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Stochastic portfolio optimization of generation self-scheduling in deregulated electricity markets

Master of Science Thesis

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CHALMERS UNIVERSITY OF TECHNOLOGY
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Abstract

In this thesis, different optimization models are formulated to provide a unit commitment schedule and a bidding strategy on several electricity markets. Both the deterministic and stochastic unit commitment formulations are presented from the perspective of a generation company (GENCO). In the formulation, the stochasticity is due to the uncertainty in the day-ahead forecasts of the spot market price and wind farm energy production.

Thus, the main goal of this thesis is to incorporate the uncertainty the GENCO's wind power production into the price-based unit commitment scheduling for bidding in the day-ahead spot market. We have used Lagrangian relaxation, dynamic programming, mixed integer linear programming and stochastic programming for that purpose.

We started with applying our algorithms to the unit commitment of six thermal units. We then added to those thermal units a wind farm which had a stochastic production output. We tested our algorithms with different forecast methods and wind penetration (25% and 45% of the GENCO installed production capacity being wind farm power).

As a result, a clear difference is observed between the optimal solutions from the deterministic program and from the stochastic program. This indicates that ignoring stochasticity may lead to a non-optimal schedule in reality, i.e. lower profits for the GENCO.

Finally, tests show that the error levels of the forecasts, in both spot prices and wind farms production, impact significantly the possible income of the GENCO. Thereby, reducing the error of those forecasts lead to increased profits. Moreover, our tests seems to show that increasing the wind penetration lead to increased profits for the GENCO, but this last result still have to be proved.

Index Terms: Deterministic optimization, economic power dispatch, stochastic programming, unit commitment scheduling in deregulated electricity markets.

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Pierre Rousseau
Göteborg, Sweden, 2011

Glossary

- **CDF**: Cumulative Distribution Function
- **PDF**: Probability Distribution Function
- **LB**: Lower Bound
- **UB**: Upper Bound
- **GENCO**: Generation Company (Electricity generation)
- **Nordic area**: Area composed of Norway, Sweden, Finland and Denmark.
- **Spot market**: Day-ahead electricity exchange
- **LP**: Linear Programming
- **MILP**: Mixed Integer Linear Programming
- **LR**: Lagrangian relaxation
- **DynP**: Dynamic programming
- **PBUC**: Price-Based Unit Commitment
- **EPD**: Economic Power Dispatch
- **RDG**: Relative Duality Gap

List of symbols

Indicators

- **i**: the indicator of the generation unit
- **t**: the indicator of the time period
- **s**: the indicator of a given scenario (a given set of realization of the stochastic variables)

Functions

- **f** and **F**: objective functions
- **h**: a constraint function
- **L**: a Lagrangian function
- **q** : a dual Lagrangian function
- $\mathbf{C_i^q}$: the quadratic cost function of generator i of the form $C_i^q(P) = a(i) + b(i)\ P + c(i)\ P^2$
- $\mathbf{C_i^l}$: the 3 steps linear cost function of generator i of the form $C_i^l(P) = C_A(i) + C_{B1}(i)\ P_1 + C_{B2}(i)\ P_2 + C_{B3}(i)\ P_3$
- $\mathbf{f}_c$: a backward cumulative objective function
- **g**: the global objective function in the stochastic programming (First and Second stage)
- $\mathbf{f}_{sp}$: the first stage problem in the stochastic programming
- **Q**: the optimal solution of the second stage problem in the stochastic programming
- $\mathbf{q^{sp}}$: the expectancy of Q

Constants

- **X**: a set of feasibility
- **T**: a matrice
- **W**: a matrice
- $\mathbf{h_{sp}}$: a vector
- $\mathbf{q_{vect}}$: a vector
- **Z**: a binary vector
- **N**: the number of the generation units
- **T**: the number of scheduling periods
- **S**: the number of scenarios
- $\mathbf{100(1-\alpha)}$: a confidence percentage in %

- $\mathbf{U_{N'}(\hat{x})}$: an upper bound for $g(\hat{x})$
- $\mathbf{L_{N,H}}$: a lower bound for $\mathbb{E}[\hat{v}_N]$
- λ, $\mu_{\mathbf{G}}$ and $\mu_{\mathbf{R}}$: Lagrangian multipliers
- α: a step coefficient
- $\mathbf{P_{min}(i)}$: the minimum power that can be produced by generator i
- $\mathbf{P_{max}(i)}$: the maximum power that can be produced by generator i
- $\mathbf{SU(i)}$: the start up cost of generator i
- $\mathbf{SD(i)}$: the shut down cost of generator i
- $\mathbf{RU(i)}$: the ramp up rate of generator i
- $\mathbf{RD(i)}$: the ramp down rate of generator i
- $\mathbf{P_B(t)}$: the bilateral contracts for period t
- $\mathbf{\bar{D}(t)}$: the expected value of the forecasted load at period t
- $\mathbf{\bar{R}(t)}$: the expected value of the forecasted reserve needed by the global system at period t
- $\mathbf{M}$: a number big enough compared to power production (100000 for example)
- $\mathbf{MUT(i)}$: the minimum ON time of generator i
- $\mathbf{MDT(i)}$: the minimum OFF time of generator i
- $\mathbf{a(i)}$: a cost coefficient of the quadratic cost function C_i^q
- $\mathbf{b(i)}$: a cost coefficient of the quadratic cost function C_i^q
- $\mathbf{c(i)}$: a cost coefficient of the quadratic cost function C_i^q
- $\mathbf{C_A(i)}$: a cost coefficient of the linear cost function C_i^l
- $\mathbf{C_{B1}(i)}$: a cost coefficient of the linear cost function C_i^l
- $\mathbf{C_{B2}(i)}$: a cost coefficient of the linear cost function C_i^l
- $\mathbf{C_{B3}(i)}$: a cost coefficient of the linear cost function C_i^l
- $\mathbf{E_1(i)}$: the upper bound of the variable P_1 of the linear cost function above
- $\mathbf{E_2(i)}$: the upper bound of the variables $P_1 + P_2$ of the linear cost function above
- $\mathbf{I0(i)}$: the status of generator i before the first period of scheduling
- $\mathbf{TH(i)}$: the time ON or OFF ($TH < 0$ if OFF time, $TH > 0$ if ON time) of generator i before the first period of scheduling
- $\mathbf{P0(i)}$: the power output of generator i before the first period of scheduling
- $\rho_{\mathbf{G}}\mathbf{(t)}$: the expected value of the spot price forecast for period t
- $\rho_{\mathbf{R}}\mathbf{(t)}$: the expected value of the UP regulation price forecast for period t
- $\rho_{\mathbf{B}}\mathbf{(t)}$: the value of the bilateral price for period t

Decision variables

- $\mathbf{x}$: a vector of decision variables (In the case of stochastic programming, it represents the first stage decision variables)

- $\mathbf{y}$: a vector of decision variables (In the case of stochastic programming, it represents the second stage decision variables)
- $\mathbf{x^R}$: a vector of decision variables where the integer variable have been rounded.
- $\mathbf{P^G(i,t)}$: the power produced by generator i at period t
- $\mathbf{P^R(i,t)}$: the spinning reserve for generator i at period t
- $\mathbf{P^B(i,t)}$: the power for generator i at period t that is going to be sold to bilateral contract
- $\mathbf{I(i,t)}$: the status of the generator i at period t (0 if OFF/1 if ON)
- $\mathbf{Xon(i,t)}$: the number of period the generator i has been ON in a row until period t
- $\mathbf{Xoff(i,t)}$: the number of period the generator i has been OFF in a row until period t
- $\mathbf{P_{spot}(t)}$: the amount of power that will be bid on the SPOT market for delivery at period t
- $\mathbf{x_{cstrt}(i,t)}$ see [9] for description
- $\mathbf{y_{cstrt}(i,t)}$ see [9] for description
- $\mathbf{z_{cstrt}(i,t)}$ see [9] for description

Second stage decision variables

- $\mathbf{P^+(i,t,\xi)}$: the amount of power by which the GENCO increase the scheduled power $P^G(i,t)$ of generator i at period t
- $\mathbf{P^-(i,t,\xi)}$: the amount of power by which the GENCO decrease the scheduled power $P^G(i,t)$ of generator i at period t
- $\mathbf{w(t,\xi)}$: the net costs from the regulation market
- $\mathbf{v(t,\xi)}$: the amount of power under regulation (can be positive or negative)

Stochastic variables

- ξ: a possible outcome of the uncertain data for the second stage problem in the stochastic programming
- $\rho_{\mathbf{spot}}(\mathbf{t},\xi)$: the spot price at period t
- $\rho^{\mathbf{rg}}_{\mathbf{up}}(\mathbf{t},\xi)$: the up regulation price at period t
- $\rho^{\mathbf{rg}}_{\mathbf{down}}(\mathbf{t},\xi)$: the down regulation price at period t
- $\mathbf{P_{wind}(t,\xi)}$: the wind power at period t
- $\Delta\mathbf{P(t,\xi)}$: the imbalance of the power system (Generation-Consumption) at period t

Contents

Chapter 1

Introduction

1.1 Background

1.1.1 Electricity generation companies

All the work performed during this thesis has been made from the generation company's (GENCO) point of view. A generation company, who need to decide whether or not to buy/sell power from/to the spot and regulation markets, thus it has to determine the generation schedule of their power plants so that it fulfills its bilateral contracts while maximizing their own profits.

1.1.2 The deregulation of power system in Sweden [10]

Sweden until 1991 is a special case: the electricity sector was not completely centralized or nationalized, and tens of companies were generating and/or transporting electricity to their own clients. So there was no centralized dispatch of generation scheduling. For example, Vattenfall was the main and the most important one, they also controlled the 400kV and 220kV transmission lines. Due to deregulation, swedish government decided in 1991 to create a dedicated company that will be in charge of the national transmission lines (thus removed from Vattenfall). This company is called Svenska Kraftnat and its main missions are to manage the national transmission lines and to promote the competition in the electricity market. (see [3])

The Nordpool is created in 1996 between Sweden and Norway. It's a power exchange that operates on two types of markets:

- The **future market**, in which actors can buy or sell base or peak load power from a week to three years in advance.
- The **spot market**, in which actors can buy or sell power on an hourly basis for the next day.

Finland joined the Nordpool in 1998 and Denmark completely joined the Nordpool in 2000.
Thus every generation company, distribution/transmission company, and traders can buy and sell energy in those markets in addition to their own bilateral contracts.

1.1.3 Selling/Buying electricity

Nordpool spot market Elspot

The spot market enables the actors to buy and/or sell energy on an hourly basis for one day ahead. According to the Nordpool spot market website [1]:

"Contracts are made between seller and buyer for delivery in the next day, the price is set and the trade is settled.

12:00 CET is the time of gate closure for bids with the delivery tomorrow. Elspot feeds the information into its computer system and the price is calculated based on an advanced algorithm. Simply put, the price *is set where the curves for sell price and buy price intersect. The price is announced to the market at approximately 13:00 CET, and after that the trades are settled. From 00:00 CET the next day, contracts are delivered physically hour by hour according to the contracts entered.*

As soon as the noon deadline for participants to submit bids has passed, all purchase and sell orders are aggregated into two curves for each delivery hour; an aggregate demand curve and an aggregate supply curve. The system price for each hour is determined by the intersection of the aggregate supply and demand curves which are representing all bids and offers for the entire Nordic region." (see figure 1.1)

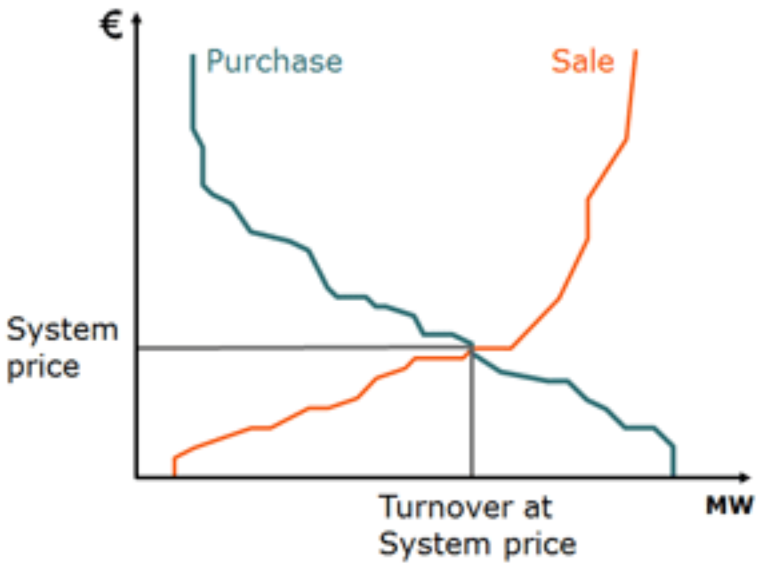


Figure 1.1: System price settlement [1]

We can notice that there is another type of exchange on the Nordpool called Elbas (We do not use it in this thesis), which is a balance adjustment market thus covers the period between the end of the spot market Elspot bidding and the delivery hour.

Regulation market exchange

The regulation market is operated by Svenska Kraftnat [6] and behaves differently from the Nordpool spot market. On the regulation market, actors can submit selling bids for downward regulation reserve (Capacity to reduce its energy output) and/or for upward regulation reserve (Capacity to increase its energy output). But there is no buying bids. It is Svenska Kraftnat who decides to buy the necessary reserve.

Bilateral contracts

We define the bilateral contracts of a GENCO as the contracts specifying for each hour the amount of energy that the GENCO has to provide to a specific client.

In this thesis, these are the three opportunities taken into account to enable the generation companies to reach their objectives (maximizing its benefits).

