets close to zero around \( P = 0.7 \ \text{pu} \) and for the other around \( P = 1 \ \text{pu} \). In this point in the PQ-curve the PQ-ratio is matched with the XR-ratio of the grid. This means that the voltage change that comes from a change in P is counteract of the voltage change that comes from Q. It is thus possible to use slightly different generators to achieve a certain grid voltage influence.
2.3.2 Quasi stationary versus dynamic grid network model

In the previous section, the grid reactance has always been used instead of the grid inductance. As will be shown in this section this is an acceptable approximation when the power quality impact of wind turbines is determined. To determine the dynamic voltage fluctuations the static model in chapter 2.2.1 will be used. This can be done because the spectrum of the voltage is not very wide. The assumption made when the quasi-static model is used is that all frequency components in the voltage and the current is affected equal by the grid. In reality this is not true because the reactance change value when the frequency changes. This means that different frequency components are affected differently. The transfer function from voltage \( u_v - jU_u \) to the current \( i_v \) for the simple model in chapter 2.2.1 may be expressed as:

\[
G(\omega) = \frac{I(\omega)}{U(\omega)} = \frac{1}{R_{\text{tot}}} \frac{1}{1 + j \frac{L_{\text{tot}}}{R_{\text{tot}}} \omega}
\]

With 3 dB cut-off frequency

\[
f_0 = \frac{R_{\text{tot}}}{2\pi L_{\text{tot}}} \text{ (Hz)}
\]

The cut-off frequency is dependent on the XR-ratio, this means that the deviation of the transfer function around 50 Hz is dependent on the XR-ratio. This is seen in figure 2.8 where the bode plot of the transfer function in equation 2.8 is show for three different XR-ratios.

Figure 2.8. Bode plot of equation 2.8 with XR-ratio equal to 0.5(solid), XR-ratio equal to 1 (dashed) and XR-ratio equal to 5 (dotted).
From figure 2.8 it is seen that with higher XR-ratio the magnitude decrease more strongly around 50 Hz. This means that the approximation with the static model gets less accurate with increasing XR-ratio, because the static model has a constant reactance value for all frequencies. Another parameters that affect the accuracy of the approximation is the width of the voltage spectrum. The voltage and current from a wind turbine can be expressed as:

\[ u_v(t) = A_u(t) \sin(2\pi f + \varphi_u(t)) = (U_0 + u(t))\sin(2\pi f + \varphi_u(t)) \]  
(2.9)

\[ i_v(t) = A_i(t) \sin(2\pi f + \varphi_i(t)) = (I_0 + i(t))\sin(2\pi f + \varphi_i(t)) \]  
(2.10)

Where:
- \( f \) frequency (50 Hz)
- \( t \) time (seconds)
- \( U_0 \) and \( I_0 \) main frequency component of voltage and current
- \( \varphi_u(t) \) and \( \varphi_i(t) \) time varying phase angles (radians)
- \( u(t) \) and \( i(t) \) time varying amplitude of the voltage and current.

The spectrum for \( i(t) \) is shown in figure 2.9 and the spectrum for \( u(t) \) looks the same but with another scale on the amplitude axis.

![Figure 2.9. FFT of \( i(t) \), grey curve is for the static model and black curve is for the dynamic model.](image)

As it can be seen in figure 2.9 the spectrum has large components up to approximately 2 Hz and some smaller components up to 15 Hz. This means that the spectrum of the voltage is around 48 to 52 Hz (35 to 65 Hz). Because the 50 Hz main frequency is amplitude modulated with this spectrum and therefore the spectrum becomes twice as wide. If the spectrum is increased in width the approximation would be less accurate.
because the lowest and highest frequency component would not be affected equally. For the approximation to be good, the transfer function should be flat around 50 Hz and the spectrum of the voltage should be narrow. A calculation of the current, P, Q and the $P_{st}$ value for a dynamic model and a static model was done for two different XR-ratios, 0.5 and 5. The relative error between the different models was for P, Q and the currents less than 1.5% and for the $P_{st}$ value less than 1%. The worse case was for the highest XR-ratio as expected. The difference between the models is for the higher frequencies as it can be seen in figure 2.9. For frequencies above 10 Hz the models do not agree well. Because the difference between the results from the dynamic and static model is small, the static model will be used for the dynamic calculations of the $P_{st}$ value.

2.3.3 Calculation of ideal XR-ratio

In chapter 2.3 it is shown that the voltage variation is dependent on the PQ relation of the generator and on which impedance the grid has. To find the XR-ratio that minimises the voltage variations at the wind turbine a Matlab function was designed. In figure 2.10 a flow chart of the function is shown.
In the list below the action taken place in each block in figure 2.10 is described.

(1) To find the XR-ratio that minimises the stationary voltage variations the input to the function shall be active and reactive power from the PQ-curve for the generator, with active power from 0 to 1 pu. Var shall be equal to 1 and $U_{ref}$ equal to $U_e$. In order to
find the XR-ratio that minimises flicker the input shall be measured active and reactive power from the wind turbine and $Var$ equal to 2.

(2) Definition of a start vector with XR-ratios.

(3) Calculation of the voltage for the different XR-ratios in the XR vector. The voltage is calculated with equation 2.6 with the P, Q, $Z_{bel}$ and the XR-ratio as input.

(4) If $Var$ is equal to 1 then the voltage reference vector is equal to the $U_{ref}$ value.

(5) If $Var$ is equal to 2 then the voltage reference vector is calculated with equation 2.6 with the mean value of the P and Q vector, $Z_{bel}$ and the XR-ratio as input. This means that the reference value is set to almost the mean value of the voltages calculated in block 3.

(6) Calculation of the quadratic voltage error.

(7) Definition of a new XR vector. The XR value in the old vector that give the smallest error is placed at position 3 in the new XR vector. The old XR values on each side of it is placed on position 1 and 5. The two new XR values on position 2 and 3 is calculated as the mean value of the values at position 1 and 3 respective 3 and 5.

(8) If all five XR-ratios is equal in the 2 first decimals then the function is finish and the ideal XR-ratio is set to be the XR-ratio in position 3 in the XR vector. If not the function jumps to block 3 and loops until the XR-ratios are equal in the 2 first decimals.

If the calculation takes to long time the accuracy of the function can be decreased. This means for example that the XR-ratios in block 8 shall be equal by 1 decimal. When the flicker is minimised ($Var = 2$) the reference voltage is calculated as the mean voltage of the voltages in block 3. This means that the XR-ratio is tuned to minimise the voltage variations around that mean voltage, minimise the relative voltage changes. From chapter 2.1.2 it is known that it is these relative voltage changes that the flicker meter calculates the flicker value from. Therefore by minimising the error the flicker value is minimised.

**2.4 Line capacity**

To choose realistic values of the short circuit ratio and the XR-ratio for the calculations, knowledge of which values that can be expected in the point of common connection is needed. From figure 2.2 it is observed that the network model consist of a supplying grid, a distribution transformer, a distribution line and the local wind turbine installation.

**2.4.1 Supplying grid impedance**

The supplying grid can have different voltage levels, for example 132, 66, 45, 33 or 20 kV. In general the short circuit power increases with increasing voltage level. In reality the short circuit power varies much and it is therefore difficult to set a general level on it for each voltage level. But often the short circuit power of the supplying grid is high enough so that the impedance of it can be neglected when comparing it to the impedance of the distribution transformer and distribution line.
2.4.2 Distribution transformer

The impedance for the distribution transformer is not easy to determine generally. The amount of copper used is depending on which temperature class the transformer has, transformers with high temperature class has thick copper conductors and lower classes has thinner conductors. The series resistance of the transformer is depending on the thickness of the conductor. The transformers in Sweden should have the same temperature class and therefore approximately the same resistive power losses per rated power. In table 2.1 examples of different distribution transformers are present. The impedances in table 2.1 is calculated for the low voltage side of the transformer, the $R_{kTr}$ and $X_{kTr}$ impedance is the series impedance shown in figure 2.4.

Table 2.1. Impedance of distributions transformers.

| Voltage [kV] | $S_n$ [MVA] | $P_l$ [kW] | $Z_b$ [Ω] | Impedance $Z_k$ | $X_{kTr}$ [mΩ] | $Z_{kTr}$ [mΩ] | $X_{kTr}/R_{kTr}$ | $|Z_{kTr}|/Z_b$ [%] | $P_l/S_n$ [%] |
|-------------|-------------|-----------|----------|----------------|----------------|----------------|------------------|----------------|-------------|
| 135/11      | 40          | 186.6     | 3.025    | 14.1          | 478            | 33.9           | 15.8             | 0.47            |
| 135/11      | 39          | 205.4     | 3.103    | 16.3          | 482            | 29.6           | 15.5             | 0.53            |
| 66/11.86    | 25          | 86        | 5.63     | 19.4          | 619            | 31.9           | 11               | 0.34            |
| 66/11.34    | 20          | 124       | 6.43     | 39.8          | 706            | 17.7           | 11               | 0.62            |
| 22.5/11.5   | 12          | 60        | 11.02    | 55.1          | 880            | 16.0           | 8                | 0.5             |
| 22.5/11.5   | 10          | 56        | 13.23    | 74.1          | 923            | 12.5           | 7                | 0.56            |
| 22.5/11.5   | 6.3         | 35.9      | 21.0     | 120           | 1466           | 12.2           | 7                | 0.57            |
| 45/11.5     | 4           | 26        | 33.1     | 215           | 2304           | 10.7           | 7                | 0.65            |
| 33/11       | 2.5         | 25        | 48.4     | 484           | 2912           | 6.0            | 6.1              | 1               |

In table 2.1 $Z_b$ is the base impedance of the transformer and it is calculated as:

$$Z_b = \frac{U_n^2}{S_n}$$

(2.11)

$P_l$ is the load power losses in the transformer, the losses in the resistance $R_{kTr}$, at rated current. From table 2.1 it is seen that the ratio between the load power losses and the rated power dose not vary much. Another thing that is noticed is that the XR-ratio of the transformer increases with increasing rated power.