With the increase in renewable energy production, the uncertainty in the power output increases. In our case we will only take into account wind farms, but the reasoning in this thesis can be adapted to every uncertain electricity production plants. So, the GENCO has to engage itself to buy/sell electricity/reserve on the markets before knowing the electricity prices and its wind farm production. Making an optimal decision is not obvious, it is why a framework of algorithms and tools has to be developed to help the decision making process.

This Master Thesis focuses on the analysis of the electricity portfolio day-ahead problem of a big producer. We thus use stochastic programming with a particular attention to the uncertainty in the wind-farms power production and in the spot prices.

1.2 Problem formulation

The objective of the study is to determine a unit commitment schedule and corresponding bidding strategy that maximize the profits of the company with respect to the system and unit constraints. The tasks of this project include:

- Review standard **unit commitment** algorithms
- Review basics of market operation in Nordic countries
- Data specification concerning the power plants and the system
- Implement an optimization algorithm for **deterministic unit commitment**
- Implement a model of stochastic optimization algorithm for **stochastic unit commitment**
- Create annexe programs for **statistical inference** and **Monte Carlo methods**
- Use a realistic set of generation units to see the validity, robustness and computational speed of the optimization algorithm.

1.3 Methodology

The framework of tools used in this project consists of stochastic analysis, optimization and power market management. The optimization is the main tool of this study. It is a mathematical framework that enables us to find the best way to use our resources (minimum cost or maximum profits) while respecting the technical constraints required. The stochasticity of the optimization problem is caused by the uncertainty of electricity price forecasts and wind farm power forecasts.

1.4 Contribution and limits of the work

The contribution of this work is on the formulation of a two-stage linear stochastic programming model for incorporating wind power forecasting error in generation scheduling in the Nordic power market.

Its limits are the absence of use of proper risk management methods and utility function in the stochastic optimization algorithms. This create lead to an exposure to extreme but rare event such as very high electricity spot price.

Chapter 2

Overview of the optimization methods for price-based unit commitment

As explained in the introduction, the main mathematical framework we have used in this thesis is optimization. This chapter is to provide a basic overview of programming techniques used in this project. The goal of this chapter is not to provide a detailed description on those subjects. For that, the reader can check the references in the end.

In the field of optimization we can distinguish two parts. The first one, **mathematical modeling** aims at modeling our practical problem as a standardized optimization formulation. Such an optimization is composed of:

- **Decision variables**: representing the choice to be made (i.e. in our case, the unit commitment, power production, power bid on spot market...etc.)
- **Objective function**: representing the total net cost of making a choice of decision variables
- **Constraints**: representing values that decision variables can take. For example, a variable can be restricted to binary values.

The second one, **mathematical programming** aims at solving the mathematical model so that optimal solution(s) can be found. To do so, different algorithms are available for different categories of models, e.g. linear, nonlinear, etc. In the following sessions, we describe the optimization algorithms used in this thesis..

2.1 Lagrangian relaxation

This section has been written using [2]

Lagrangian relaxation enables us to avoid solving difficult constraints directly, (e.g., equality constraint linking several variables together). Assume the following primal optimization problem:

$$\begin{aligned} \underset{x\in\mathbb{R}^n}{\text{minimize}} \quad & f(x) \\ \text{subject to} \quad & h(x) \leq 0 \end{aligned}$$

To relax the constraints $h(x) \leq 0$ (Assume that it is hard to implement), we introduce a variable λ and create the relaxed Lagrange formulation where $\lambda \geq 0$ is given:

$$\underset{x\in\mathbb{R}^n}{\text{minimize}} \quad L(x|\lambda) = f(x) + \lambda \cdot h(x)$$

This problem is now easier to solve. Assuming that we know the value of λ, we look for the maximum of the Lagrangian Relaxation function. This is equivalent to the assumption that the constraints not relaxed are not active in the problem. Thus, we can solve for a corresponding optimal solution x^*:

$$\frac{dL}{dx} = \frac{df}{dx} + \lambda \frac{dh}{dx} = 0 \Rightarrow x^* \tag{2.1}$$

If the solution x^* obtained is outside the feasible set defined by the non relaxed constraints, then we fix the value of x^* corresponding to this constraints and recompute x^*.

According to the relaxation theorem:

$$\forall \lambda : min\{L(x|\lambda)\} \leq min\{L(x|\lambda), h(x) \leq 0\} \leq min\{f(x), h(x) \leq 0\} \tag{2.2}$$

So if we find a solution of x^* and λ^* such that $L(x^*|\lambda^*) = f(x^*)$, we know that x^* is a global optimal solution to the primal problem. Define:

$$\begin{aligned} q(\lambda) =& \underset{x \in \mathbb{R}^n}{\text{minimize}}\ L(x|\lambda) = f(x) + \lambda h(x) \\ =& L(x^*|\lambda) \\ =& f(x^*) + \lambda \cdot h(x^*) \end{aligned} \tag{2.3}$$

This define the dual problem:

$$q^* = \underset{\lambda \in \mathbb{R}^+}{\text{maximize}} \quad q(\lambda)$$

As most of the time it is hard to have an analytical expression of $q(\lambda)$, we use the sub gradient method to find λ^* ($\lambda^{(k)}$ being he value of lambda at iteration k):

$$\lambda^{(k+1)} = \max\left(0, \lambda^{(k)} + \alpha_k \left[\frac{dq(\lambda)}{d\lambda)}\right]\right. \tag{2.4}$$

Where α_k is the step size, and $\frac{dq(\lambda)}{d\lambda} = h(x^*)$

2.2 Lambda-iteration technique

This section has been written using [11]

The goal of this algorithm is to dispatch the amount of bilateral contracts P^B in MWh among all the N units that are ON (If the unit i is ON then $I(i) = 1$, if it is OFF then $I(i) = 0$). The idea of this method is that the incremental costs of all the ON units are the same and equal to λ. Knowing the cost functions of the production units for a given amount of energy P, $C_i^q(P(i)) = a(i) + b(i)P(i) + c(i)P(i)^2$, the incremental cost for unit i is:

$$\frac{dC_i}{dP(i)} = b(i) + 2c(i)P(i) \tag{2.5}$$

The algorithms main idea is that for all ON units:

$$\frac{dC_i}{dP(i)} = \lambda \tag{2.6}$$

so we compute P(i) as follows:

$$P(i) = \frac{\lambda - b(i)}{2c(i)} \tag{2.7}$$

As long as $\lambda \leq \rho_G$ hold, we then change the value of λ until having:

$$\sum_{i=1}^{N} P(i) = P_B \tag{2.8}$$

As we are dealing with a market environment, we stop updating λ when we reach the spot market price ρ_G. This means that it is now better to buy the remaining energy from the spot market to fulfill the bilateral contract.

At iteration n, $\lambda_.$ is updated by:

$$\lambda_n = \frac{-e_{n-2}(\lambda_{n-2} - \lambda_{n-1})}{e_{n-2} - e_{n-1}} + \lambda_{n-2} \tag{2.9}$$

Where $e_n = \sum_{i=1}^{N} P(i) - P_B$ is the error function. We assume that the error function of λ is a linear function, and we update Lambda such that this linear function is equal to 0.

The block diagram of the Lambda-iteration technique is shown on figure 2.1.

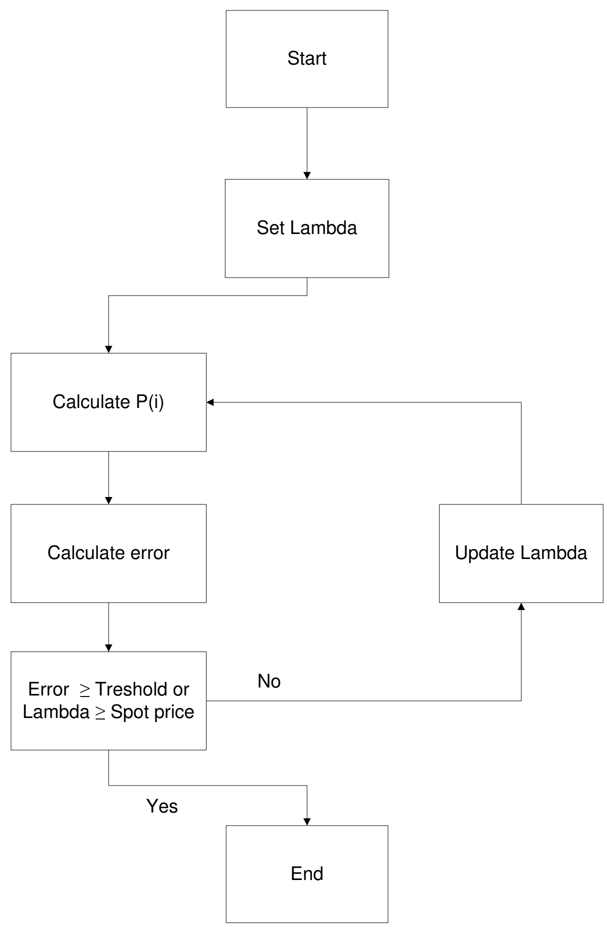


Figure 2.1: Lambda iteration algorithm

2.3 Dynamic programming

In this thesis we perform a backward dynamic programming. We define a minimization problem with $f(t)$ as objective function at hour t and the states $I(t)$ for $t \in \{0, ..., T\}$ as decision variables. In our case we say that t is a period of 1 hour, and the state $I(t)$ is a binary vector corresponding to the status of our N units at hour t. For instance, in the case of N=4, I(1)=[0 1 1 0], It means that unit 1 and 4 are OFF and unit 2 and 3 are ON during hour 1. As each unit has 2 state (ON or OFF), there are in total 2^N states at each hour. This leads to a dimensional problem as N increases.

For simplicity reason let's consider only one units, thus for every stage t we have 2 possible states([0] or [1]).

Now we can decompose the dynamic programming algorithm in three phases:

1. **Initialization** For this first phase, we are at the $T-1$ stage of the problem (Because we perform a backward method, it would have been different if we had perform a forward method).
   Now at this stage:
   - Assuming the unit is ON at hour $t = T-1$:
     - We compute the value of $f(t)+f(t+1)$ if unit is ON at hour $t+1$
     - We compute the value of $f(t)+f(t+1)$ if unit is OFF at hour $t+1$
     - We store the value of the state of the unit at hour T that produces the lower value of $f(t)+f(t+1)$ as OptimalPath(ON,t)
   - Assuming the unit is OFF at hour $t = T-1$:
     - We compute the value of $f(t)+f(t+1)$ if unit is ON at hour $t+1$
     - We compute the value of $f(t)+f(t+1)$ if unit is OFF at hour $t+1$
     - We store the value of the state of the unit at hour T that produce the lower value of $f(t)+f(t+1)$ as OptimalPath(OFF,t)
2. **Iterations** We now do the same for every stages recursively (i.e. t going from $T-2$ to 0) using the value of $f(t)+f(t+1)+...+f(T)$ instead of just $f(t)+f(t+1)$ to estimate the best path.
3. **Termination** Given that we know the state of the system just before the scheduling period (Hour 0), we can now choose the optimal path (That minimize the global objective function) between states and stages from the information found during the iterations. For that we use the values stored in OptimalPath, proceeding as follow:
   - Knowing the unit is ON at hour 0, according to the OptimalPath(ON,0) if unit is OFF at hour 0) if the unit should be ON or OFF at hour 1. Similar procedure if the unit is OFF at hour 0.
   - We repeat the same procedure until reaching hour T-1 where OptimalPath(OFF or ON,T-1) tell us if the unit should be ON or OFF at hour T
   - We have thus defined the optimal status of the unit at every hour of the scheduling period

The Bellman principle ensures that this algorithm gives the optimal solution to our problem.

Unit commitment dynamic programming in this thesis

We define our minimization problem F with the objective function f for each unit and decision variable $P^G(i,t)$, $P^R(i,t)$, $P^B(i,t)$ and $I(i,t)$ (see List of symbols). If we assume $I(i,t) = 1\ \forall(i,t)$ (all the units are ON, all the time) then we can compute $P^G(i,t)$, $P^R(i,t)$, $P^B(i,t)$ by using different EPD technique (part 2.2 and part 3.2.3 of the thesis).

We can now perform the dynamic programming for every unit because the objective is split accordingly and this only requires two states at each stage of the dynamic programming. So in our programming problem, there are only 2 states (I(i,t)=1 when the unit is ON during hour t, I(i,t)=0 when the unit is OFF during hour t). It is important, regarding the computation time, to be able to split the global objective function in independent part corresponding to each unit. That is why when a problem contains coupling constraints we can use Lagrangian relaxation to form a Lagrangian function that can be decoupled into independent part of each unit. We perform a backward dynamic programming, which means that we are going to begin our algorithm from the last stage and go recursively. The hours considered go from 1 to T.