2.4.3 Distribution wire

The distribution wires should at least have a cross section adequate to meet the system requirements for power transmission capacity. The evaluation of the overall cost of a distribution system should include the capitalised cost of losses, both load and no-load losses. Since the cost of losses is normally evaluated based on the marginal costs of energy and installed power, overall optimisation may often lead to using larger conductor cross sections than the minimum ones meeting current carrying requirements.
To represent distribution cables, XLPE cables from ABB is used. For cables no-load losses are basically dielectric losses in the cable insulation and load losses are basically the ohmic losses in the conductors and the metallic screen. The XLPE cables can be loaded continuously to a conductor temperature of 90˚ C. However, in order to keep a safety margin, or to keep losses lower, or to avoid possible thermal instability due to drying out the surrounding soil, it may be advantageous to limit the operating temperature to 65˚ C.

The continuous current rating given in table 2.2 is calculated according to IEC Publ. 287 and the following conditions:
- Unarmoured three-core XLPE cable
- Ground temperature 20˚ C
- Laying depth 1.0m
- Ground thermal resistivity 1.0 Km/W

**Table 2.2. Rated currents for three-core unarmoured XLPE cables, in ground, conductor temperature 65˚ C, Ground temperature 20˚ C, Laying depth 1.0m, Ground thermal resistivity 1.0 Km/W. The resistance is dc resistance at 20˚ C.**

<table>
<thead>
<tr>
<th>Conductor [mm$^2$]</th>
<th>Screen [mm$^2$]</th>
<th>Rated current</th>
<th>Resistance at 20˚ C</th>
<th>Inductance</th>
<th>Reactance</th>
<th>Capacitance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al [A]</td>
<td>Cu [A]</td>
<td>Al [Ω/km]</td>
<td>Cu [Ω/km]</td>
<td>[mH/km]</td>
<td>[Ω /km]</td>
<td>[μF/km]</td>
</tr>
<tr>
<td>50</td>
<td>16</td>
<td>135</td>
<td>175</td>
<td>0.641</td>
<td>0.387</td>
<td>0.32</td>
</tr>
<tr>
<td>70</td>
<td>16</td>
<td>165</td>
<td>210</td>
<td>0.443</td>
<td>0.268</td>
<td>0.30</td>
</tr>
<tr>
<td>95</td>
<td>25</td>
<td>195</td>
<td>250</td>
<td>0.320</td>
<td>0.193</td>
<td>0.29</td>
</tr>
<tr>
<td>120</td>
<td>25</td>
<td>220</td>
<td>285</td>
<td>0.253</td>
<td>0.153</td>
<td>0.28</td>
</tr>
<tr>
<td>150</td>
<td>25</td>
<td>245</td>
<td>315</td>
<td>0.206</td>
<td>0.124</td>
<td>0.28</td>
</tr>
<tr>
<td>185</td>
<td>35</td>
<td>280</td>
<td>355</td>
<td>0.164</td>
<td>0.0991</td>
<td>0.27</td>
</tr>
<tr>
<td>240</td>
<td>35</td>
<td>320</td>
<td>410</td>
<td>0.125</td>
<td>0.0754</td>
<td>0.26</td>
</tr>
<tr>
<td>300</td>
<td>35</td>
<td>365</td>
<td>460</td>
<td>0.100</td>
<td>0.0601</td>
<td>0.25</td>
</tr>
</tbody>
</table>

For overhead-lines no-load losses are the ohmic losses in the conductor. In table 2.3 the rated current and the resistance for wires of different materials are presented. The resistance is given at a conductor temperature of 20˚ C and the rated continuous current is calculated under the following conditions.
- Sunshine
- Wind speed 0.6 m/s
- Ambient temperature 35˚ C
- Copper conductor temperature 70˚ C
- Other conductor temperature 80˚ C

Al/steel is a wire with a core of steel and aluminium conductors twisted around it. This is done to increase the mechanical strength of the aluminium wire and to keep the god conductivity of the aluminium. The aldrey wire is a alloy between aluminium, magnesium and silicon. The aldrey wire has a mechanical strength comparable with the Al/steel wire. In modern networks it is mainly Al/steel or aldrey conductors that are used and in older installations it is mainly copper wires. Aluminium wires are mainly used in substations because of the requirement for high mechanical strength is low there.
Table 2.3. Rated current and resistance for overhead-lines, aldrey wires consist of the alloy of Al, Mg and Si. The rated current is for wind speed 0.6 m/s and sunshine for an ambient temperature of 35˚ C and the following ultimate conductor temperature copper 70˚ C and aluminium, aldrey aluminium/steel 80˚ C.

<table>
<thead>
<tr>
<th>Cross section</th>
<th>Rated current</th>
<th>Resistance at 20˚ C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cu, aldrey and Al</td>
<td>Cu</td>
</tr>
<tr>
<td>[mm²]</td>
<td>[mm²]</td>
<td>[A]</td>
</tr>
<tr>
<td>50</td>
<td>50/8</td>
<td>250</td>
</tr>
<tr>
<td>70</td>
<td>70/12</td>
<td>310</td>
</tr>
<tr>
<td>95</td>
<td>95/15</td>
<td>380</td>
</tr>
<tr>
<td>120</td>
<td>120/20</td>
<td>440</td>
</tr>
<tr>
<td>150</td>
<td>150/25</td>
<td>510</td>
</tr>
<tr>
<td>185</td>
<td>185/30</td>
<td>585</td>
</tr>
<tr>
<td>240</td>
<td>240/40</td>
<td>700</td>
</tr>
<tr>
<td>300</td>
<td>300/50</td>
<td>800</td>
</tr>
</tbody>
</table>

The inductance and the capacitance of an overhead-line installation are dependent on the physical position of the wires, the distance between the conductors and the conductor radius. To even out the inductance and capacitance variations between the three phases the conductors are twisted regularly. In figure 2.11 an illustration of a twisted three phases wire is shown and the definitions of the distances between the wires.

Figure 2.11. Illustration of a twisted three phase wire and a definition of the distances between the conductors.

With twisted wires means that the phase conductors switch places at intervals of even distances. The inductance and shunt capacitance for a twisted three phase wire can be calculated as follow with the distances a, b and c defined in figure 2.11 and r is the radius of the conductor.
\[
L = 2 \cdot 10^{-4} \ln \frac{3\sqrt{abc}}{r} \quad [H/km]
\]

\[
C = \frac{1}{18 \ln \frac{3\sqrt{abc}}{r}} \quad [\mu F/km]
\]  \hspace{1cm} (2.12)

Where: 
- \(a\) is the distance between wire B and C 
- \(b\) is the distance between wire A and C 
- \(c\) is the distance between wire A and B 
- \(r\) is the radius of the wires, see figure 2.11.

It shall be noted that the equation for the capacitance is an approximation. Since the physical position of the wires differs from installation to installation an exact value for the inductance and capacitance are difficult to calculate. A god approximation for the mean value of the inductance is 1.3 mH/km and for the shunt capacitance 9 nF/km. This gives that the approximation for the reactance becomes 0.41 \(\Omega/km\) and it is this value of the reactance that is used for all over-head wires in this report. The influence on the calculations of power quality from the shunt capacitance is small due to the fact that the variations in the voltage level are low and due to the low value of the capacitance. Therefore the capacitance is neglected in the calculations.
3 Measuring sites

In the calculations measured quantities are used to get more accurate results, because it is difficult to simulate a wind turbine with sufficient accuracy. Measurements were made on two different types of wind turbines: A stall-regulated wind turbine and a pitch-regulated wind turbine. Both wind turbines have their generators directly connected to the grid.

3.1 Alsvik measurement site, stall-regulated wind turbine

The stall-regulated wind turbine studied, is a part of a group of 4 turbines located on the west shoreline of the island of Gotland in the Baltic Sea. In figure 3.1 the location of the wind turbines and the wind speed measurement mast is presented. The wind turbines are three-bladed with a rated power of 180 kW each. They are 30 m high, have a 23 m rotor diameter and rotate at 42 rpm. Of course, various turbines produce different power fluctuations, but the power pulsation from constant-speed stall-regulated turbines are fairly similar, therefore the results delivered in this thesis is most likely valid also for other constant speed stall-regulated turbines.

![Figure 3.1. Location of the wind turbines and the wind mast.](image)
This site was designed especially that wake operation of wind turbines could be studied. Therefore turbines 1 to 3 are placed with different distances from turbine 4. The distances between the turbines are presented in table 3.1.

**Table 3.1. Distance between turbine 4 and the other turbines.**

<table>
<thead>
<tr>
<th>Turbine 4</th>
<th>Distance in rotor diameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbine 1</td>
<td>9,5 (218,5 m)</td>
</tr>
<tr>
<td>Turbine 2</td>
<td>5  (115 m)</td>
</tr>
<tr>
<td>Turbine 3</td>
<td>7  (161 m)</td>
</tr>
</tbody>
</table>

Measurements are done on turbine 4 where active and reactive powers are collected with a data acquisition system. The signals are first anti alias filtered before sampled with a sample rate of 62.5 Hz. The anti-alias filter has a cut of frequency at 37,5 Hz. In the wind mast the wind speed is measured on three levels, 18, 31 and 41 meters. The wind direction is measured at 31 meters. The wind speed and wind direction is sampled with a sample rate of 2 Hz. All the measurements at the Alsvik site were sorted into three different cases, A to C. Case A the wind comes from land, case B is non disturb wind from the sea and case C contains the three cases when turbine 4 is in wake of the other turbines. In table 3.2 the direction for the different cases are chown.