At hour t, we need to make an assumption on $t-1$ state to take into account the start-up cost of unit if needed (Turning ON a unit at period t). At each iteration, we now look at hours $t-1$, t and $t+1$, thus we have 8 cases to deal with (as shown in Figure 2.2):

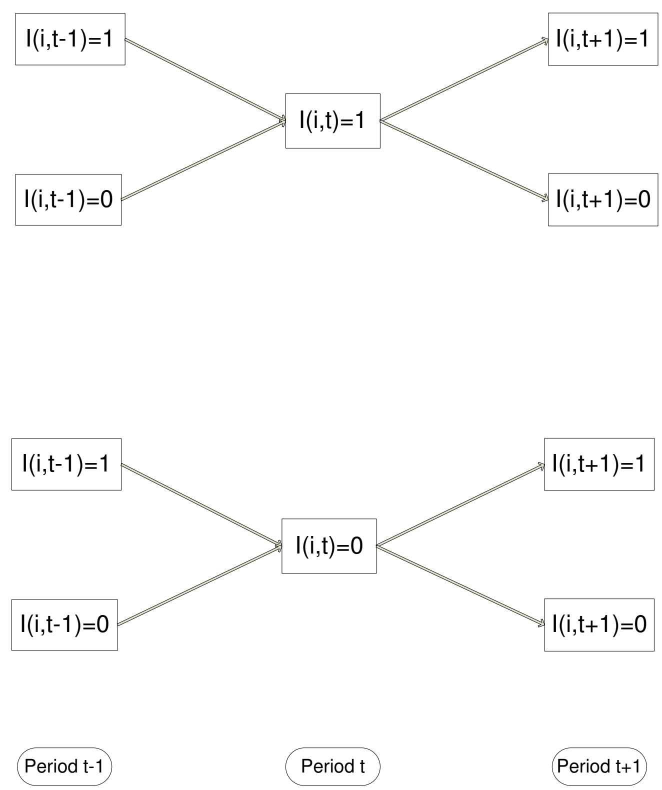


Figure 2.2: Possible states of dynamic programming.

For each one of those cases, we compute the cumulative objective function for the optimum path "until now from hour T". For a given unit i and optimum path from hour t to hour T, the cumulative objective function is defined as:

$$f_c^*(i,t) = f(i,t) + f(i,t+1) + ... + f(i,T) \tag{2.10}$$

We call it f_{c1}^* if I(i,t)=1 and f_{c0}^* if I(i,t)=0.

Knowing the state of each unit before the first scheduling hour we can then choose the best path. Hour 1 is the first hour where we have to make a decision, the status of the units at hour 0 is supposed to be known.

The flowchart of the dynamic programming can be found on figure 2.3.

2.4 Branch and Bound method

This section has been written using [4]
Branch and Bound method is based on the idea that the general set of solutions can be split into subsets of

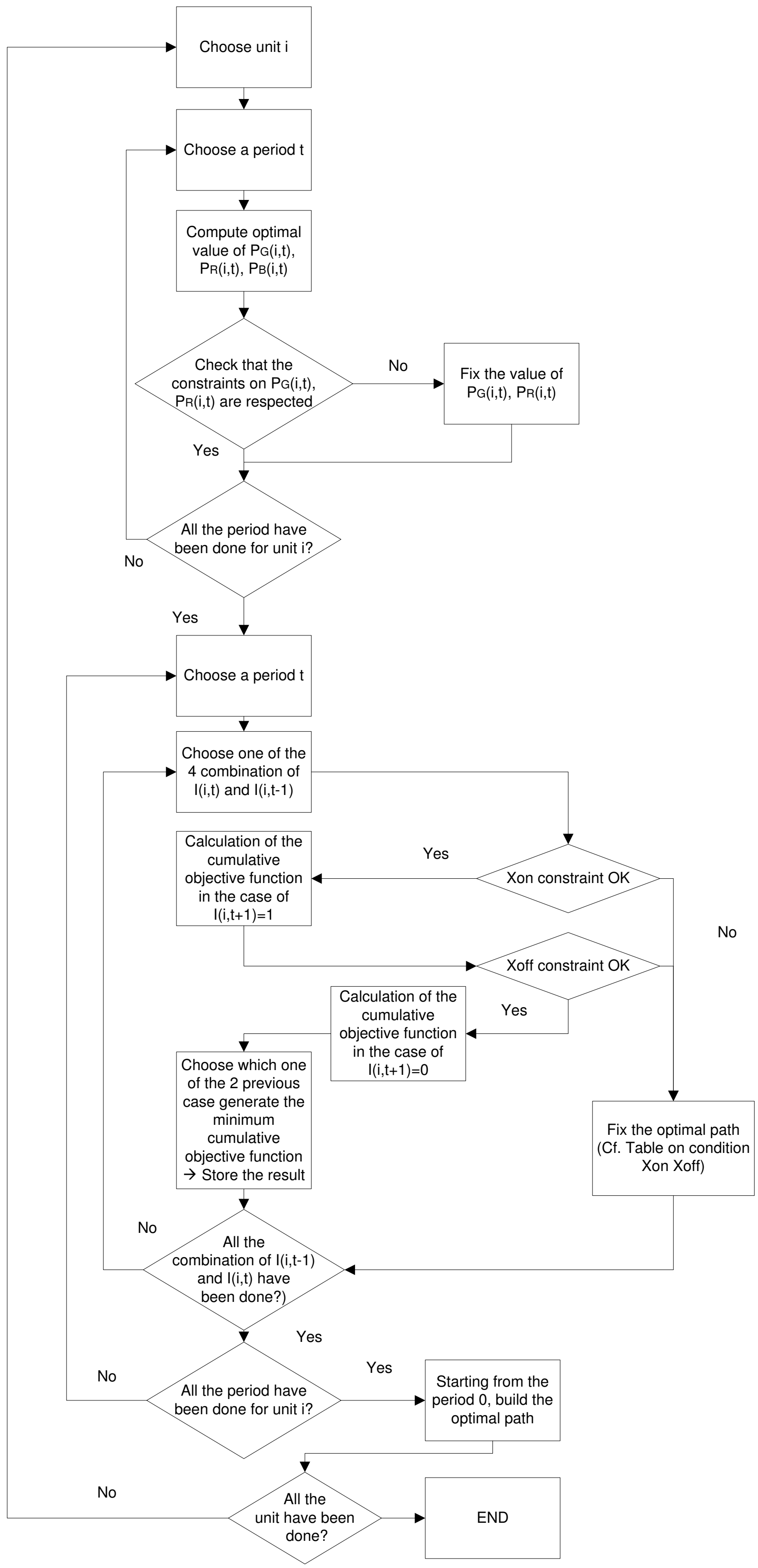


Figure 2.3: Dynamic programming algortihm

solutions. This method can be used to solve integer linear programming (i.e. LP where the variable are integer variables) in addition to a classic linear programming (For example Simplex algorithm). Considering the following optimization problem:

$$\underset{x \in X}{\text{maximize}}\ f(x) \tag{2.11}$$

These are the steps of the method:

Initialisation

We first solve the LP problem just like if the variable where not binary variables, the solution is called x^*. We thus obtain a upper bound UB that is $f(x^*)$. We now rounded x^* that give the solution x^R, and thus have a lower bound LB that is $f(x^R)$. (for example $x^* = 2,45$ give $x^R = 2$). We must be careful to keep the feasibility of x^R (i.e. $x^R \in X$). The optimal solution will be between LB and UB.

Iterations

We now choose the variable that is the farthest from its rounded-value (We call it $x_1 = 5,56$) and one constraint to our linear programming problem. We make two cases, one by adding $x_1 \leq 5$ and the other by adding $x_1 \geq 6$. We resolve the two new LP problems, round the solution and compute the LB and UP for each of them. We keep the best LB. If both rounded solution are feasible, we choose the to keep the constraints for which the UP is the highest (The maximum value that can be possibly reach is the highest). We now repeat this step.

Terminations

The iterations terminate when UB=LB.

In the case of mixed integer linear programming (i.e. some variable are integer, some are continuous in the same LP problem), we only round the integer variables.

2.5 Two-stage stochastic linear programming

This section have been written with the help of [8]

2.5.1 Formulating the problem

Stochastic programming is an approach that enables us to solve optimization problem subject to uncertainty. It enables us to deal with parameters whose values are unknown at the time a decision should be made (In our case, such parameters include the spot market price, the wind power production,...etc.).

The basic idea of a two-stage stochastic programming is that the optimal solution given by our program should not solely depend on a specific future realization of the stochastic parameters. The optimal solution should only be made on data available at the time of the decision.
Two stage in the title refers to:

- **First stage**: that is the time of the decision when we still do not know the exact realization of some parameters (e.g. at time $t-24h$, the spot price at time t is unknown but we have to make a unit commitment scheduling)
- **Second stage**: that is the time where the unknown parameters take a given value (e.g. at time t, the spot price at this time t is knows)

We put the two-stage stochastic linear programming problem in the following form:

$$\underset{x\in X}{\text{minimize}} \; \{g(x) = f_{sp}(x) + \mathbb{E}[Q(x,\xi)]\} \tag{2.12}$$

Where $\mathbf{x} \in \mathbb{R}^n$ is the first-stage decision vector, X is a polyhedral set defined by a finite number of linear constraints, f_{sp} is a linear function and $Q(x,\xi)$ is the optimal value of the second-stage problem:

$$\begin{aligned} \underset{y}{\text{minimize}} \quad & \mathbf{q_{vect}}^T \mathbf{y} \\ \text{subject to} \quad & \mathbf{Tx} + \mathbf{Wy} \leq \mathbf{h}_{sp} \end{aligned} \tag{2.13}$$

Where $\mathbf{y} \in \mathbb{R}^n$ is the second-stage decision vector and $\xi = \{q_{vect}, T, W, h_{sp}\}$ is a random vector that contain the uncertain data. One specific realization of ξ is called a scenario ξ_k

So we have to make a decision x before knowing the realization of ξ. The second stage problem can be seen as the optimal behavior once the uncertain data are revealed. We must be careful in the modeling to ensure that $Q(x,\xi) = -\infty$ never happen for all $x \in X$ and ξ_k. Finally, if for some $x \in X$ and a scenario ξ_k the system $\mathbf{Tx} + \mathbf{Wy} \leq \mathbf{h}_{sp}$ is infeasible, we usually take $Q(x,\xi) = \infty$.

2.5.2 Approximating the solution

As the vector ξ is a continuous vector for which we suppose to know the probability distribution function, there is an infinite number of scenarios. We thus use Monte Carlo simulation to reduce the scenario set to a manageable size. Given a sample $\xi_1, ..., \xi_N$ where each ξ_j, $j \in \{1, ..., N\}$ are independently identically distributed of each other and has the same distribution as ξ we can use $\hat{q}_N^{sp}(x)$ to approximate $q^{sp}(x) = \mathbb{E}[Q(x,\xi)]$:

$$\hat{q}_N^{sp}(x) = \frac{1}{N} \sum_{j=1}^{N} Q(x,\xi_j) \tag{2.14}$$

And thus our general problem (2.12) become:

$$\underset{x\in X}{\text{minimize}} \left\{ \hat{g}_N(x) = f_{sp}(x) + \frac{1}{N} \sum_{j=1}^{N} Q(x,\xi_j) \right\} \tag{2.15}$$

This method is known as the *sample average approximation* method. Once the problem is formulated as in the formulation (2.15) we can solve it by a classic linear programming technique.

2.5.3 Estimation of the optimum found

As we use Monte Carlo simulation to estimate $q^{sp}(x) = \mathbb{E}[Q(x,\xi)]$ (See section 2.5.2), there is no certainty on the global optimality of the solution $\hat{x} \in X$ obtained by solving the sample average approximation problem. If $v^* = \min\limits_{x\in X} g(x)$ is the optimal solution of the real problem, we have $g(\hat{x}) \geq v^*$. We thus want to measure the quality of this solution based on $gap(\hat{x}) = g(\hat{x}) - v^*$. But as it is not computable in our case, we will estimate it with a statistical procedure.
By Monte Carlo sampling we generate N' independently identically distributed random sample ξ_j, $j = 1, ..., N'$ of ξ and thus have:

$$\hat{g}_{N'}(\hat{x}) = f_{sp}(\hat{x}) + \hat{q}_{N'}^{sp}(\hat{x}) \tag{2.16}$$

And the sample variance of $\hat{q}_{N'}^{sp}(\hat{x})$ is:

$$\hat{\sigma}_{N'}^2(\hat{x}) = \frac{1}{N'(N'-1)} \sum_{j=1}^{N'} [Q(\hat{x},\xi_j) - \hat{q}_{N'}^{sp}(\hat{x})]^2 \tag{2.17}$$

As the computation of $Q(\hat{x},\xi_j)$ is made by only solving the second stage problem, we can use a large value of N'.
We now define a $100(1-\alpha)\%$ confidence upper bound for $g(\hat{x})$:

$$U_{N'}(\hat{x}) = \hat{g}_{N'}(\hat{x}) + z_\alpha \hat{\sigma}_{N'}(\hat{x}) \tag{2.18}$$

Where $z_\alpha = \Phi^{-1}(1-\alpha)$, with $\Phi(z)$ the CDF of the normal distribution. Those expressions and bound come from the Central Limit Theorem.
We now want to give a lower bound to v^*. We define $\hat{v}_N$ as the optimal value of the SAA complete problem (First and second stage) above. As we know, the sample average is an unbiased estimator of the expectation, thus $\mathbb{E}[\hat{g}_N(x)]$=g(x). So for $x \in X$, we have:

$$g(x) = \mathbb{E}[\hat{g}_N(x)] \geq \mathbb{E}[\inf_{x' \in X} \hat{g}_N(x')] = \mathbb{E}[\hat{v}_N] \tag{2.19}$$

Thus $v^* \geq \mathbb{E}[\hat{v}_N]$. To estimate $\mathbb{E}[\hat{v}_N]$ we solve the SAA problem to optimality H times (H problems, each based on a specific sample of size N that are all independently generated). We note $\hat{v}_N^1, ..., \hat{v}_N^H$ the computed optimal solution for each of these problems. We use the sample average $\bar{v}_{N,H}$ as an unbiased estimator of $\mathbb{E}[\hat{v}_N]$:

$$\bar{v}_{N,H} = \frac{1}{H}\sum_{j=1}^{H} \hat{v}_N^j \tag{2.20}$$

The variance of $\bar{v}_{N,H}$ is estimated by the sample variance:

$$\hat{\sigma}^2_{N,H} = \frac{1}{H(H-1)}\sum_{j=1}^{H} [\hat{v}_N^j - \bar{v}_{N,H}]^2 \tag{2.21}$$

So, the same way we proceed for the upper bound, we define a lower bound for $\mathbb{E}[\hat{v}_N]$ by:

$$L_{N,H} = \bar{v}_{N,H} + t_{\alpha,\nu}\hat{\sigma}_{N,H} \tag{2.22}$$

Where $\nu = H-1$and $t_{\alpha,\nu}$ is the α-critical value of the t-distribution with ν degrees of freedom. In application small values of H are sufficient (H=5 or 10).