**Table 3.2. Wind direction for the different cases.**

<table>
<thead>
<tr>
<th>Case</th>
<th>From angle</th>
<th>To angle</th>
<th>Number of files</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0°</td>
<td>90°</td>
<td>44</td>
<td>Wind from land</td>
</tr>
<tr>
<td>B</td>
<td>270°</td>
<td>298°</td>
<td>60</td>
<td>Wind from sea</td>
</tr>
<tr>
<td>C1</td>
<td>298°</td>
<td>328°</td>
<td>64</td>
<td>Turbine 4 in wake of turbine 1</td>
</tr>
<tr>
<td>C2</td>
<td>240°</td>
<td>270°</td>
<td>57</td>
<td>Turbine 4 in wake of turbine 2</td>
</tr>
<tr>
<td>C3</td>
<td>195°</td>
<td>225°</td>
<td>83</td>
<td>Turbine 4 in wake of turbine 3</td>
</tr>
</tbody>
</table>

In the left plot in figure 3.2 the 1-minute average values of the wind speed for the different cases is presented. In the right plot the standard deviation of the wind speed for case A and B is shown.
Figure 3.2. In the left plot 1-minute average of the wind speed for different directions, grey is for case A and C and the black is for case B, see in table 3.2 for a definition. In the right plot the standard deviation of the wind speed for case A and B. The grey dots is for land wind, case A, and the black dots is for sea wind, case B. The black curves is the floating average of the dots.

From the left plot in figure 3.2 it is seen that for land wind the wind speed newer exceed 15 m/s this is due to that the wind from land is not as strong as the wind from sea and due to that winds from this direction is rare. In case C2 there is a lack of files with wind speeds around 15 m/s. In the right plot in figure 3.2 it is noticed that the land wind has higher variations then the sea wind. This due to that trees and hills on land distort the wind so that it gets more turbulent and therefore the variations is higher in the land wind. In figure 3.3 the generator PQ-curve and the power-wind speed curves for the wind turbine are shown.
Figure 3.3. In the left plot the generator $PQ$-curve is shown, the black dots are one minutes mean values of active and reactive power and the grey curve is the floating mean of the dots. In the right plot the turbine power-wind speed curve for different cases is shown. Black solid curve consists of the sea wind and land wind cases (case A and B), dashed when turbine 4 is in wake of turbine 1 (case C1), grey solid when turbine 4 is in wake of turbine 2 (case C2) and dotted when turbine 4 is in wake of turbine 3 (case C3).

In the left plot in figure 3.3 the black dots are 1-minutes mean values of the active and reactive power and the grey curve is the floating mean value of the dots. In the right plot the curves are floating mean values of the different cases. The black solid curve consists of the sea wind and land wind cases (case A and B), because these cases give the same result. The dashed, grey and dotted curve is for the different wake cases (case C). The power-wind speed curves for the wake cases differs from the sea and land cases because when the wind comes from these directions turbine 1, 2 or 3 dose not affect the wind that is measured in the wind mast, they only affect the wind that hits turbine 4, as seen in figure 3.1. This means that the measured wind speeds is higher than the wind speeds of the wind that affect wind turbine 4. This lead to the result seen in figure 3.3 that it needs a higher wind speed for the wake case before the maximum power for turbine 4 is reached.

As could be expected the case when turbine 4 is in wake of turbine 2 (solid grey) differs most from the sea wind and land wind cases (solid black) and the case when turbine 4 is in wake of turbine 1 (dashed) is almost the same as the sea wind and land wind cases. This could be expected because turbine 2 is closest to turbine 4, which results in that the wind do not recover from turbine 2 before it hits turbine 4. When the wind comes from turbine 1 it is almost recovered before it hits turbine 4 and therefore the curve for the case when turbine 4 is in wake of turbine 1 (dashed) do not differ much from the non disturb curve for the sea and land cases (solid black).
3.2 Hjärtholmen measurement site, pitch-regulated wind turbine

The wind turbine studied, is a part of a group of three turbines located on the island Hjärtholmen in the coast of Gothenburg. The wind turbines are two bladed and pitch-regulated with a rotor diameter of 27 m and a nacelle height of 30 m. The wind turbines are equipped with pole-changing induction generators, 6/8 poles, with a rating of 225 kW at the higher rotor speed and 50 kW at the lower rotor speed. At low wind speeds the wind turbine operates at the lower rotor speed. Unfortunately there is no wind measuring mast at the site, therefore a meteorological measuring mast about 1.4 km away is used to collect data over wind speeds, wind directions and the standard deviation of wind speed and wind direction. In figure 3.4 the location of the wind turbines and the wind measuring mast is presented. In table 3.2 the distances from turbine 1 to turbine 2, to turbine 3 and to the wind mast is presented.

![Diagram showing wind turbine locations](image)

*Figure 3.4. Location of the wind turbines and the wind measuring mast.*

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>Distance [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbine 1</td>
<td>Turbine 2</td>
<td>150</td>
</tr>
<tr>
<td>Turbine 1</td>
<td>Turbine 3</td>
<td>210</td>
</tr>
<tr>
<td>Turbine 1</td>
<td>Wind mast</td>
<td>1400</td>
</tr>
</tbody>
</table>

Table 3.2 Distances from turbine 1 to turbine 2, turbine 3 and to the wind measuring mast.

From figure 3.4 it is seen that turbine 1 is in wake of turbine 2 at wind directions around 140° and in wake of turbine 3 at wind directions around 46°. For the wind measuring the
wind sensors are disturbed from the mast for wind directions around 90° and they are disturbed from a nearby located wind turbine for wind directions around 170°. Therefore wind directions between 35° to 180° are neglected, because of the uncertain wind measuring and to avoid wake cases.

The data acquisition system for the wind turbine is a sample and hold system with a sample rate of 256 Hz/channel and an 8 pole anti alias filter. The quantities collected are the three phase voltages and currents, pitch angle, generator speed and the acceleration of the nacelle both in X- and Y-axis. From measured voltages and currents the active and reactive power are calculated by:

\[
P = U_r I_r + U_s I_s + U_t I_t
\]

\[
Q = \frac{1}{\sqrt{3}} ((U_s - U_r)I_r + (U_t - U_r)I_s + (U_r - U_t)I_t)
\]

In figure 3.5 the measured PQ-curve of the generator is shown with no-load reactive compensation and the power-wind speed curve.

![Figure 3.5](image)

**Figure 3.5.** In the left plot the measured PQ-curve for the generator with no-load compensation of reactive power. In the right plot the power-wind speed curve for the wind turbine is shown. The black dots are the one-minute mean values and the grey curves are the floating mean of the dots.

In the left plot in figure 3.5 the one-minute mean value of measured active and reactive power is shown and the grey curve is the floating mean of the dots. In the right plot the black dots are one-minute mean values of the active power as function of wind speed and the grey curve is the floating mean of the dots. From the right plot it is seen that the mean power is held almost constant when the wind speed is over 14 m/s, this is due to the pitch regulation of the blades.
4 Analysis
The analysis of the power quality impact that wind turbines has on the network is performed for the two wind turbine types presented in chapter 3. The data on the different grid components presented in chapter 2.4 is analysed to determine what grid parameter combinations that can be expected in a wind turbine connection point to the grid. The different grid parameters that the analysis is performed for are presented in table 4.1.

Table 4.1. Chosen grid parameters for the analysis.

<table>
<thead>
<tr>
<th>XR-ratio</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short circuit ratio</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>Equivalent network voltage [kV]</td>
<td>10.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

All parameter combinations are used in the analysis of the steady state voltage level and for the flicker level.

4.1 Steady-state calculations
The equations in chapter 2 are used to calculate the impact that the two different wind turbines have on the static voltage level. The impact is investigated for different grid parameters, to see the difference. The XR-ratio that gives the lowest impact on the static voltage level is also calculated.

4.1.1 Alsvik steady-state calculations
The steady state quantities that are investigated are the voltage change, the current and the power losses in the grid impedance when the grid parameters are changed. The input to all calculations is the measured PQ-curve for generator 4 on Alsvik presented in figure 4.1 and as the grey curve in the left plot in figure 3.3.
From figure 4.1 and figure 3.3 it is noticed that the generator is not fully compensated at no-load operation, this due to avoid self-magnetisation if a fault occur and the connection to the grid is lost. In figure 4.2 the voltage change caused by the wind turbine, the current and the power losses in the supplying grid are shown for different XR-ratios and short circuit ratio equal to 10.
The different colours in figure 4.2 stands for different XR-ratios see in the middle plot for a definition. The yellow curve is with the ideal XR-ratio calculated with the function \(\text{min}_\Delta Uv\) described in chapter 2.3.3. For this case with short circuit ratio equal to ten the ideal XR-ratio is 2.62. This can be compared with the value of 2.74 presented by J. Olav and G. Tande in [5]. From the left plot in figure 4.1.2 it is seen that for the case with a XR-ratio less than the ideal the voltage is increasing with increasing input power and the cases with greater ratio the voltage decreases with increasing power. This is explained by equation 2.6 where it can be seen that if \(X_{tot}\) is small the consumption of reactive power must be large to keep the voltage down and if \(X_{tot}\) is large the consumption can be smaller and still the voltage variations can be small. In the cases with low XR-ratio (\(X_{tot}\) small) the generator consume to little reactive power to keep the voltage down and for the cases with high XR-ratio it consumes too much. The two horizontal black lines in the left plot in figure 4.2 are the ±2.5% limit of the voltage change. To ensure good power quality the static voltage shall be within these limits. If one curve in the left plot is studied, for example the yellow one, it can be observed that the voltage first increases until \(P=0.5\) pu and after that it drops. This is explained by the fact that the ratio between active and reactive power is constantly increasing when the active power increase, see figure 4.1. For low production of active power the slope is low, which means that when the active power increases the increase in reactive power is small, and this leads to that the voltage increases. For high production of active power the slope is higher and this leads to that
when the active power increases the increase in reactive consumption is high. This leads to that the voltage decrease. This phenomenon is also seen in the left plot in figure 4.2 for XR-ratio equal to 2 and higher. It is not seen for XR-ratio equal to 1 and 0.5 because for these cases the grid is too resistive that the increasing reactive power consumption is to small to stop the increasing voltage. In the middle plot in figure 4.2 the current for the different cases is shown. As it can be observed the current dose not differ much from the different cases. It is also noticed that for the case with the lowest voltage the current is the highest and for the case with the highest voltage the current is the lowest, because the same power is transferred to the grid. In the right plot in figure 4.2 the active power losses in the grid impedance are shown. It can be noticed from the plot that the case with the biggest resistance (lowest XR-ratio) has the highest losses even though this case has the lowest current. This depends on that the difference in current between the cases is small so the losses are mainly dependent on the resistance value.

Figure 4.3 present the voltage change and the current for 1 pu active power as function of short circuit ratio and for different XR-ratios. See in the right plot for a definition of the colours.

![Figure 4.3. Left plot static voltage change from $U_e$ in per cent and the right plot the current in per unit of rated current for 1 pu active power as function of short circuit ratio and for different XR-ratios.](image)

It should be noticed that for the yellow curve the XR-ratio is constantly changed when the short circuit ratio is changed. As is observed from the left plot in figure 4.3 the voltage change decreases with increasing short circuit ratio. This depends on that the grid impedance decreases when the short circuit ratio increase and therefore the active and the reactive power influence on the voltage decrease, se equation 2.6. In figure 4.4 the ideal XR-ratio for the generator is presented. The curve for the ideal XR-ratio is calculated
with the function \texttt{min\_delta\_Uv} described in chapter 2.3.3. The inputs to the function are the PQ-curve in figure 4.1 and $U_{\text{ref}} = U_e$ (Var=1).

![Ideal XR-ratio for the generator.](image)

From the figure it is seen that the XR-ratio increases with increasing short circuit power, this is seen in equation 2.6. When the short circuit ratio ($K_{sk}$) increases the term $\frac{P^2 + Q^2}{K_{sk}}$ that helps to keep the voltage down decrease. This leads to that the XR-ratio must increase to compensate for this.

4.1.2 Hjärtholmen steady-state calculations
The steady state quantities that are investigated are the voltage change, the current and the power losses in the grid impedance when the grid parameters are changed. The input to all calculations is the measured PQ-curve for the large generator on Hjärtholmen presented in figure 4.5 and as the grey curve in the left plot in figure 3.5.
By comparing figure 4.1 with 4.5 it is seen that the generator on Hjärtholmen consumes more reactive power than the one on Alsvik, -0.37 compared with -0.42 pu, this is mainly due to the smaller no-load compensation of reactive power that is made on Hjärtholmen. In figure 4.6 the voltage change caused by the turbine, the current and the power losses in the supplying grid are shown for different XR-ratios.
Figure 4.6. Left plot static voltage change from $U_e$ in per cent, middle plot current in per unit of rated power and right plot active power losses in the grid impedance in per cent of the rated power of the wind turbine for short circuit ratio equal 10 and for different XR-ratios.

In principle, the curves presented in figure 4.6 are very similar to these presented in figure 4.2. If figure 4.6 is compared with the same figure 4.2 for the Alsvik case it is seen that the voltage rise for the low XR-ratios is less on Hjärtholmen than on Alsvik and the voltage drop for the high XR-ratios is greater on Hjärtholmen than on Alsvik. This due to the higher consumption of reactive power on Hjärtholmen. The lower voltage on Hjärtholmen leads to a higher current and higher power losses on the grid.

Figure 4.7 presents the voltage change and the current for 1 pu active power as function of the short circuit ratio for different XR-ratios. See in the right plot for a definition of the colours.
Figure 4.7. In the left plot the static voltage change from $U_e$ in per cent is shown and in the right plot the current in per unit of rated current for 1 pu active power as function of short circuit ratio and for different XR-ratios.

From the left plot in figure 4.7 it is observed that the static voltage change decrease with increasing short circuit ratio. Due to the higher reactive power consumption for the Hjärtholmen turbine compared to the Alsvik turbine the short circuit ratio needs to be higher for the larger XR-ratios before the voltage change is less than 2.5% compared to Alsvik. On the other hand, for the lower XR-ratios a smaller short circuit ratio can be accepted than in the Alsvik case.

In figure 4.8 the ideal XR-ratio for the generator is presented. The curve for the ideal XR-ratio is calculated with the function $\min\_\Delta\_U\_V$ described in chapter 2.3.3. The input to the function are the PQ-curve in figure 4.5 and $U_{Re\_f} = U_e$ (Var=1).
From figure 4.8 it is noticed that the ideal XR-ratio increases with increasing short circuit ratio as discussed in chapter 4.1.1. If figure 4.8 is compared with figure 4.4 it is seen that the ideal XR-ratio for Hjärtholmen is smaller for the same short circuit ratio than the ideal XR-ratio for Alsvik. This due to the higher consumption of reactive power on Hjärtholmen.

### 4.2 Flicker calculations

The model used to calculate the emissions of fast voltage variations (flicker) from wind turbines is the stationary model described in chapter 2.3.1. In chapter 2.3.2 it is explained why this model can be used to approximate the dynamic behaviour of the grid when calculating flicker emissions from wind turbines. The analysis of flicker emissions is made for the two types of wind turbines in chapter 3 and for different grid parameters. The routine for calculating the flicker value ($P_{st}$) is presented in chapter 2.1.2 and more in detail in appendix A. From figure 2.1 of the flicker meter it is seen that on the input of the flicker meter there is a variable gain amplifier which is used to normalise the input voltage to a reference value. This means that the flicker routine only uses the relative voltage change to calculate the flicker value, the change relative the floating mean value of the RMS voltage. To investigate how the voltage varies around a mean value, a first order Taylor expansion of equation 2.6 is used, it can be expressed as:
\[
\left[ u'_e(P_{pu} + \Delta P_{pu} Q_{0pu}, \Delta Q_{pu}) \right]^2 = \left[ u'_e(P_{pu} Q_{0pu}) \right]^2 + \frac{\partial u'_e(P_{pu} Q_{0pu})}{\partial P} \Delta P_{pu} + \frac{\partial u'_e(P_{pu} Q_{0pu})}{\partial Q} \Delta Q_{pu} \quad (4.1)
\]

Where

\[
\frac{\partial \left[ u'_e(P_{pu} Q_{0pu}) \right]^2}{\partial P} = \frac{U_e^2}{K_{Sk} \sqrt{1 + XR_{ratio}^2}} \left\{ \frac{1}{2} + \frac{P_{pu} + Q_{0pu} XR_{ratio}}{K_{Sk} \sqrt{1 + XR_{ratio}^2}} \right\} \frac{1}{K_{Sk} \sqrt{1 + XR_{ratio}^2}} - \frac{P_{pu}^2}{K_{Sk}^2} X R_{ratio}^2 \\
\frac{\partial \left[ u'_e(P_{pu} Q_{0pu}) \right]^2}{\partial Q} = \frac{U_e^2}{K_{Sk} \sqrt{1 + XR_{ratio}^2}} \left\{ \frac{1}{2} + \frac{P_{pu} + Q_{0pu} XR_{ratio}}{K_{Sk} \sqrt{1 + XR_{ratio}^2}} \right\} \frac{X R_{ratio}}{K_{Sk} \sqrt{1 + XR_{ratio}^2}} - \frac{Q_{0pu}^2}{K_{Sk}^2} X R_{ratio}
\]

In equation 4.1 \( P_{pu} \) and \( Q_{0pu} \) defines the working point and \( \Delta P_{pu} \) and \( \Delta Q_{pu} \) are the variations around that point in per unit of the rated power of the wind turbine. In the partial derivatives in equation 4.1 it is seen that if the grid is strong (\( K_{Sk} \) large) the last term in the partial derivatives (\( \frac{Q_{0pu}}{K_{Sk}^2} \) respective \( \frac{P_{pu}}{K_{Sk}^2} \)) can be neglected. If the last terms are neglected the partial derivatives are equal apart from the variable \( X R_{ratio} \) in the partial derivative of the reactive power. From this it is seen that an approximation to minimise the voltage variations when the grid is strong is to set:

\[
\Delta P_{pu} = -\Delta Q_{pu} X R_{ratio} \Rightarrow \frac{X_{tot}}{R_{tot}} = -\frac{\Delta P_{pu}}{\Delta Q_{pu}} \quad (4.2)
\]

But the ratio \( \frac{\Delta P_{pu}}{\Delta Q_{pu}} \) is not constant it changes with the working point. The ratio \( \frac{\Delta P_{pu}}{\Delta Q_{pu}} \) is approximate equal to the inverse slope of the PQ-curve of the generator. This means that there is an ideal XR-ratio fore each working point that minimises the voltage variations. This leads to that the ideal XR-ratio for minimising flicker in the following chapters are changing with changing mean wind speed.