We now can give a statistically valid bound (confidence of at least $1-2\alpha$) on the true $gap(\hat{x})$:

$$\widehat{gap}(\hat{x}) = U_{N'}(\hat{x}) - L_{N,H} \tag{2.23}$$

Chapter 3

Deterministic price-based unit commitment scheduling

3.1 PBUC: Without unit coupling constraints

In this model, we assume that all the power produced is sold to the spot market. Bilateral contracts are not considered and the following constraints for each unit are considered:

- Minimum MDT (Minimum number of periods that the production units must stay OFF when turned OFF) and MUT (likewise with ON)
- Ramp down and ramp up rates
- Shut-down and start-up costs
- Minimum (P_{min}) and maximum power output (P_{max}) for each unit

3.1.1 Optimum production of each unit during each period

With the decision variables being $\forall(i,t)$:

- $\mathbf{P^G(i,t)}$, the energy produced during period t by unit i
- $\mathbf{I(i,t)}$, the status of the unit i at period t (0 if OFF, 1 if ON)

And the linked variables being $\forall(i,t)$:

- $\mathbf{SD(i,t)}$, the start-up cost of unit i at period t
- $\mathbf{SU(i,t)}$, the shut-down cost of unit i at period t
- $\mathbf{Xon(i,t)}$ being the number of period the generator i has been ON in a row until period t
- $\mathbf{Xoff(i,t)}$ being the number of period the generator i has been OFF in a row until period t

With $\forall(i,t)$:

$$\begin{cases} SD(i,t) = SD(i) & \text{If I(i,t-1)=1 and I(i,t)=0} \\ SD(i,t) = 0 & \text{Otherwise} \end{cases}$$

$$\begin{cases} SU(i,t) = SU(i) & \text{If I(i,t-1)=0 and I(i,t)=1} \\ SU(i,t) = 0 & \text{Otherwise} \end{cases}$$

And

$$\begin{cases} Xon(i,t) = 0 & \text{If the unit i is Off at period t} \\ Xon(i,t) = Xon(i,t+1)+1 & \text{If the unit i is On at period t} \end{cases}$$

$$\begin{cases} Xoff(i,t) = 0 & \text{If the unit i is On at period t} \\ Xoff(i,t) = Xoff(i,t+1)+1 & \text{If the unit i is Off at period t} \end{cases}$$

The objective function that we minimize is:

$$\begin{aligned} F = \sum_{t=1}^{24} [& \underbrace{\sum_{i=1}^{N} C_i^q(P^G(i,t))\, I(i,t)}_{\text{Generation cost function}} + \underbrace{\sum_{i=1}^{N} SU(i,t)}_{\text{Start up costs}} \\ & + \underbrace{\sum_{i=1}^{N} SD(i,t)}_{\text{Shut down costs}} - \underbrace{\sum_{i=1}^{N} \rho_G(t)\, P^G(i,t)}_{\text{Income from spot market}}] \end{aligned} \tag{3.1}$$

F represents the net cost for the GENCO over a 24 hours period. We can take the summation over all units out:

$$F = \sum_{t=1}^{T}\sum_{i=1}^{N} f(i,t) \tag{3.2}$$

where

$$f(i,t) = C_i^q(P^G(i,t))\, I(i,t) + SU(i,t) + SD(i,t) - \rho_G(t)\, P^G(i,t) \tag{3.3}$$

Knowing C_i^q(P)=a(i)+b(i) P+c(i) P^2 is the generation cost function of unit i.

This objective function $f(i,t)$ is subject to the following constraints:

$$\begin{aligned} & P_{min}(i).I(i,t) \le P^G(i,t) \le P_{max}(i)\, I(i,t) && \text{Generation constraint} \\ & P^G(i,t) - P^G(i,t-1) \le RU(i) && \text{Ramp up rate constraint} \\ & P^G(i,t-1) - P^G(i,t) \le RD(i) && \text{Ramp down rate constraint} \end{aligned} \tag{3.4}$$

In the particular case of $RD(i) > P_{min}(i)$, the unit will need at least two periods to completely turn off. The model does not take care of that implicitly, thus we must take care of it with the help of the constants $MDT(i)$ and $SD(i)$. A similar procedure must be done to take care of the case $UR(i) < P_{min}(i)$ and $MUT(i)$, $SU(i)$.

The following constraints is taken into account:

If $I(i,t+1) = 1$ having $I(i,t) = 0$ is possible only if $Xon(i,t+1) \ge MUT(i)$
If $I(i,t+1) = 0$ having $I(i,t) = 1$ is possible only if $Xoff(i,t+1) \ge MDT(i)$

We first found the minimum of the function f $\forall i,t$:

$$P^G(i,t) = \frac{\rho_G(t) - b(i)}{2\, c(i)} \tag{3.5}$$

Taking into account the constraints (3.4), we correct the value of P(i,t) with the following method:

First step: We create a binary vector Z (Which looks like [1011...]) where:

1. The first binary variable representing $P_{min}(i) \le P^G(i,t)$ when equal to 1, the opposite when equal to 0
2. The second binary variable representing $P^G(i,t) \le P_{max}(i)$ when equal to 1, the opposite when equal to 0
3. The third binary variable representing $P^G(i,t) - P^G(i,t-1) \le RU(i)$ when equal to 1, the opposite when equal to 0

4. The fourth binary variable representing $P^G(i,t-1) - P^G(i,t) \leq RD(i)$ when equal to 1, the opposite when equal to 0

5. And so on...

Second step: After computation of the optimal $P^G(i,t)$ in (3.5), we compute the value of this vector Z and correct the values of P^G through a "filter" as below:

$$\begin{array}{ll} P^G(i,t) = P_{min}(i) & \text{if } Z \in A \\ P^G(i,t) = P_{max}(i) & \text{if } Z \in B \\ P^G(i,t) = P^G(i,t-1) - RD(i) & \text{if } Z \in C \\ P^G(i,t) = RU(i) + P^G(i,t-1) & \text{if } Z \in D \end{array} \tag{3.6}$$

With A, B, C, D being sets of binary vectors representing possible combinations of conditions.

The constraints on MUT and MDT is managed in the dynamic programming part (see part 2.3) by deleting the impossible combinations of variable $I(i,t)$. At period t, those constraints are respected by forcing the value of the optimal path at period t to the only feasible one. For each case, we have the following table 3.1 (In this table the "X" represents an impossible case, in the cases columns 10|1 means $I(t-1) = 1$, $I(t) = 0$, $I(t+1) = 1$ and so on):

Table 3.1: Feasibility of the different combinations of states

Cases	Xon(t+1) ≥MUT	Xoff(t+1) ≥ MDT	Xon(t+1) < MUT	Xoff(t+1) < MDT
11|1	Feasible	X	Feasible	X
11|0	X	Feasible	X	NOT Feasible
10|1	Feasible	X	NOT Feasible	X
10|0	X	Feasible	X	Feasible if MDT-Xoff(t+1)=1
01|1	Feasible	X	Feasible if MUT-Xon(t+1)=1	X
01|0	X	Feasible	X	NOT Feasible
00|1	Feasible	X	NOT Feasible	X
00|0	X	Feasible	X	Feasible

We now add the unit coupling constraints to this model.

3.2 PBUC: With unit coupling constraints

3.2.1 Description

We now add to the first model the following elements in the **objective function**:

- A possibility of buying power from the spot market
- A possibility of selling UP regulation spinning reserve

The **new constraints** that couple different units together, for each hour:

- The amount $P_B(t)$ of bilateral contract power must be fulfilled
- The sum of the GENCO production must not be greater than the forecasted load
- The sum of the GENCO reserve bids must not be greater than the forecasted reserve

3.2.2 The mathematical formulation

We define the following minimization objective function F for a 24h period:

$$F = \sum_{t=1}^{24}[\sum_{i=1}^{N} \underbrace{[C_i^q(P^G(i,t)+P^R(i,t))I(i,t)}_{\text{Generation costs}} + \underbrace{SU(i,t)+SD(i,t)}_{\text{Start-up and Shut-down costs}} \\ \underbrace{-\rho_R(t)\ P^R(i,t)}_{\text{Income from reserve market}} \ -\underbrace{\rho_G(t)\ (P^G(i,t)-P^B(i,t))}_{\text{Income from spot market}} - \underbrace{\rho_B(t)\ P^B(i,t)}_{\text{Income from bilateral contracts}} \ ]] \tag{3.7}$$

With the added decision variables, $\forall(i,t)$:

- $\mathbf{P^R}(i,t)$ being the UP regulation spinning reserve for unit i at period t
- $\mathbf{P^B}(i,t)$ being the power for unit i at period t that is going to be sold to bilateral contract. (Knowing that $\sum_{i=1}^{N} P^B(i,t) = P_B(t)$) $P_B(i,t)$ comes from generator production or power buy on spot market or both.

This objective function is subject to the following constraints:

$$\begin{aligned} &\forall(i,t) : P_{min}(i)\ I(i,t) \leq P^G(i,t) \leq P_{max}(i)\ I(i,t) && \text{Generation constraint} \\ &\forall(i,t) : P^G(i,t)+P^R(i,t) \leq P_{max}(i)\ I(i,t) && \text{Generation constraint} \\ &\forall(i,t) : 0 \leq P^R(i,t) \leq (P_{max}(i)-P_{min}(i))\ I(i,t) && \text{Generation constraint} \end{aligned} \tag{3.8}$$

$$\forall(i,t) : [P^G(i,t-1)+P^R(i,t-1)]-[P^G(i,t)+P^R(i,t)] \leq RD(i) \quad \text{Ramp down rate constraint} \tag{3.9}$$

$$\forall(i,t) : [P^G(i,t)+P^R(i,t)]-[P^G(i,t-1)+P^R(i,t-1)] \leq RU(i) \quad \text{Ramp up rate constraint} \tag{3.10}$$

$$\forall t : \textstyle\sum_{all\ unit\ i} P^G(i,t)I(i,t) \leq \bar{D}(t) \quad \text{Energy demand of the global system constraint} \tag{3.11}$$

$$\forall t : \textstyle\sum_{all\ unit\ i} P^R(i,t)I(i,t) \leq \bar{R}(t) \quad \text{Reserve demand of the global system constraint} \tag{3.12}$$

$$\forall t : \textstyle\sum_{all\ unit\ i} P^B(i,t) = P_B(t) \quad \text{Bilateral contracts constraint} \tag{3.13}$$

And the constraints on MUT and MDT (see part 3.1.1):

If I(i,t+1)=1 having I(i,t)=0 is possible only if $Xon(i,t+1) \geq MUT(i)$
If I(i,t+1)=0 having I(i,t)=1 is possible only if $Xoff(i,t+1) \geq MDT(i)$

3.2.3 Solving the model

As previously we use dynamic programming to solve this problem. But this time, because of the unit coupling constraints (3.11), (3.12) we can not solve the problem for each unit. So if we keep only using dynamic programming, we would have a 2^N stages problem. This mean that for a 20 power units' problem, we would have approximately 1 million of stages (i.e. possible state of the system) at each period. This would have some serious computation time issues. We thus want to perform the dynamic programming for each unit as previously and therefore introduce the Lagrangian relaxation of the coupling constraints.

It is interesting to notice that the constraint (3.13) is a unit coupling constraint but does not affect the dynamic programming part as we obtain the value of $P^B(i,t)$, through an Economic Power Dispatch, before the dynamic programming (EPD, see lambda iteration technique 2.2).

By relaxing the constraints (3.11), (3.12) we have the following Lagrangian function:

$$
\begin{aligned}
L = \sum_{t=1}^{24}[\sum_{i=1}^{N}[C_i^q(P^G(i,t)+P^R(i,t))\ I(i,t)+SU(i,t)+SD(i,t) \\
-\rho_R(t)\ P^R(i,t)-\rho_G(t)\ (P^G(i,t)-P^B(i,t))-\rho_B(t)\ P^B(i,t)]] \\
+\underbrace{\mu_R(t)\ (\sum_{i=1}^{N}(P^R(i,t)I(i,t))-\bar{R}(t))}_{\text{Relaxation term 1}} \\
+\underbrace{\mu_G(t)\ (\sum_{i=1}^{N}(P^G(i,t)I(i,t))-\bar{D}(t))}_{\text{Relaxation term 2}}
\end{aligned}
\tag{3.14}
$$

As $\bar{R}(t)$ and $\bar{D}(t)$ are constants at every period t, we can remove them from Lagrangian function. However we must use the global expression for the update of the Lagrangian multipliers (see part 3.2.4).
To reduce the number of state of the dynamic programming we can solve the optimal schedule for each unit i at period t:

$$
\begin{aligned}
l(i,t) =& C_i(P^G(i,t)+P^R(i,t))\ I(i,t)+SU(i,t)+SD(i,t) \\
&-\rho_R(t)\ P^R(i,t)-\rho_G(t)\ (P^G(i,t)-P^B(i,t))-\rho_B(t)\ P^B(i,t) \\
&+\mu_R(t)\ P^R(i,t)I(i,t) \\
&+\mu_G(t)\ P^G(i,t)I(i,t)
\end{aligned}
\tag{3.15}
$$

We work with an iterative algorithm, so it means that for each iteration all the μ are fixed (We will see later how to update those μ between each iteration).