4.2.1 Alsvik flicker calculations

To analyse the flicker emissions from a stall-regulated wind turbine the measurements of active and reactive power from Alsvik is used as input to equation 2.6. The flicker level for different levels of turbulence in the wind are analysed and the XR-ratio that minimises the flicker level is calculated. In figure 4.9 the calculated one minute flicker values, \( P_{st} \), from measured active and reactive power are shown for grids with different XR-ratios, for short circuit ratio equal to ten and for sea wind, wind case B in table 3.2.
For the curve with the ideal XR-ratio in figure 4.9 the XR-ratio is constantly changing according to the discussion in chapter 4.2. From figure 4.9 it is seen that the ideal XR-ratio gives the lowest $P_{st}$ value and that the curves with XR-ratio near the ideal also gives a low $P_{st}$ value. This shows that the routine described in chapter 2.3.3 can be used to calculate the XR-ratio that minimises the flicker value. For the other cases with constant XR-ratio it is the case with XR-ratio equal to two that gives the lowest flicker value. As it can bee noticed in figure 4.9 the $P_{st}$ value for the low XR-ratios is increasing with increasing wind speed but the curves for XR-ratios over 2 the $P_{st}$ value increases up to 15 m/s and for wind speeds over 15 m/s the value is almost constant. To easier understand this behaviour of the $P_{st}$ curves, a linear relation between the variations in active power and the variations in the voltage is needed. For this the first order Taylor expansion in equation 4.1 is used. To relate the variations in the reactive power to the variations in the active power the grey curve in figure 4.10 is used. In figure 4.10 the relation between $\Delta P$ and $\Delta Q$ as function of active power is shown.
Figure 4.10. Relation between the variations in active power and variations in reactive power as function of active power. The black dots are the calculated variations from measured active and reactive power and the grey curve is the floating mean of the dots.

If this relation described as the grey curve in figure 4.10 and the first order Taylor expansion in equation 4.1 are used the Taylor expansion can be simplified to equation 4.3. The equation describes the voltage variations as function of the variations in active power ($\Delta P$) and for the mean value in active and reactive power ($P_0, Q_0$).

$$
\Delta u_v(P_0, Q_0, \Delta P(P_0)) = \left[ \frac{\partial u_v(P_0, Q_0)}{\partial P} \right]^2 + \left[ \frac{\partial u_v(P_0, Q_0)}{\partial Q} \right]^2 f(P_0) \Delta P(P_0) = G(P_0, Q_0) \Delta P(P_0) \quad (4.3)
$$

Where $f(P_0) = \frac{\Delta Q(P_0)}{\Delta P(P_0)}$ = The grey curve in figure 4.10.

The function $G(P_0, Q_0)$ relates the variations in active power to variations in voltage. If $|G(P_0, Q_0)|$ is big then variations in active power gives a large variation in voltage and if $|G(P_0, Q_0)|$ is small then the variations in the voltage is small. This is only an approximation because the first order Taylor expansion is only valid for small variations ($\Delta P$). In figure 4.11 the function $G(P_0, Q_0)$ is shown for different XR-ratios and $K_{sl} = 10$ and 3.
From figure 4.11 it is seen that the function $G(P_0, Q_0)$ decreases with increasing active power. For XR-ratios bigger or equal to two it is seen that the function is equal to zero for some active power. It is also noticed that the function has larger values for lower short circuit ratios. To determine the voltage variations the power variations must be known. To get the power variations, the standard deviation of the power is calculated for sea wind (case B). In the left plot in figure 4.12 the standard deviation of the active and reactive power for the case with wind from the sea is shown. In the right plot the power-wind speed curve is shown for sea wind.
Figure 4.12. The left plot presents the standard deviation of the active (black dots) and reactive (grey dots) power for sea wind (case B). The black curves is the mean value of the dots. In the right plot the active power as function of wind speed is presented for sea wind. The black dots are one-minute mean value, and the grey curve is the mean.

From figure 4.12 it can be observed that the variations in the active power increases up to 11 m/s and after that it is almost constant. This can be explained by looking in the right plot in figure 3.2 and the right plot in figure 4.12. In figure 3.2 the standard deviation of the wind speed is shown and from this figure it is observed that the variations is increasing with increasing wind speed. In figure 4.12 the active power as function of the wind speed for the turbine is shown. For wind speeds below 11 m/s the power-wind speed graph has almost a constant derivative and the variations in the wind speed are increasing. This leads to that the variations in active power increases. As it can be noticed from figure 4.12 the derivative is smaller for wind speeds over 13 m/s than for wind speeds under 13 m/s. This leads to that the variations in active power are low although the variations in wind speed are constantly increasing with increasing wind speed. The magnitude of the standard deviation in figure 4.12 gives an indication of how large the variations in the active power (ΔP) is for different wind speeds. Because the standard deviation is the mean deviation from the mean value:

$$\text{std}(P) = \sqrt{\frac{1}{N-1} \sum_{i=0}^{N} (P_i - \overline{P})^2}$$  \hspace{1cm} (4.4)

Where \( P_i \) is the i:th sample of \( P \) and \( \overline{P} \) is the mean value of \( P \).

One approximation for the magnitude of \( \Delta P \) is

\( \Delta P = \pm \text{std}(P) \)
If this approximation is used with the left plot in figure 4.11, the function \( G(P_0, Q_0) \), and figure 4.12 this will explain the behaviour of the \( P_{st} \) value in figure 4.9. From figure 4.9 it is seen that for XR-ratio equal to 0.5 and 1 the \( P_{st} \) value is increasing with increasing wind speed. From the right plot in figure 4.12 it is seen that the power increases with increasing wind speed from 5 to 14 m/s. From figure 4.11 it is seen that the function \( G(P_0, Q_0) \) is decreasing for increasing power and from the left plot in figure 4.12 it is seen that the magnitude of \( \Delta P \) is increasing. This results in that the total variation in \( u_v \) for wind speeds in the interval 5 to 14 m/s is increasing with increasing wind speed which leads to increasing \( P_{st} \) value. For wind speeds over 14 m/s it is seen from figure 4.12 that the active power is decreasing with increasing wind speed. From figure 4.12 it is seen that the magnitude of \( \Delta P \) is almost constant. But the function \( G(P_0, Q_0) \) in figure 4.11 is increasing for decreasing power. Which leads to that the voltage variations increases even that the power variations is almost constant for increasing wind speed over 14 m/s and increasing voltage variations gives increasing \( P_{st} \) values for increasing wind speed. This explains why the \( P_{st} \) value increases with increasing wind speed for XR-ratios equal to 0.5 and 1. For XR-ratios greater or equal to 2 it is seen from figure 4.9 that the \( P_{st} \) value increases with increasing wind speeds up to 15 m/s and after this it is almost constant for increasing wind speed. This is explained by looking in figure 4.11 where the function \( G(P_0, Q_0) \) decreases with increasing power and for some power it is equal to zero for XR-ratios greater or equal to 2. Even that the factor is equal to zero the \( P_{st} \) value is not zero, this depends on that this is a first order Taylor expansion that is used. That the first term gets equal to zero dose not mean that the higher terms in the Taylor expansion are equal to zero. Therefore the \( P_{st} \) value is not zero when the function \( G(P_0, Q_0) \) is zero. For XR-ratio greater or equal to 2 the function \( G(P_0, Q_0) \) decreases with increasing wind speed to zero and after that it decreases again. Because of the increasing magnitude of \( \Delta P \) and that the other terms in the Taylor expansion are not zero the \( P_{st} \) value increases with increasing wind speed up to 15 m/s. For wind speeds over 14 m/s the power decreases with increasing wind speed. The decreasing active power leads to that the function \( G(P_0, Q_0) \) increases toward zero and combined with that the variations in active power is almost constant for wind speeds over 14 m/s this leads to that the \( P_{st} \) value is almost constant for increasing wind speeds over 15 m/s.

In figure 4.13 the maximum \( P_{st} \) value for different XR-ratios and short circuit ratios are presented, for wind case B. In the left plot the \( P_{st} \) value as function of the short circuit ratio is shown for different XR-ratios. In the right plot the \( P_{st} \) value as function of XR-ratio is shown for different short circuit ratios.
Figure 4.13. Maximum $P_{st}$ value for different XR-ratios and short circuit ratios for sea wind (case B). In the left plot as function of short circuit ratio and in the right as function of XR-ratio.

It shall be noticed that the curves in the left plot in figure 4.13 are curve fitting of equation 4.5 to the calculated values, the stars represent the calculated values.

$$P_{st} = \frac{A}{B + K_{Sk}}$$  \hspace{1cm} (4.5)

From figure 4.13 and equation 4.5 it is seen that the $P_{st}$ value is inversely proportional towards the short circuit ratio. From equation 4.1 this is expected because when the short circuit ratio increase, the partial derivatives decrease and when the derivatives decrease the influence on the voltage from the power variations decrease. This is also seen if the two plots in figure 4.11 of the function $G(P_0, Q_0)$ for short circuit ratio equal to 10 and 3 are compared. It is seen that the function values are smaller for short circuit ratio equal to 10 than for short circuit ratio equal to 3. In table 4.2 the A and B parameters for equation 4.5 are presented for different XR-ratios and for the Alsvik case with wind from sea.

<table>
<thead>
<tr>
<th>XR-ratio</th>
<th>Parameter A</th>
<th>Parameter B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>7.4079</td>
<td>2.1863</td>
</tr>
<tr>
<td>1</td>
<td>4.7896</td>
<td>3.7025</td>
</tr>
<tr>
<td>2</td>
<td>1.1649</td>
<td>-1.8653</td>
</tr>
<tr>
<td>3</td>
<td>1.6824</td>
<td>-2.1114</td>
</tr>
<tr>
<td>4</td>
<td>2.1732</td>
<td>-2.1204</td>
</tr>
<tr>
<td>5</td>
<td>2.498</td>
<td>-2.1392</td>
</tr>
</tbody>
</table>
In the right plot in figure 4.13 it is seen that the minimum for the $P_{st}$ value for a short circuit ratio greater than or equal to 5 is for XR-ratios between 1 and 3 and for short circuit ratio equal to 3 the minimum is for XR-ratios between 0.5 and 2. For weak grids the ratio shall be smaller then for strong grids. This is also seen in figure 4.14 where the ideal XR-ratio that minimises the flicker level is calculated for different short circuit ratios.

![Graph showing calculated ideal XR-ratio based on measured active and reactive power from Alsvik for different short circuit ratios.](image)

*Figure 4.14. Calculated ideal XR-ratio based on measured active and reactive power from Alsvik for different short circuit ratios.*

The ideal XR-ratio is calculated with the routine described in chapter 2.3.3. The input for the calculations are ten minutes measurements of active and reactive power, different short circuit ratios and $Var = 2$ (minimise the fast voltage variations). The curves that are shown in the plot are floating mean values of the calculated ideal XR-ratios. If the approximation in equation 4.2 is used on the grey curve in figure 4.10 approximately the ideal XR-ratio is reach. As it can be noticed the approximation agrees best with the curve for the strongest grid (dotted) as discussed before.

From figure 4.9 it is noticed that the highest $P_{st}$ values for the ideal XR-ratio curve (yellow) is for wind speeds between 17 m/s and 20 m/s which from the right plot in figure 4.12 gives active power around 0.73 pu. In table 4.3 the ideal XR-ratios for active power equal to 0.73 pu is shown. It is the values in table 4.3 that is the most favourable if a constant XR-ratio that reduces the flicker emissions from the wind turbine is desired.
Table 4.3. The ideal XR-ratio that minimises the flicker emissions from Alsvik.

<table>
<thead>
<tr>
<th>Short circuit ratio</th>
<th>Ideal XR-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1.3</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>20</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Another factor that affects the level of flicker is how turbulent the wind is. If the wind is turbulent then the flicker level is higher. In this report low turbulent wind is represented by sea wind (case B) and the turbulent wind are represented by land wind (case A) and the wake cases (case C). In figure 4.15 the 1-minuter $P_{st}$ value for different wind cases and with short circuit ratio equal to 10 and XR-ratio equal to 2 are presented.

Figure 4.15. The 1-minuter $P_{st}$ value for different wind cases and for grid parameters $K_{sk} = 10$ and $XR_{ratio} = 2$. The dashed curve is for land wind, case A. The solid black curve is for sea wind, case B. The dotted curve is for wake case 1, case C1. The dash-dotted curve is for wake case 2, case C2. The solid grey curve is for wake case 3, case C3.

From figure 4.15 it is seen that the sea wind gives the lowest $P_{st}$ value except in the interval 12 to 15 m/s. That the sea case does not give the lowest $P_{st}$ value in this interval can depend on that the different cases do not have the same power-wind speed curve, as can be seen in the right plot in figure 3.3. This gives that the different cases reach their maximum power at different wind speeds. It is also noticed that wake case 1 gives the lowest $P_{st}$ value of the wake cases and highest $P_{st}$ value comes from wake case 2. This can be expected from figure 3.1 where it is seen that turbine 2 is nearest turbine 4 and therefore turbine 2 is distorting turbine 4 most. Turbine 1 is far away from turbine 4 which leads to that the wind almost has recovered from turbine 1 before it comes to turbine 4 and therefore turbine 1 does not disturb turbine 4 as much as turbine 2 do. This
can also be seen in the right plot in figure 3.3 where the power-wind speed curve for wake case 1 is almost the same as for the sea case. From this the conclusion can be made that the wind has almost recovered. From figure 4.15 it is notice that the land wind has higher $P_{st}$ value then the sea wind. This is explained by that the wind from land is more turbulent then the sea wind, se in the right plot in figure 3.2 where the standard deviation for the land and sea wind is shown. From figure 3.2 it is seen that the land wind has higher variations then the sea wind for the same wind speed, this give the higher $P_{st}$ value.

4.2.2 Hjärtholmen flicker calculations

To analyse the flicker emissions from a pitch-regulated wind turbine measured active and reactive power from Hjärtholmen is used as input to equation 2.6. The flicker level for different types of grid parameters is analysed and the XR-ratio that minimises the flicker level is calculated. In figure 4.16 the one-minute flicker value is shown for different XR-ratios and for short circuit ratio equal to ten.

![Figure 4.16. 1 minute flicker value for different XR-ratios and for short circuit ratio equal to ten.](image)

For the yellow curve in figure 4.16 it shall be noticed that the XR-ratio is constantly changing according to the discussion in chapter 4.2. It shall also be noticed that for higher wind speeds the yellow curve gives the lowest $P_{st}$ value. From figure 4.16 it is observed that the XR-ratio that gives the lowest $P_{st}$ value is between 1 and 2 for short circuit ratio equal to ten. From figure 4.16 it is seen that the $P_{st}$ value is increasing for increasing wind speed for all XR-ratios. This can be explained with the first order Taylor expansion presented in equation 4.1. The relation between the variations in active and reactive power is achieved in the same way as in chapter 4.2.1, by fitting a function to the measured relation between $\Delta P$ and $\Delta Q$. The figure over the relation between $\Delta P$ and
ΔQ as function of active power P is not presented because it is similar to the relation from Alsvik presented in figure 4.10 in chapter 4.2.1. The function that describes the relation between ΔP and ΔQ as function of active power P is called f(P₀). If this function is used the Taylor expansion can be simplified to:

$$\Delta u_v(P_0, Q_0, \Delta P(P_0)) = \left[\frac{\partial u_v(P_0, Q_0)}{\partial P} \Delta P(P_0) + \frac{\partial u_v(P_0, Q_0)}{\partial Q} \Delta Q(P_0)\right] f(P_0) = G(P_0, Q_0) \Delta P(P_0)$$ (4.6)

Where $f(P_0) = \frac{\Delta Q(P_0)}{\Delta P(P_0)}$

The function $G(P_0, Q_0)$ relates the variations in active power to variations in voltage. If $|G(P_0, Q_0)|$ is big then variations in active power gives a large variation in voltage and if $|G(P_0, Q_0)|$ is small then the variations in the voltage is small. This is only an approximation because the first order Taylor expansion is only valid for small variations (ΔP). In figure 4.2.4 the function $G(P_0, Q_0)$ is shown for different XR-ratios and $K_{sk} = 10$ and 3.

![Figure 4.17. Function $G(P_0, Q_0)$ for different XR-ratios. The left plot is for $K_{sk} = 10$ and the right plot is for $K_{sk} = 3$. The different colours are for different XR-ratios, see the right plot for a definition.](image)

By comparing figure 4.17 with figure 4.11 in chapter 4.2.1 it is seen that they are almost the same, this because the relation between ΔP and ΔQ are similar. The function $G(P_0, Q_0)$ is decreasing with increasing power and for XR-ratios larger or equal to 2 the function passes zero in both cases. This means that the difference in the $P_{sk}$ value between
the stall-regulated and pitch-regulated wind turbine is not depending on the function $G(P_0, Q_0)$. The difference must depend on differences in active power variations and differences in the power-wind speed relation. To determine the magnitude of the variations in active power the measured standard deviation of the active power is used. In the left plot in figure 4.18 the standard deviation of the active and reactive is shown and in the right plot the power-wind speed curve is shown.

![Figure 4.18. The left plot standard deviation of the active (black dots) and reactive (grey dots) power. The black curves is the mean value of the dots. In the right plot the active power as function of wind speed is presented. The black dots are one-minute mean value, and the grey curve is the mean.](image)

In the left plot in figure 4.18 the black dots are the standard deviation over one minute of active power and the grey dots are the one minute standard deviation of reactive power, the black curves are the floating mean value. From the left plot it is seen that the standard deviation is constantly increasing with increasing wind speed. In the right plot the one-minute mean value of the active power is shown as function of wind speed. From the right plot it is seen that the active power is almost constant for wind speeds over $13\text{m/s}$. With the discussion in chapter 4.2.1 this would lead to that the variations in active power would be small for wind speeds over $13\text{m/s}$, due to the low slope of the power-wind speed curve. But as noticed in the left plot the variations is increasing with increasing wind speed. This depends on that the mean active power is held to a constant value with the pitch-regulation. When the pitch angle is changed, the power-wind speed curve is changed, this is illustrated in the right plot in figure 4.19 and in the left plot the measured pitch angle is shown as function of wind speed.
Figure 4.19. In the left plot the one minute mean value of the pitch angle is shown and in the right plot a illustration of the power-wind speed curve for different pitch angles.