To perform our dynamic programming, we need at each iteration an optimal value P^{R*}, P^{G*} and P^{B*} assuming that all the units are ON. To find those optimal values, we perform two different economic power dispatch (EPD). To find P^{B*} we use a lambda iteration technique (see 2.2). For P^{R*} and P^{G*} we proceed as follow.

We first want to find the optimal P^{R*} at a given iteration when the unit is ON. For that we search for the minimum of $l(i,t)$ in the direction of P^R for all unit i and period t:

$$\frac{\partial l}{\partial P^R}(i,t) = \mu_R(t)+b(i)+2c(i)(P^G(i,t)+P^R(i,t))-\rho_R(t)=0 \tag{3.16}$$

We do the same for P^{G*}

$$\frac{\partial l}{\partial P^G}(i,t) = \mu_G(t)+b(i)+2c(i)(P^G(i,t)+P^R(i,t))-\rho_G(t)=0 \tag{3.17}$$

Where $C_i^q(P)=a(i)+b(i)P+c(i)P^2$ is the generation cost function of unit i.
As the system is not perfectly solvable and we are not considering the DOWN regulation market, we approximate the results in the case of $\rho_R(t)\geq\rho_G(t)$ by:

$$P^{R*}(i,t)=\frac{\rho_R(t)-b(i)-\mu_R(t)}{2c(i)}-P_{min}(i) \tag{3.18}$$

$$P^{G*}(i,t)=\frac{\rho_G(t)-b(i)-\mu_G(t)}{2c(i)}-P^{R*}(i,t) \tag{3.19}$$

But the results above are not necessarily respecting the constraints (3.8), (3.9), (3.10). After computing the theoretical optimal value we set these values to the limit. For that we just give to every constraint a binary value equal to 0 if the constraints is not fulfilled. So we have 7 inequality constraints in the equations (3.8), (3.9), (3.10). That mean theoritically $2^7 = 128$ possible combinations. By eliminating all the impossible cases we reduce the number of combinations to 31. We then fix the value of $P^{R*}(i,t)$ and $P^{G*}(i,t)$ depending on the state of the constraints (Similar to the approach in equation (3.6)).

Finding the optimal value P^{B*} is a bit more complex as this time the Lagrangian function is linear on P^B direction. So to dispatch the amount of bilateral contracts among the N units, we use Lambda iteration technique, with the key idea that the incremental cost of any generator must not be greater than the spot market price. If it is, we had better buy the power on the spot market to fulfill the bilateral contracts requirements.

3.2.4 Updating the Lagrangian multipliers

After iteration of the algorithm, we update the Lagrangian multipliers with the sub-gradient method.

$$\forall t : \mu_R(t) = max(\ 0\ ,\ \mu_R(t) - \alpha^R(t)\ (\sum_{all\ unit\ i} P^R(i,t) - \bar{R}(t))) \tag{3.20}$$

$$\forall t : \mu_G(t) = max(\ 0\ ,\ \mu_G(t) - \alpha^G(t)\ (\sum_{all\ unit\ i} P^G(i,t) - \bar{D}(t))) \tag{3.21}$$

With the step size α_k computed using the following heuristic rule (Suitable in our case):
In the case of $\alpha_R(t)$ (idem for $\alpha_G(t)$)
If $Grad_R(t) < 0$:

$$\alpha_R(t) = 0,002 \tag{3.22}$$

Else:

$$\alpha_R(t) = 0,01 \tag{3.23}$$

With:

- $Grad_R(t) = \sum_{all\ unit\ i}(P^R(i,t) - \bar{R}(t))$
- $Grad_G(t) = \sum_{all\ unit\ i}(P^G(i,t) - \bar{D}(t))$

3.2.5 Computation of the relative duality gap

To define if we have to stop the algorithm, we compare at every iteration the relative duality gap (RDG) with a threshold predefined. The idea is:

"We know that if we find a couple x^* and λ^* such that $L(x^*|\lambda^*) = f(x^*)$, x^* is a global optimal solution to the primal problem."

Thus the RDG is a measure of the distance between $f(x)$ and $L(x|\lambda)$ at a given couple (x,λ). When $RDG \to 0$ the algorithm can be stopped.

The relative duality gap at a given iteration is defined as:

$$RDG = \left| \frac{F(P^G,P^R,P^B,I) - L(P^G,P^R,P^B,I,\mu_R,\mu_G)}{L(P^G,P^R,P^B,I,\mu_R,\mu_G)} \right| \tag{3.24}$$

The flowchart of the algorithm can be found in figure 3.1

3.3 Results

By running the program described in part 3.2 of this thesis on a 24h period, we obtain the following results.

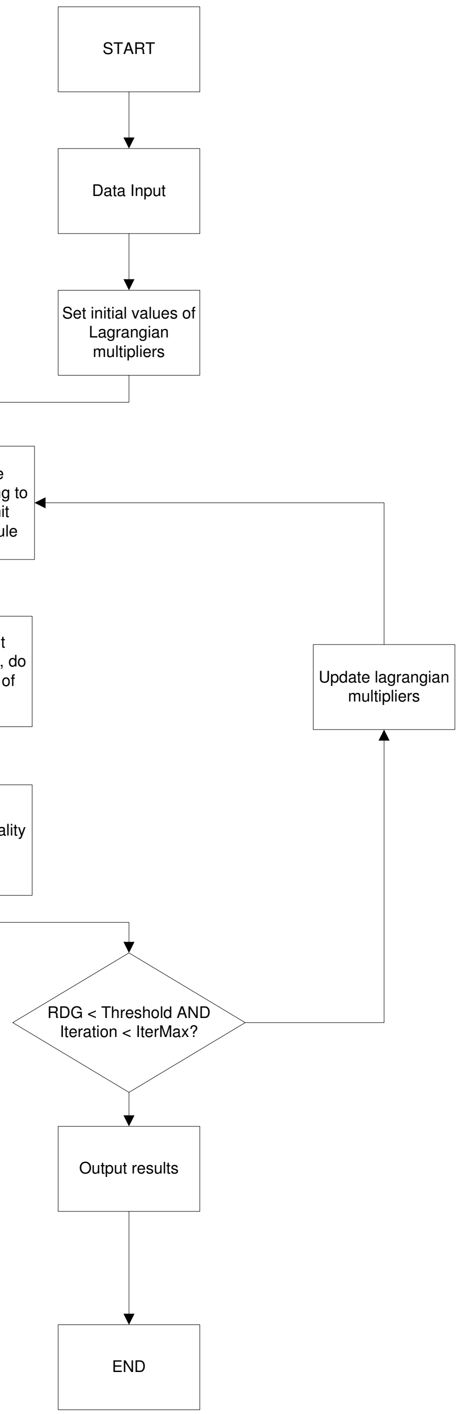


Figure 3.1: Lagrangian relaxation and Dynamic programming algortihm

In figure 3.2, we can see the unit commitment schedule (Solid point when the unit is ON, no point when the unit is OFF) and the spot prices for a 24 h period. We can notice that when the spot price is low, some units are turned OFF.

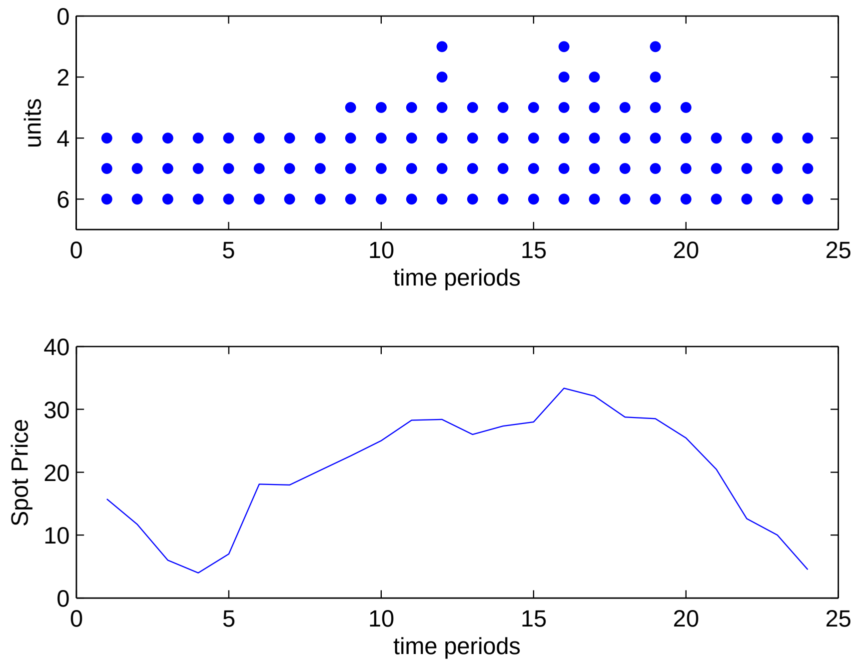


Figure 3.2: Unit commitment schedule

In figure 3.3, we can observe the corresponding sum of energy dispatch. *Pr* represents the sum of energy sold to up regulation market and *Pg* represents the sum of energy planned to be produced. We can notice that the maximum generation capacity 1200MW is reached when the Spot price reach peaks (period 12 and 16). We can observe the initial condition of production at period 0 and the fact that the algorithm prefer to sell reserve energy because it is better paid. The constraints (3.12) forces the algorithm not to sell irrealistic amount of reserve.

To illustrate the convergence of our algorithm, we can see the plot of the relative duality gap (RDG) in the figure 3.4. We can see that the program does converge in a few number of iterations (approximately 20 iterations). The growing RDG between iteration 0 and 3 represents the calibration of the RDG. It is important to notice that this does not necessarily represent a convergence to the optimal solution of the global PBUC problem. It only represents a convergence in our Lagrangian relaxation. As we are using other optimization technics coupled with Lagrangian, this can not be a perfect proof of optimality. However, knowing that the dynamic programming always give an optimal solution, we can be quite confident on the optimality of the solution.

The conclusions of using LR-DynP algorithm to solve PBUC problems are:

- This algorithm is not very efficient in computation time (Computation time $\approx 2s$ for 45 iterations)
- This algorithm implemented is very problem specific to this PBUC formulation, changing a few characteristics in the optimization problem and a complete re-design will be needed (Specially in the EPD part)

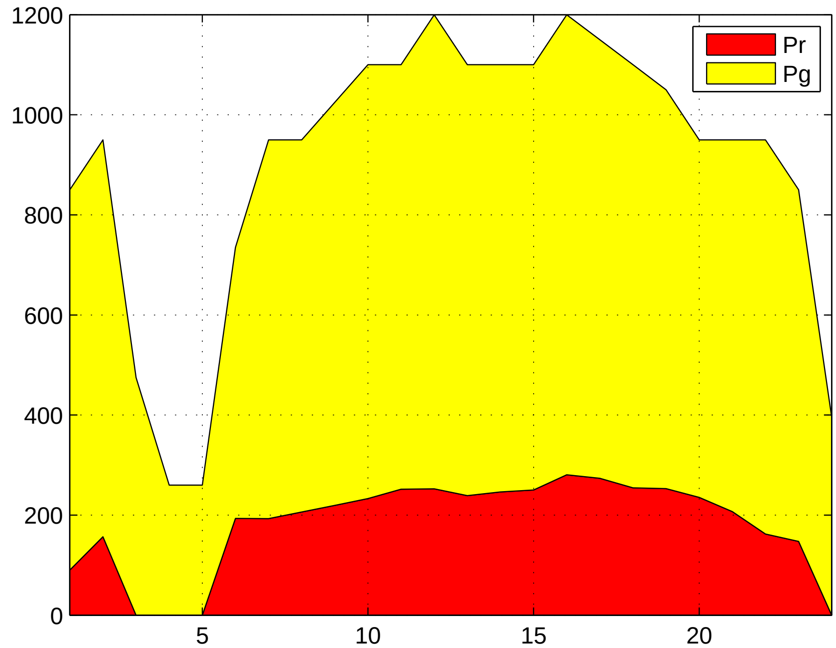


Figure 3.3: Power production planned

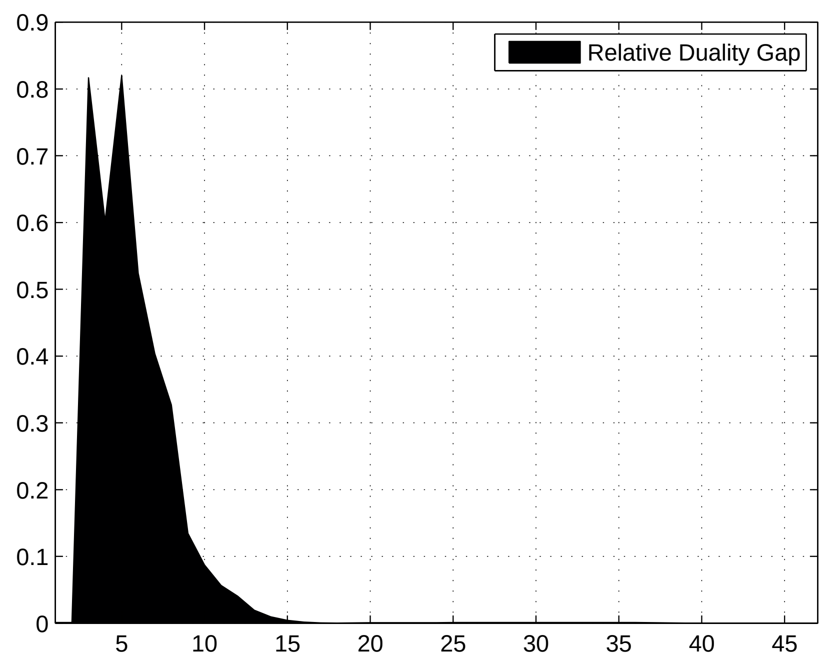


Figure 3.4: Relative Duality Gap

Chapter 4

Stochastic price-based unit commitment

4.1 Description

In this chapter, we introduce the parameter uncertainty into our PBUC formulation. For that we consider that the spot price ρ_{spot}, the wind power production P_{wind} and system imbalance ΔP (At the system scale, too much production $\Delta P > 0$ or too much consumption $\Delta P < 0$) are stochastic variables for which we know the distribution functions. In addition, we add 2 dependent stochastic variables, i.e. the down regulation price $\rho^{rg}_{down} = 0.1\rho_{spot}$ and the up regulation price $\rho^{rg}_{up} = 10\rho_{spot}$.

Using the stochastic programming framework described in section 2.5, we formulate a stochastic PBUC model to take into account the foregoing uncertainties. As seen in the previous chapter, using LR-DynP algorithm may have a disadvantage in computational speed. Therefore, we will use a MILP solver from the Xpress Optimization Suite. In order to use MILP, we have to formulate our stochastic PBUC problem in a linear form.

4.2 FICO™Xpress Optimization Suite

FICO™Xpress Optimization Suite is a suite of software using Xpress-Optimizer and Xpress-Mosel to solve optimization problems. Xpress-Optimizer is the optimization engine that can solve the following types of optimization problems [5]:

- Linear programming LP
- Mixed integer linear programming MILP
- Quadratic programming QP
- Mixed integer quadratic programming MILP
- Quadratically constrained programming QCQP
- Mixed integer quadratically constrained programming MIQCQP
- Convex general non-linear programming NLP

The solver uses simplex and barrier methods, a branch-and-bound algorithm, but also heuristics methods. Xpress-Mosel is the language used to formulate our problem. As we will see it later, it is very powerful to solve our problem.

4.3 Assumption on the energy bids

We assume in this thesis that:

- We bid at 0 $/MWh

- We are a price-taker GENCO (Our bids do not impact the electricity market price)
- We act in an electric system where the sum of all the energy bids of the price-taker GENCOs is always negligible compared to the amount of energy selected by the market

Thus, it is reasonable to assume that all energy bids made on the different markets are accepted.

4.4 Linearizing the problem

In order to be able to use MILP technique, we need to formulate our PBUC formulation in a linear form, both for the objective function and the constraints. In particular, quadratic cost function of generators need to be linearized.

4.4.1 Generation costs functions

In our deterministic models, we have used a quadratic generation cost function C_i^q of the form:

$$\forall i : C_i^q(P) = a(i) + b(i)P + c(i)P^2 \tag{4.1}$$

A linearized form of (4.1) can be expressed as:

$$\forall i : C_i^l(P) = C_A(i) + C_{B1}(i)P_1 + C_{B2}(i)P_2 + C_{B3}(i)P_3 \tag{4.2}$$

with the constraints:

$$\begin{aligned} \forall (i,t) &: P_{min}(i)I(i,t) \leq P^{G1}(i,t) \leq E_1(i)I(i,t) \\ \forall (i,t) &: 0 \leq P^{G2}(i,t) \leq [E_2(i) - E_1(i)]I(i,t) \\ \forall (i,t) &: 0 \leq P^{G3}(i,t) \leq [P_{max}(i) - E_2(i)]I(i,t) \end{aligned} \tag{4.3}$$

We now have $P^G = P^{G1} + P^{G2} + P^{G3}$.

According to the quadratic cost function, the coefficients of the linear cost function in (4.2) and (4.3) can be determined by:

$$\begin{aligned} \forall i : E_1(i) &= \frac{[P_{max}(i) - P_{min}(i)]}{3} + P_{min}(i) \\ \forall i : E_2(i) &= 2\frac{[P_{max}(i) - P_{min}(i)]}{3} + P_{min}(i) \end{aligned} \tag{4.4}$$

$$\forall i : C_{B1}(i) = \frac{[C_i^q(E_1(i)) - C_i^q(P_{min}(i))]}{E_1(i) - P_{min}(i)} \tag{4.5}$$

$$\forall i : C_{B2}(i) = \frac{[C_i^q(E_2(i)) - C_i^q(E_1(i))]}{E_2(i) - E_1(i)} \tag{4.6}$$

$$\forall i : C_{B3}(i) = \frac{[C_i^q(P_{max}(i)) - C_i^q(E_2(i))]}{P_{max}(i) - E_2(i)} \tag{4.7}$$

$$\forall i : C_A(i) = C_i^q(P_{min}(i)) - C_{B1}(i)P_{min}(i) \tag{4.8}$$

4.4.2 Particular case of linearizing a non-linear constraint

Most of the time, non-linear constraints can be formulated into several linear constraints. In our case, we have non-linear constraints involving the product of a binary decision variable b and a real continuous decision variable r in an inequality constraint (see constraints (4.43), (4.45), (4.47), (4.50), (4.52) and (4.54)):

$$p \leq b\, r \tag{4.9}$$

If we suppose that the decision variable p has some upper bound U, then the non-linear inequality (4.9) is equivalent to the two linear inequalities (4.10) and (4.11):

$$p \leq r \tag{4.10}$$

$$p \leq b\, U \tag{4.11}$$

4.5 Mathematical formulation

In the following part of this section, for the initial conditions we have $\forall(i,\xi)$: $\begin{pmatrix} I(i,0) = I0(i) \\ P^{G}(i,0) = P0(i) \\ P^{+}(i,0,\xi) = 0 \\ P^{-}(i,0,\xi) = 0 \end{pmatrix}$

4.5.1 First-stage formulation

The **main first stage decision variables** are $\forall(i,t)$:

$$\begin{pmatrix} I(i,t) \\ P^{G}(i,t) \\ P^{G1}(i,t) \\ P^{G2}(i,t) \\ P^{G3}(i,t) \\ P_{spot}(t) \end{pmatrix}$$

The **auxiliary first stage decision variables** are $\forall(i,t)$:

$$\begin{pmatrix} x_{cstrt}(i,t) \\ y_{cstrt}(i,t) \\ z_{cstrt}(i,t) \end{pmatrix}$$

The auxiliary variables do not have physical meanings, but are introduced for the linear formulation of the program.

We define the following **objective function** F for a 24 h period:

$$F = \sum_{t=1}^{24} [\underbrace{\sum_{i=1}^{N} [f(i,t)]}_{\text{Costs of the first stage decision}} + \underbrace{\mathbb{E}[Q(x,\xi)]}_{\text{Expectancy of the optimal solution of the second stage problem}} \tag{4.12}$$

With

$$\begin{aligned} f(i,t) = & \underbrace{I(i,t) * C_{A}(i) + C_{B1}(i)P^{G1}(i,t) + C_{B2}(i)P^{G2}(i,t) + C_{B3}(i)P^{G3}(i,t)}_{\text{Generation costs of the first stage decision}} \\ & + \underbrace{SU(i)(I(i,t) - x_{cstrt}(i,t))}_{\text{Start up costs}} + \underbrace{SD(i)(I(i,t-1) - x_{cstrt}(i,t))}_{\text{Shut down costs}} \end{aligned} \tag{4.13}$$

This objective function will be minimized and is subject to the following constraints [9]:

Binary constraints

$$\forall(i,t): x_{cstrt}(i,t) \in \{0,1\} \tag{4.14}$$

$$\forall(i,t): y_{cstrt}(i,t) \in \{0,1\} \tag{4.15}$$

$$\forall(i,t): z_{cstrt}(i,t) \in \{0,1\} \tag{4.16}$$

$$\forall(i,t): I(i,t) \in \{0,1\} \tag{4.17}$$

Free variables

$$\forall(t): P^{spot}(t) \in \mathbb{R} \tag{4.18}$$

Unit generation constraints on P^G

$$\forall(i,t): P_{min}(i) I(i,t) \leq P^{G1}(i,t) \leq E_1(i)\, I(i,t) \tag{4.19}$$

$$\forall(i,t): 0 \leq P^{G2}(i,t) \leq [E_2(i) - E_1(i)]\, I(i,t) \tag{4.20}$$

$$\forall(i,t): 0 \leq P^{G3}(i,t) \leq [P_{max}(i) - E_2(i)]\, I(i,t) \tag{4.21}$$

$$\forall(i,t): P^G(i,t) = P^{G1}(i,t) + P^{G2}(i,t) + P^{G3}(i,t) \tag{4.22}$$

Unit start-up and shut-down constraints

$$\forall(i,t): x_{cstrt}(i,t) \leq I(i,t) \tag{4.23}$$

$$\forall(i,t): x_{cstrt}(i,t) \leq I(i,t-1) \tag{4.24}$$

$$\forall(i,t): x_{cstrt}(i,t) \geq I(i,t) + I(i,t-1) - 1 \tag{4.25}$$

Minimum ON and OFF time constraints

$$\forall t \in [1,...,MUT(i) - TH(i)], MUT(i) > TH(i) > 0 : I(i,t) = 1 \tag{4.26}$$

$$\forall t \in [2,...,T-1], \begin{cases} I(i,t) - I(i,t-1) \leq I(i,t+1) \\ I(i,t) - I(i,t-1) \leq I(i,t+2) \\ \quad \vdots \\ I(i,t) - I(i,t-1) \leq I(i, min\{t + MUT - 1, T\}) \end{cases} \tag{4.27}$$

$$\forall t \in [1,...,MDT(i) + TH(i)], MDT(i) < TH(i) < 0 : I(i,t) = 1 \tag{4.28}$$

$$\forall t \in [2,...,T-1], \begin{cases} I(i,t-1) - I(i,t) \leq 1 - I(i,t+1) \\ I(i,t-1) - I(i,t) \leq 1 - I(i,t+2) \\ \quad \vdots \\ I(i,t-1) - I(i,t) \leq 1 - I(i, min\{t + MUT - 1, T\}) \end{cases} \tag{4.29}$$

Ramp rate constraints

$$\forall(i,t): 1 - (I(i,t) - I(i,t-1)) \leq M(1 - y_{cstrt}(i,t))\} \tag{4.30}$$

$$\forall(i,t): 1 - (I(i,t-1) - I(i,t)) \leq M(1 - z_{cstrt}(i,t))\} \tag{4.31}$$

$$\forall(i,t,\xi) : P^G(i,t-1) - P^G(i,t) \leq max\{P_{min}(i), DR(i)\} \tag{4.32}$$

$$\forall(i,t,\xi) : P^G(i,t) - P^G(i,t-1) \leq max\{P_{min}(i), UR(i)\} \tag{4.33}$$

$$\forall(i,t,\xi) : P^G(i,t) - P^G(i,t-1) \leq My_{cstrt}(i,t) + UR(i)\} \tag{4.34}$$

$$\forall(i,t,\xi) : P^G(i,t-1) - P^G(i,t) \leq Mz_{cstrt}(i,t) + DR(i)\} \tag{4.35}$$

4.5.2 Second-stage formulation

For a given set of first stage variables and a possible realization ξ, the **main second stage decision variables** are $\forall(i,t)$:

$$\begin{pmatrix} P^+(i,t,\xi) \\ P_1^+(i,t,\xi) \\ P_2^+(i,t,\xi) \\ P_3^+(i,t,\xi) \\ P^-(i,t,\xi) \\ P_1^-(i,t,\xi) \\ P_2^-(i,t,\xi) \\ P_3^-(i,t,\xi) \end{pmatrix}$$

And the **auxiliary second-stage decision variables** are $\forall(i,t)$:

$$\begin{pmatrix} v(t,\xi) \\ w(t,\xi) \end{pmatrix}$$

If at the period t for a possible outcome ξ, the amount of power bids on the spot market $P_{spot}(t)$ is different from the actual production available $P_{wind}(t,\xi) + \sum_{i=1}^{N}[P^G(i,t)] - P_B(t)$, the corrective actions must enable the GENCO to fulfill the commitment made on the Spot market. There are two possibilities to fill the gap between the commitment on the Spot market and the actual production available (See constraint (4.64)):

- Reschedule the GENCO's units internally with the help of $P^-(i,t,\xi)$ and $P^+(i,t,\xi)$, $\forall(i,t,\xi)$.
- Purchase balancing power from the regulation market. $\forall(t,\xi)$, $v(t,\xi)$ represents the power that will be balanced by the regulation marker ($v(t,\xi) < 0$ if the GENCO under produces and $v(t,\xi) > 0$ if the GENCO over produces)

In the case we resort to the regulation market, the costs of the recourse action are described as follows:

- In the case of **over production of the GENCO**, the amount of surplus is sold to the regulation market at the price $\rho^+(t,\xi) \leq \rho_{spot}(t,\xi)$, where $\rho^+(t,\xi)$ is defined in the constraints (4.60) and (4.61)
- In the case of **under production of the GENCO**, the amount of deficit is bought from the regulation market at the price $\rho^-(t,\xi) \geq \rho_{spot}(t,\xi)$, where $\rho^-(t,\xi)$ are defined in the constraints (4.62) and (4.63)

The net imbalance costs from the regulation market is thus ($\mathbb{E}$ being the expectation):

$$\forall t:\ \mathbb{E}[\rho^-(t,\xi)(v(t,\xi))^+ - \rho^+(t,\xi)(-v(t,\xi))^+] = \mathbb{E}[Cost_{regulation}(t,\xi)] \tag{4.36}$$

with:

$$\forall(t,\xi):\ (v(t,\xi))^+ = \begin{cases} v(t,\xi) & \text{if } v(t,\xi) \geq 0 \\ 0 & \text{else.} \end{cases} \tag{4.37}$$

So for a possible scenario ξ,

$$\forall(t,\xi):\ Cost_{regulation}(t,\xi) = \begin{cases} \rho^-(t,\xi)v(t,\xi) & \text{if } v(t,\xi) \geq 0 \\ \rho^+(t,\xi)v(t,\xi) & \text{if } v(t,\xi) < 0 \end{cases} \tag{4.38}$$

As $\forall(t,\xi)$, $\rho^-(t,\xi) \geq \rho^+(t,\xi)$, (4.38) can be formulated as a simple optimization problem $\forall(t,\xi)$:

$$\begin{aligned} Cost_{regulation}(t,\xi) = & \underset{w(t,\xi)\in\mathbb{R}}{\text{minimize}} && w(t,\xi) \\ & \text{subject to} && w(t,\xi) \geq \rho^-(t,\xi)v(t,\xi) \\ & && w(t,\xi) \geq \rho^+(t,\xi)v(t,\xi) \end{aligned}$$

This leads to the constraints (4.65) and (4.66) later.