In the right plot in figure 4.19 an illustration of how the power-wind speed curve can look like for a pitch-regulated wind turbine. For wind speed less than the nominal wind speed the active power follows the curve to the left. When the power exceeds the nominal the turbine starts to pitch and this means that the power will follow a curve to the right. When the turbine pitch the power decreases, by changing the pitch angle the mean active power can be held at an almost constant value, the grey line. But from the right plot it is seen that the derivatives are almost constant for the different power-wind speed curves. This combined with the fact that the variations in the wind speed is increasing with increasing mean wind speed gives that the power variations increases with increasing mean wind speed. By combining the plots for active power variations, power-wind speed and the plots of $G(P_0, Q_0)$ the behaviour of the $P_{st}$ plot can be explained. From the left plot in figure 4.17 it is noticed that the value of the function $G(P_0, Q_0)$ is decreasing with increasing active power. When the wind speed increases the function value decreases and when the wind speed reaches the nominal wind speed the function has almost a constant value for increasing wind speeds bigger than the nominal value. This is due to that the mean active power is almost constant for wind speeds over nominal wind speed. Because of that the power variations increases more then the function value of $G(P_0, Q_0)$ decreases for increasing wind speed the voltage variations increases with increasing wind speed which gives that the $P_{st}$ value increases with increasing wind speed.

In the left plot in figure 4.20 the $P_{st}$ value as function of the short circuit ratio is shown for different XR-ratios. In the right plot the $P_{st}$ value as function of XR-ratio is shown for different short circuit ratios.
The curves in the left plot in figure 4.20 is a least square fitting of equation 4.7 to the calculated maximum $P_{st}$ values presented as stars in the plot.

$$P_{st} = \frac{A}{B + K_{sk}}$$  \hspace{1cm} (4.7)

From the left plot and from equation 4.10 it is noticed that the maximum $P_{st}$ value is inversely proportional towards the short circuit ratio as noticed before. In table 4.4 the A and B parameter values for the different XR-ratios are presented.

**Table 4.4. Parameter A and B for the least square fitting of equation 4.10 to the stars in the left plot in figure 4.20**

<table>
<thead>
<tr>
<th>XR-ratio</th>
<th>Parameter A</th>
<th>Parameter B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>12.7888</td>
<td>3.5428</td>
</tr>
<tr>
<td>1</td>
<td>7.1171</td>
<td>4.0783</td>
</tr>
<tr>
<td>2</td>
<td>3.1297</td>
<td>-2.3203</td>
</tr>
<tr>
<td>3</td>
<td>4.7416</td>
<td>-2.477</td>
</tr>
<tr>
<td>4</td>
<td>5.8422</td>
<td>-2.5391</td>
</tr>
<tr>
<td>5</td>
<td>6.5706</td>
<td>-2.5695</td>
</tr>
</tbody>
</table>

From the right plot in figure 4.20 it is seen that for short circuit ratio equal to 3, 5 or 7 the flicker value has a minimum for XR-ratio between 0.5 and 2. For the higher short circuit ratios the minimum is for XR-ratios between 1 and 3. The stars in the right plot indicate the calculated values.
In figure 4.21 the calculated ideal XR-ratio for turbine 1 on Hjärtholmen is shown for different short circuit ratios.

\[ \frac{X}{R} = \frac{-\Delta P}{\Delta Q} \]

From figure 4.21 it is seen that the ideal XR-ratio is decreasing with increasing active power. This is because the slope of the PQ-curve increases with increasing power. It is also noticed that the ideal XR-ratio is increasing with increasing short circuit ratio as discussed before in chapter 4.2.1. The XR-ratio that minimises the flicker level is the value at 1 pu active power. This is due to the fact that the \( P_{st} \) values increases with increasing wind speed and that the power is almost constant for wind speeds over the rated wind speed of the turbine. This is seen in figure 4.16 where the curve for the ideal XR-ratio gives the smallest \( P_{st} \) value and for that curve the XR-ratio follows the dashed curve in figure 4.21 and therefore for wind speeds over 14 m/s it is almost constant to 1.6. For grids with short circuit ratio equal to 3 it is noticed that the ideal XR-ratio is 1. This can also be noticed in the right plot in figure 4.17 where the blue curve for XR-ratio equal to 1 gives that the function \( G(P_0, Q_0) \) is equal to zero for 1 pu active power. From this it is observed that the XR-ratio that minimise the flicker level is the one that sets the function \( G(P_0, Q_0) \) to zero for 1 pu active power. The dash-dotted curve in figure 4.21 shows the approximation of the ideal XR-ratio shown in equation 4.2. As noticed the approximation is valid for strong grids, high short circuit ratios.

**Figure 4.21.** The solid, dashed and dotted curves show the calculated ideal XR-ratio for different short circuit ratios. The dash-dotted curve is the approximation for the ideal XR-ratio.
**4.3 Line capacity**

The analysis of the grid parameters presented in chapter 2.4 is performed in the following way. To each transformer a distribution line with a length between 1 and 30 km is connected and the XR-ratio and short circuit ratio in the end of the distribution line is calculated. The short circuit ratio in this case is the relation between the short circuit power in the end of the distribution line and the rated power of the distribution line (= the rated current multiplied with 10.5 kV). In figure 4.22 the result of this calculation is shown. The green curves is for a aluminium/steel overhead-lines, the magenta curves are for aldrey overhead-lines, the red curves are for copper ground cables and the blue curves are for aluminium ground cables.

![Figure 4.22](image)

*Figure 4.22. In the left plot different grid parameters are calculated for transformers with rated power between 20 and 40 MVA. In the right plot different grid parameters are calculated for transformers with rated power between 2.5 and 12 MVA. The green curves is for a aluminium/steel overhead-lines, the magenta curves are for aldrey overhead-lines, the red curves are for copper ground cables and the blue curves are for aluminium ground cables.*

The right plot in figure 4.22 is for transformers with a rated power between 2.5 and 12 MVA and in the left plot the rated power is between 20 and 40 MVA. The right end of the curves in figure 4.22 is for a distribution wire length of 1 km and the left end is for a length of 30 km. From the figure it can be noticed that for short cables length the XR-ratio is manly depending on the XR-ratio of the transformer and for long cables length the XR-ratio approaches the XR-ratio of the cable. It is also noticed that the short circuit ratio decreases with increasing cable length, this is due to that the total impedance increases with increasing cable length, which leads to a decreasing short circuit power. From the figure it is seen that it is mainly the ground cables that gives the low XR-ratios
and it is mainly the overhead-lines that gives the high XR-ratios. If the two plots are compared it is seen that the highest XR-ratios with low short circuit ratio is only reached in the left plot with the large transformers. This depends on that the transformer XR-ratio is increasing with increasing rated power as seen in table 2.2. The main conclusion that shall be drawn from figure 4.22 is that the parameters of the grid in a connection point can vary much.

4.4 Result analysis

From figure 4.9 and figure 4.16, flicker as function of wind speed, it is noticed that for almost all XR-ratios that differs from the ideal XR-ratio the \( P_{st} \) values are over the limit for the \( P_{st} \) value, 0.25. This indicates that the short circuit ratio for grids with these XR-ratios has to be higher than 10 in order to get the \( P_{st} \) values under the limit. But if many wind turbines are connected to the grid in the same connection point the total flicker emission from all wind turbines can be calculated with the following equation, this equation is taken from AMP [1]:

\[
P_{st} = \sqrt{\sum_{i=1}^{N} P_{st}^2} = \left[ P_{st} = P_{st} \right] = \sqrt{N} P_{st} \quad (4.8)
\]

Where \( P_{st} \) is the flicker contribution from the i:th turbine.

If the flicker contribution is the same for all turbines the simplification is valid. From chapter 4.2.1 and 4.2.2 it is known that the flicker emission is inversely proportional against the short circuit ratio according to:

\[
P_{st} = \frac{A}{B + K_{sk}} \quad (4.9)
\]

If \( N \) wind turbines are connected to the grid and the ratio between the short circuit power in the connection point and the rated power of the wind farm is held constant, the flicker emission from one wind turbine can be calculated with:

\[
P_{st} = \frac{A}{B + \frac{S_{sk1}N}{P_{max}}} \quad (4.10)
\]

Where \( S_{sk1} \) is the short circuit power when only one turbine is connected to the grid and \( P_{max} \) is the rated power for one turbine.

The total flicker emission from the whole wind farm can be calculated by using equation 4.8 and 4.10. Equation 4.11 presents the flicker emission from the wind farm as function of the number of turbines (\( N \)) and the short circuit ratio.

\[
P_{st} = \frac{A\sqrt{N}}{B + \frac{S_{sk1}N}{P_{max}}} = \frac{A\sqrt{N}}{B + K_{sk1}N} \quad (4.11)
\]

From equation 4.11 it is seen that when the number of turbines increases the total flicker emission from the wind park decreases. This is due to that the ratio between the short
circuit power in the connection point and the rated power of the wind farm ($NP_{\text{max}}$) is held constant. From equation 4.11 it is also seen that it is better from a flicker point of view to install many small wind turbines instead of one large with the same rated power that of the wind farm.

The analysis of the results is presented as limiting curves in a short circuit ratio versus XR-ratio diagram. In figure 4.23 the limiting curves for Alsvik is shown.

![Figure 4.23. Limiting curves for Alsvik. The solid curve is for the static voltage level, the dashed curve is for flicker with one turbine and the dotted curve is for flicker with a wind farm consisting of 3 wind turbines.](image)

From the limiting curves it can be seen for which combinations of grid parameters that no problems occur with the static voltage level and with the flicker level. If the grid parameter combination is over the limiting curve no problem with that limit will occur but if the combination is under the curve power quality problems will occur. For example if one grid combination is under the dashed curve and over the solid curve in figure 4.23 there will be problems with the flicker emissions from the wind turbine but there will not be any problem with the static voltage level. Unfortunately the limiting curves for flicker emissions can only be calculated at the given XR-ratios in table 4.1, therefore the resolution is bad. The calculated values are shown with stars in figure 4.23. From figure 4.23 it is noticed that the short circuit ratio is higher for low and high XR-ratios then for XR-ratios around 2. It is seen that for one turbine it is the flicker emission that sets the limit but when the number of turbines increases it is the static voltage that sets the limit.