The **second stage objective function**:

$$\begin{aligned} Q(x,\xi) = \sum_{t=1}^{T}[& \underbrace{w(t,\xi)}_{\text{Net costs from regulation market}} \quad \underbrace{-\ \rho_{spot}(t,\xi)P^{spot}(t)}_{\text{Income from SPOT market}} \\ & + \sum_{i=1}^{N}[\underbrace{C_{B1}(i)P_1^+(i,t,\xi) + C_{B2}(i)P_2^+(i,t,\xi) + C_{B3}(i)P_3^+(i,t,\xi)}_{\text{Costs of increasing the power production}} \\ & \underbrace{-(C_{B1}(i)P_1^-(i,t,\xi) + C_{B2}(i)P_2^-(i,t,\xi) + C_{B3}(i)P_3^-(i,t,\xi)}_{\text{Income from reduction of the power production}})]]] \end{aligned} \tag{4.39}$$

This objective function is subject to the following constraints:

Free variables

$$\forall(t,\xi) : w(t,\xi) \in \mathbb{R} \tag{4.40}$$

$$\forall(t,\xi) : v(t,\xi) \in \mathbb{R} \tag{4.41}$$

Constraints on internal up-regulation P^+

$$\forall(i,t,\xi) : 0 \leq P_1^+(i,t,\xi) \leq E_1(i) - P^{G1}(i,t) \tag{4.42}$$

$$\forall(i,t,\xi) : P_1^+(i,t,\xi) \leq MI(i,t) \tag{4.43}$$

$$\forall(i,t,\xi) : 0 \leq P_2^+(i,t,\xi) \leq [E_2(i) - E_1(i) - P^{G2}(i,t)] \tag{4.44}$$

$$\forall(i,t,\xi) : P_2^+(i,t,\xi) \leq MI(i,t) \tag{4.45}$$

$$\forall(i,t,\xi) : 0 \leq P_3^+(i,t,\xi) \leq [P_{max}(i) - E_2(i) - P^{G3}(i,t)] \tag{4.46}$$

$$\forall(i,t,\xi) : P_3^+(i,t,\xi) \leq MI(i,t) \tag{4.47}$$

$$\forall(i,t,\xi) : P^+(i,t,s) = P_1^+(i,t,\xi) + P_2^+(i,t,\xi) + P_3^+(i,t,\xi) \tag{4.48}$$

Constraints on internal down-regulation P^-

$$\forall(i,t,\xi): 0 \leq P_1^-(i,t,\xi) \leq P^{G1}(i,t) \tag{4.49}$$

$$\forall(i,t,\xi): P_1^-(i,t,\xi) \leq MI(i,t) \tag{4.50}$$

$$\forall(i,t,\xi): 0 \leq P_2^-(i,t,\xi) \leq P^{G2}(i,t) \tag{4.51}$$

$$\forall(i,t,\xi): P_2^-(i,t,\xi) \leq MI(i,t) \tag{4.52}$$

$$\forall(i,t,\xi): 0 \leq P_3^-(i,t,\xi) \leq P^{G3}(i,t) \tag{4.53}$$

$$\forall(i,t,\xi): P_3^-(i,t,\xi) \leq MI(i,t) \tag{4.54}$$

$$\forall(i,t,\xi): P^-(i,t,s) = P_1^-(i,t,\xi) + P_2^-(i,t,\xi) + P_3^-(i,t,\xi) \tag{4.55}$$

Ramp rate constraints ([9])

$$\begin{aligned}\forall(i,t,\xi): P^G(i,t-1) + P^+(i,t-1,\xi) - P^-(i,t-1,\xi) - (P^G(i,t) + P^+(i,t,\xi) - P^-(i,t,\xi)) \\ \leq max\{P_{min}(i), DR(i)\}\end{aligned} \tag{4.56}$$

$$\begin{aligned}\forall(i,t,\xi): P^G(i,t) + P^+(i,t,\xi) - P^-(i,t,\xi) - (P^G(i,t-1) + P^+(i,t-1,\xi) - P^-(i,t-1,\xi)) \\ \leq max\{P_{min}(i), UR(i)\}\end{aligned} \tag{4.57}$$

$$\begin{aligned}\forall(i,t,\xi): P^G(i,t) + P^+(i,t,\xi) - P^-(i,t,\xi) - (P^G(i,t-1) + P^+(i,t-1,\xi) - P^-(i,t-1,\xi)) \\ \leq My(i,t) + UR(i)\}\end{aligned} \tag{4.58}$$

$$\begin{aligned}\forall(i,t,\xi): P^G(i,t-1) + P^+(i,t-1,\xi) - P^-(i,t-1,\xi) - (P^G(i,t) + P^+(i,t,\xi) - P^-(i,t,\xi)) \\ \leq Mz(i,t) + DR(i)\}\end{aligned} \tag{4.59}$$

GENCO production imbalance cases
We define:

$$\forall(t,\xi \mid \Delta P(t,\xi) > 0): \rho^+(t,\xi) = \rho^{rg}_{down}(t,\xi) \tag{4.60}$$

$$\forall(t,\xi \mid \Delta P(t,\xi) \leq 0): \rho^+(t,s) = \rho_{spot}(t,\xi) \tag{4.61}$$

$$\forall(t,\xi \mid \Delta P(t,\xi) > 0): \rho^-(t,\xi) = \rho_{spot}(t,\xi) \tag{4.62}$$

$$\forall(t,\xi \mid \Delta P(t,\xi) \leq 0): \rho^-(t,\xi) = \rho^{rg}_{up}(t,\xi) \tag{4.63}$$

Regulation constraints

$$\forall(t,\xi): v(t,\xi) + \sum_{i=1}^{N}[P^+(i,t,\xi) - P^-(i,t,\xi)] = P_{spot}(t) + P_B(t) - \sum_{i=1}^{N}[P^G(i,t)] - P_{wind}(t,\xi) \tag{4.64}$$

$$\forall(t,\xi): w(t,\xi) \geq \rho^-(t,\xi).v(t,\xi) \tag{4.65}$$

$$\forall(t,\xi): w(t,\xi) \geq \rho^+(t,\xi).v(t,\xi) \tag{4.66}$$

4.6 Sample Average approximation of the objective function

By using the Sample Average Approximation technique describe in 2.5.2 with S scenarios, we thus obtain the following objective function:

$$
\begin{aligned}
F = \sum_{t=1}^{24}[& \underbrace{\sum_{i=1}^{N}[f(i,t)]}_{\text{Costs of the first stage decision}} + \frac{1}{S}\sum_{s=1}^{S}[\underbrace{w(t,s)}_{\text{Net costs from regulation market}} \underbrace{-\rho_{spot}(t,s)P^{spot}(t)}_{\text{Income from SPOT market}} \\
& + \sum_{i=1}^{N}[\underbrace{C_{B1}(i)P_1^+(i,t,s) + C_{B2}(i)P_2^+(i,t,s) + C_{B3}(i)P_3^+(i,t,s)}_{\text{Costs of increasing the power production}} \\
& \underbrace{-(C_{B1}(i)P_1^-(i,t,s) + C_{B2}(i)P_2^-(i,t,s) + C_{B3}(i)P_3^-(i,t,s)}_{\text{Income from reduction of the power production}}]]]
\end{aligned}
\tag{4.67}
$$

With

$$
\begin{aligned}
f(i,t) = & \underbrace{I(i,t)*C_A(i) + C_{B1}(i)P^{G1}(i,t) + C_{B2}(i)P^{G2}(i,t) + C_{B3}(i)P^{G3}(i,t)}_{\text{Generation costs of the first stage decision}} \\
& + \underbrace{SU(i)(I(i,t) - x_{cstrt}(i,t))}_{\text{Start up costs}} + \underbrace{SD(i)(I(i,t-1) - x_{cstrt}(i,t))}_{\text{Shut down costs}}
\end{aligned}
\tag{4.68}
$$

This is the final objective function to be solved by using the MILP solver in Xpress.

4.7 Case study: 6 thermal generation units and a wind farm

4.7.1 Scenario generation of the stochastic variables scenarios

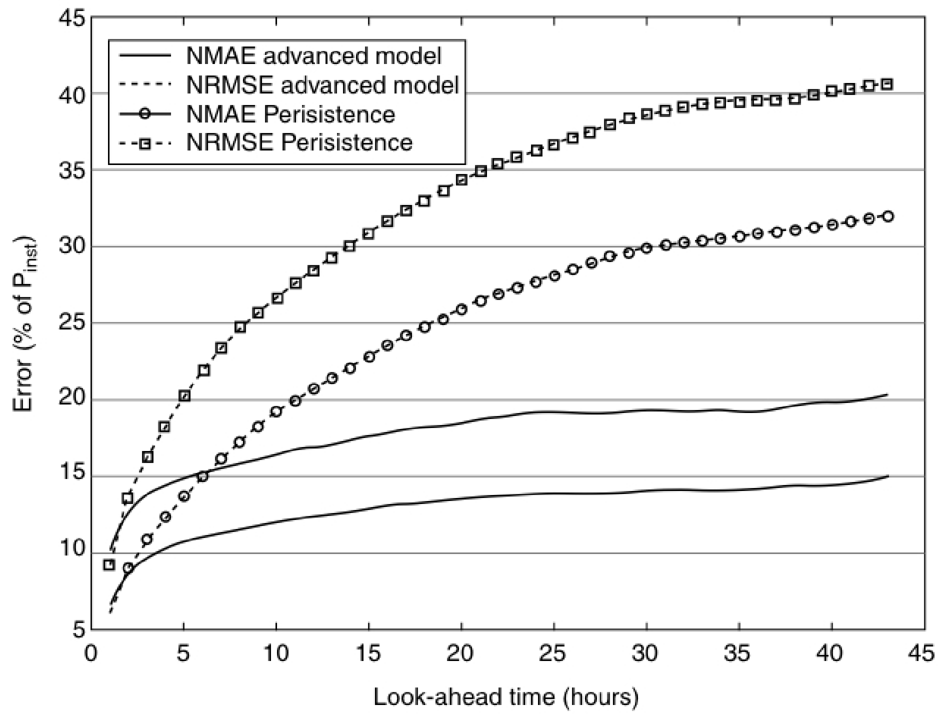
The Sample Average Approximation technique needs a set of scenarios for its random variable $\begin{pmatrix} \rho_{spot}(t,\xi) \\ P_{wind}(t,\xi) \\ \Delta P(t,\xi) \end{pmatrix}$

We thus simulate realizations of those variables as we know their distribution functions.

- $\Delta P(t,\xi)$ is simulated as a normal distributed random variable with 0 mean and 1 standard deviation. The value of the standard deviation does not really matter as we will just have interest in the sign of the variable.
- $\rho_{spot}(t,\xi)$ is simulated normally distributed with an expected value depending on the time period (see Table A.2) and a fixed standard deviation. The stochasticity in the Spot prices is not our main focus in this thesis.
- $P_{wind}(t,\xi)$ is simulated normally distributed with an expected value and a standard deviation depending on the time period. We choose to study two cases of wind power system's penetration:
  - 25% of the power installed. So 400MW of wind farm in a total of 1600MW generation capacity.
  - 45% of the power installed. So 981MW of wind farm in a total of 2181MW generation capacity.

In figure 4.1, we can see the performance of two different wind farms production forecasts with two methods of error measurement (NMAE and NRMSE).

Concerning the wind farm production forecasts scenarios, we will perform different simulations, function of the assumption on the percentage of error we assume to be realist on those forecasts. We will thus generate 2 sets of wind farm production forecasts scenarios: a forecast with the NRMSE error of the Persistence model (Worst case) and a one with the NRMSE error of the advanced model (Best case). We use the NRMSE error as it is analog to the standard deviation, and represents thus a better assumption than using the NMAE error. Moreover we use the forecast error from 12 hours ahead, as we have to bid on the Spot market a midday the day before delivery.
By plotting several scenarios together we obtain the following curves:



Use of the NMAE and the NRMSE for assessing the performance of the advanced prediction approach, and for comparison with a reference predictor (Persistence is used here).

Figure 4.1: Precision of different wind farms production forecasts [7]

- figure 4.2: an example of the Spot-price scenarios
- figure 4.3: the wind farm production forecasts scenarios with NRMSE error % of Persistence forecast model
- figure 4.4: the wind farm production forecasts scenarios with NRMSE error % of advanced model

4.7.2 The unit commitment schedule and economic power dispatch results

First case study: Effect of wind penetration level

In the case study we compare two cases with different wind penetrations (25% and 45%).