In figure 4.24 the limiting curves for Hjärtholmen is shown.
Figure 4.24. Limiting curves for Hjärtholmen. The solid curve is for the static voltage level, the dashed curve is for flicker with one turbine and the dotted curve is for flicker with a wind farm consisting of 10 wind turbines.

The stars in figure 4.24 indicate the points where the limiting curves for the flicker emission are evaluated. From figure 4.24 it is seen that it is the flicker emission that sets the limit if one turbine is connected to the grid. If many wind turbines are connected to the grid it is the static voltage that sets the limit. If figure 4.23 and 4.24 are compared it is seen that for a pitch-regulated turbine the grid has to be much stronger than for a stall-regulated turbine to avoid problems with flicker. This is also seen if the dotted curves are compared. For the stall-regulated case 3 turbines was needed to lower the flicker level so that it is almost the statically voltage that sets the limit but in the pitch-regulated case it needs 10 turbines to achieve the same result. This is one reason for that many companies have stop producing direct connected pitch-regulated wind turbines.

The flicker coefficient for a wind turbine is a coefficient that classifies the flicker emissions from a wind turbine. It is used to calculate the power quality impact from the wind turbine. When calculate the flicker coefficient the flicker emissions up to nominal wind speed shall only be used. One reason for this is that the probability that the wind speed exceeds the nominal wind speed (12-14 m/s) is low. If this way of calculating the power quality impact from wind turbine would be used when calculating the limiting curves in figure 4.23 and 4.24 the limiting curves for flicker emissions would be affected. For the stall-regulated turbine, Alsvik, it is seen from figure 4.9 that the limiting values for XR-ratios greater or equal to 2 would not be affected much, but for the lower XR-ratios a much lower short circuit ratios can be accepted. For the pitch-regulated wind turbine, Hjärtholmen, it is seen from figure 4.16 that much lower short circuit ratios can be accepted for the limiting curves for flicker then the ones presented in figure 4.24. By
comparing the flicker values in figure 4.9 and 4.16 for wind speeds around 13 m/s it is seen that the pitch-regulated turbine has higher emissions of flicker and therefore still requires a stronger grid to eliminate power quality problems from flicker. But the difference between the two types will be smaller than the difference obtained from comparing figure 4.9 with 4.16 if this way of calculating the flicker emissions is used.
5 Conclusion

The goal with this report is to investigate how much wind power that can be connected to different grids without encountering problems with the power quality at the nearest consumer. The limits that ensure a good power quality come from the Swedish connection requirements, AMP, [1].

It was found that the static voltage level is dependent on the short circuit ratio and the match between the generators PQ-curve and the XR-ratio of the grid. The static voltage level change is inversely proportional towards the short circuit capacity of the grid.

The flicker emissions from the wind turbine is dependent on the short circuit ratio, the match between the generators PQ-curve and the XR-ratio of the grid, the aerodynamics of the turbine (stall-, pitch-regulation) and the turbulence of the wind. The flicker emissions from the wind turbine are inversely proportional towards the short circuit ratio. Due to the smoother aerodynamic behaviour of the stall-regulated turbine it gives a lower emission of flicker. It is also found that the emissions increase with increasing turbulence of the wind.

Due to the fact that the power quality impact from wind turbines is depending on the match between the generator PQ-curve and the XR-ratio of the grid, it is possible by choosing generator properly to reduce the static voltage change and the flicker emission from the wind turbine.

It was also found that another way to reduce the flicker emissions is to install many small turbines instead of one large turbine with the same rated power as the rated power of the wind farm. The flicker emissions decrease approximately with the square of the number of turbines if the summation formula from AMP [1] is used.

From an analysis of different grid components, such as transformers, overhead-lines and ground cables, it was found that the grid parameters in the point of common connection could vary much.
References


Appendix A. IEC flicker meter

The flicker calculation routine used in this report is a Matlab routine based on the IEC 61000-4-15 standard of a flicker meter [2]. Variations in the intensity from a light source due to variations in the amplitude of the voltage are called flicker. The flicker meter calculates from the measured voltage level a flicker value, \( P_{st} \). This describes the probability that humans will experience light intensity variations from a 60 W coiled filament gas-filled lamp due to the measured variations in the voltage. If the \( P_{st} \) value exceeds 0.7 some humans experience light variations and if the \( P_{st} \) value exceeds 1 most experience variations in the light intensity. The flicker meter can be divided in two parts, the first simulated lamp-eye-brain response and the second is a statistical evaluation. The first part is represented by block 1 to 4 and the second by block 5 in figure A.1.

\[ v(t) = (U_o + u(t))\sin(2\pi f + \varphi(t)) \]  
(A.1)

Where:
- \( U_o \): Reference level of the main frequency component
- \( u(t) \): Time variation of the voltage amplitude
- \( f \): Main frequency (50 Hz)
- \( t \): Time (seconds)
- \( \varphi(t) \): Time variation of the phase angle

In block 2 the voltage variations are approximately recovered by squaring the amplitude-modulated voltage. In equation A.2 the square of the voltage in equation A.1 is presented.

\[ v^2(t) = U_o^2 \sin^2(2\pi f + \varphi) + u^2(t)\sin^2(2\pi f + \varphi) + 2U_o u(t)\sin^2(2\pi f + \varphi) \]

\[ \Rightarrow v^2(t) = U_o^2 \sin^2(2\pi f + \varphi) + 2U_o u(t)\sin^2(2\pi f + \varphi) \]  
(A.2)

\[ \Rightarrow v^2(t) = \frac{U_o^2}{2} + U_o u(t) - \left( \frac{U_o^2}{2} + U_o u(t) \right)\cos(2\pi f + \varphi) \]

It is the last expression in equation A.2 that is used to describe the output voltage from block 2. The approximation made is that the factor with the square of the voltage
variations \( u^2(t) \) is assumed to be small compared with the other factors so that this factor can be neglected. From figure A.2 where the FFT of the amplitude-modulated input voltage and the FFT of the squared voltage is shown it is seen that the variations are \( 10^{-3} \) times smaller than the main frequency component. Due to this there is no problem to use the approximation on equation A.2.

\[
F(s) = \frac{k \omega_0 s (1 + s / \omega_2)}{(s^2 + 2 \lambda s + \omega_0^2)(1 + s / \omega_3)(1 + s / \omega_4)} \quad (A.3)
\]

\[
k = 1.74802 \quad \lambda = 2\pi 0.05981
\]

\[
\omega_1 = 2\pi 9.15494 \quad \omega_2 = 2\pi 2.27979
\]

\[
\omega_3 = 2\pi 1.22535 \quad \omega_4 = 2\pi 21.9
\]

Figure A.2. The grey curve is the FFT of the input voltage and the black curve is the FFT of the squared voltage.

From equation A.2 and the black curve in figure A.2 it is seen that the squared signal consists of a DC-component, the voltage variations and components with twice the main frequency. This leads to that the squaring combined with the band-pass filter in block 3 is an amplitude demodulator that recover the voltage amplitude variations from the input signal. The band-pass filter in block 3 eliminates the DC-component and the double mains frequency ripple components of the output of block 2. The band-pass filter incorporates a first order high-pass filter with 3 dB cut-off frequency at 0.05 Hz and a 6:th order low-pass Butterworth filter with a 3 dB cut-off frequency at 35 Hz. The second filter in block 3 is a weighting filter that simulates the frequency response to sinusoidal voltage fluctuations of a coiled gas-filled lamp (230 V 60 W) combined with the human visual system. The response function is based on the perceptibility found at each frequency by 50% of the persons tested. The weighting filter has the transfer function:
The amplitude plot of the transfer function of the band-pass filter, the weighting filter in block 3 and the total frequency response of block 3 is shown in figure A.3.

![Amplitude plot of filters](image)

*Figure A.3. Amplitude plot of the filters in block 3. The solid curve is the band-pass filter, the dashed curve is the weighting filter and the dotted curve is the total response of block 3.*

In figure A.3 it is seen that the weighting filter has a peak at 8.8 Hz. This due to that humans are sensitive to light intensity fluctuation around 8.8 Hz. From figure A.3 it is seen that the frequencies that affect the flicker value most are in the interval 0.5 to 35 Hz.

The squaring multiplier in block 4 represents the non-linear eye-brain perception and the sliding mean averaging filter simulate the storage effect in the brain. The output of block 4 represents the instantaneous flicker level, IFL. The second part of the flicker meter, block 5, performs statistical analysis of the instantaneous flicker level in order to calculate the $P_{st}$ value. The statistical calculation is done in this way that first a time series of the output of block 4 is collected and the values are sorted in ascending order.

The $P_{st}$ value is calculated as:

$$P_{st} = \sqrt{0.0314P_{0.1} + 0.0525P_1 + 0.0657P_3 + 0.28P_{10} + 0.08P_{30}}$$  \hspace{1cm} (A.4)

Where $P_{0.1}$ stands for the sorted value that is exceeded for 0.1% of the time and $P_1$ stands for the sorted value that is exceeded for 1% of the time and so on.

Figure A.4 illustrates this.
Figure A.4. Illustration of the statistical calculation. In the left plot a 10-minute time series of the IFL values and in the right plot the IFL values are sorted in ascending order are presented.

In the left plot the time series of the IFL signal is shown and in the right plot the sorted series with the different $P_x$-values indicated. The length of the time series is dependent on which $P_{st}$ value that is wanted. If it is the one-minute value then one minute is collected and if it is the ten-minute value ten minutes is collected.