25% wind penetration case

Figure 4.5 shows the unit commitment schedule where the solid point indicates the units that should be ON at a given period of the next day. We can see here that the uncertainty in the wind power production forces more units than in the deterministic case (see figure (3.2)) to be ON to potentially compensate an over or under production of the wind farms (The stochasticity of the spot price is also important). We notice that the loss of profits due to the use of regulation forces the company to have more power units ON to operate a potential internal rescheduling (Most of the time cheaper than the regulation market).

The sum of the production planned is shown in figure 4.6. We notice that this planned generation is relatively low. This come from the fact that internal rescheduling in our model is not penalized. Thus producing X MWh cost the same if we plan directly X MWh or if we plan $\frac{X}{2}$MWh day ahead and reschedule by adding another $\frac{X}{2}$MWh just before the delivery.

The energy bids that are accepted in the spot market is also provided in figure 4.7. We notice that we bid more energy than the planned amount of energy available (We just subtract the bilateral contracts energy

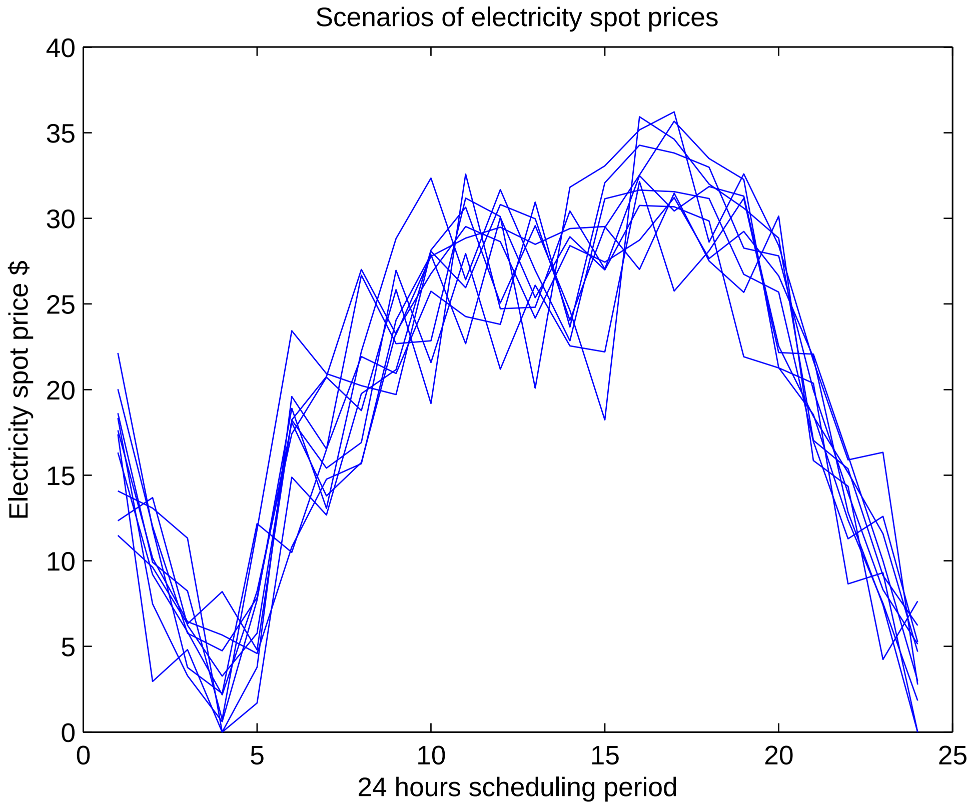


Figure 4.2: Spot price scenarios

engagement to the sum of the planned production). The algorithm is kind of anticipating the wind power production and the possibility of internal rescheduling (Option that is not more expensive than scheduling day-ahead in our case). We obtain the figure 4.8 by adding the realization of wind power to the planned amount of energy available. The difference between this new curve and the energy bids curve give the amount of energy that has to be rescheduled internally or paid to the regulation market.

For a given scenario, we observe the following curve of energy rescheduled and reserve required from regulation market in figure 4.9 to match with the engagement of the GENCO on the SPOT market and its bilateral contracts. When the energy rescheduled is positive it means that the GENCO underproduced regarding its engagement. On the opposite, when the value is negative, it means that the GENCO overproduced and could not sold it all. When the rescheduling capacity is not enough to enable the GENCO to meet its engagement, the remaining energy imbalance is managed by the regulation. It is important to notice that it will always be cheaper to reschedule inside the GENCO's generation facilities if there is unbalance in its engagement. However, when the imbalance is too big, and all the units of the GENCO are on their limits (Ramp up or down rate for example) after the rescheduling, the remaining imbalance has to come from the regulation market (The most expensive solution as long as the regulation price is not equal to the spot price).

45% wind penetration case

Figure 4.10 shows the unit commitment schedule. We notice that there is more unit ON than in the 25% wind penetration case. This is due to the fact that a bigger part of the production facilities are subject to uncertainty.

The energy bids that are accepted in the spot market is also provide in figure 4.12. We obtain the figure 4.13 by adding a random realization of wind power to the planned amount of energy available. The difference between this new curve and the energy bids curve give the amount of energy that has to be rescheduled internally or paid to the regulation market.

For a given scenario, we observe the following curve of energy rescheduled and reserve required from regulation market in figure 4.14 to match with the engagement of the GENCO on the SPOT market and its

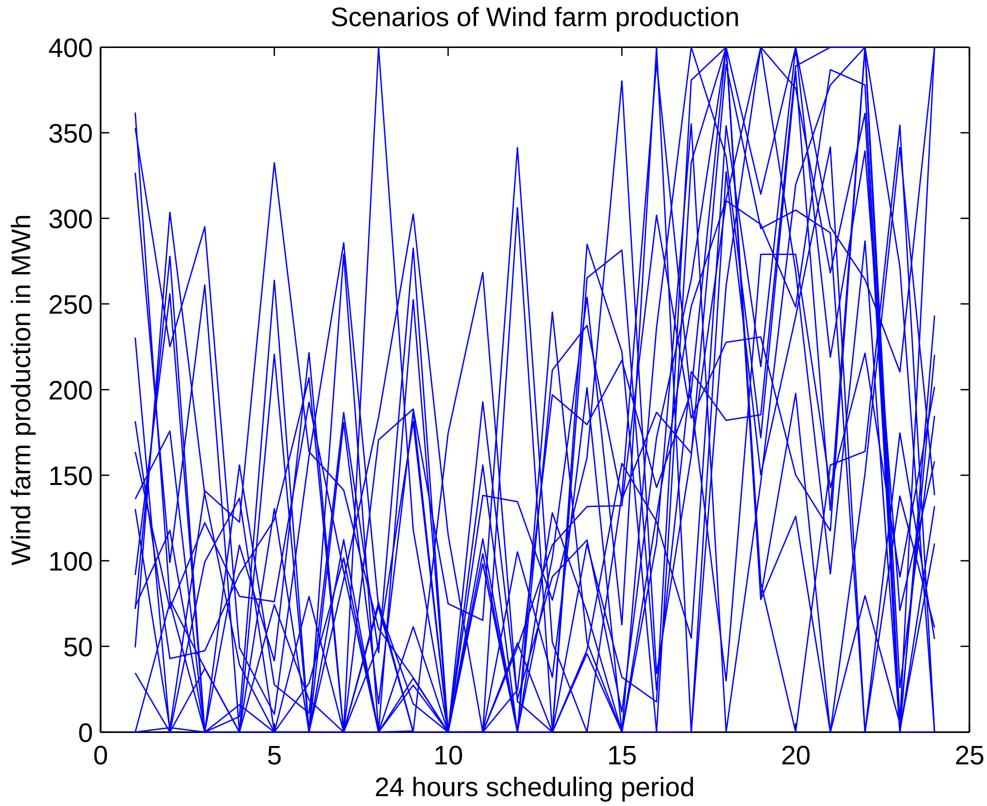


Figure 4.3: Wind farms production scenarios with NRMSE error % of a persistence forecast method

bilateral contracts. We notice here that, due to the fact that a bigger part of the GENCO's facilities is subject to uncertainty (45% against 25% previously), more energy than previously is subject to the regulation market. This last point is also explained by the fact that the total installed production capacity is bigger in the 45% case than in the 25% case.

We can also notice on Table 4.1 that using wind-power in the generation facilities of the GENCO seems to be beneficial for the GENCO. Indeed by increasing the installed capacity by $\approx 36,3\%$ with wind farm, we increase the net profits by $\approx 113,3\%$. This result can be explain by the fact that wind power in our model does not have any generation costs. Nevertheless, the absence of fuel costs seems to compensate the extra costs liked to the uncertainty of renewable energy power plants.

Table 4.1: Objective function values

Wind penetration in the GENCO's system	Wind power forecast model	Net profits (without bilateral incomes)
25% (i.e. 400MW in a total of 1600MW)	Advanced	79139,2 $
45% (i.e. 981MW in a total of 2181MW)	Advanced	168841 $

Second case study: Effect of forecast model used on the GENCO's profits

On Table 4.2, we can see the impact of the quality of the forecast on the potential income of the GENCO. Using the most advanced forecast method enable the GENCO to increase its potential profits of $\approx 5,6\%$ compare to using a basic one.

Table 4.2: Objective function values

Wind penetration in the GENCO's system	Wind power forecast model	Net profits (without bilateral incomes)
25% (i.e. 400MW in a total of 1600MW)	Persistence	74972,1 $
25% (i.e. 400MW in a total of 1600MW)	Advanced	79139,2 $

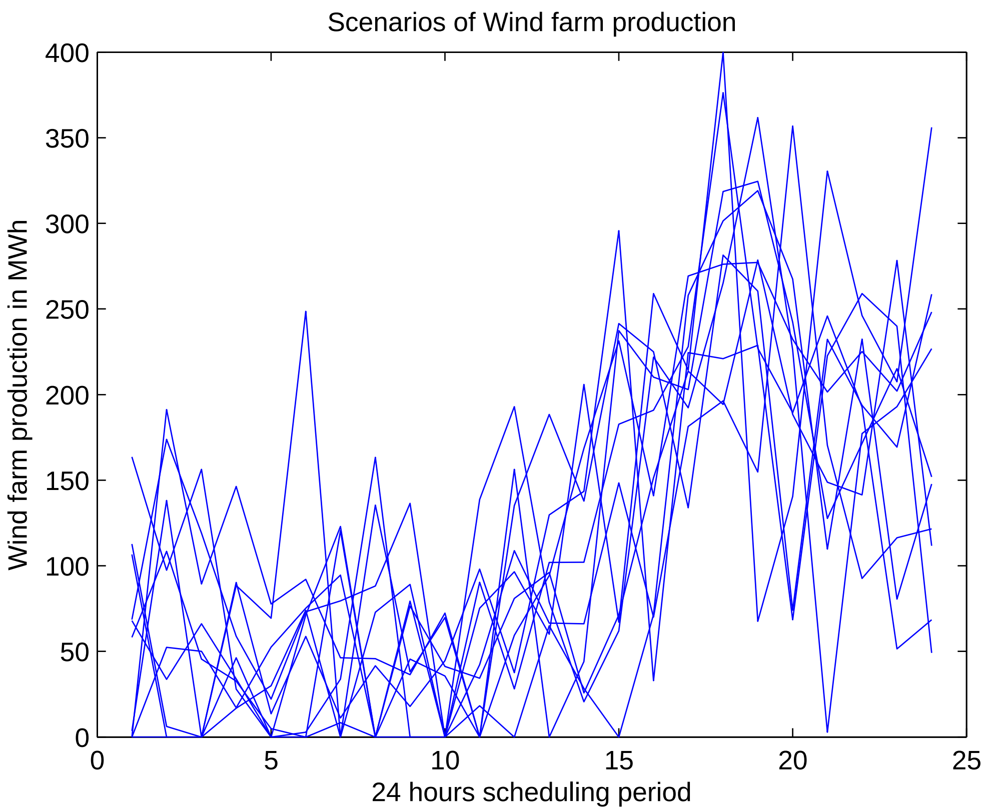


Figure 4.4: Wind farms production scenarios with NRMSE error % of an advanced model forecast method

4.7.3 Performance of the stochastic algorithm

We can test a stochastic model on its computation time and the quality of the solution it provides.

Effect of the number of scenarios on the solution quality and on the computation speed

Here we plot the estimated gap $\widehat{gap}(\hat{x})$ (This gap is in \$ and can be compared with the daily net profits of the GENCO in the 25% wind penetration and advanced forecast case) defined in 2.5.3. As shown in figure 4.15, by increasing the number of scenarios we clearly increase the confidence in our solution. However, as shown in figure 4.16, the computation time increases drastically as the number of scenario increases.

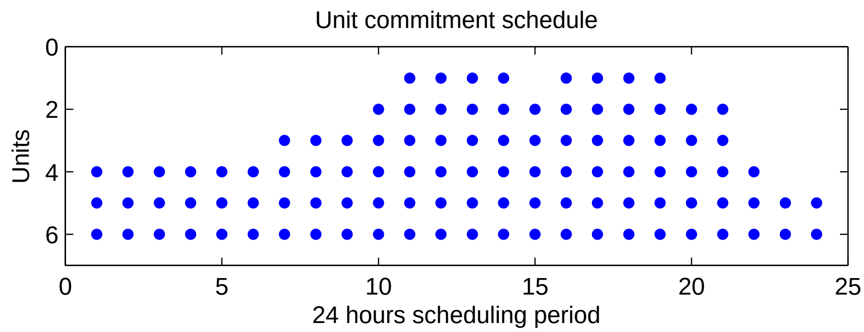


Figure 4.5: Stochastic unit commitment schedule for 25% of wind penetration

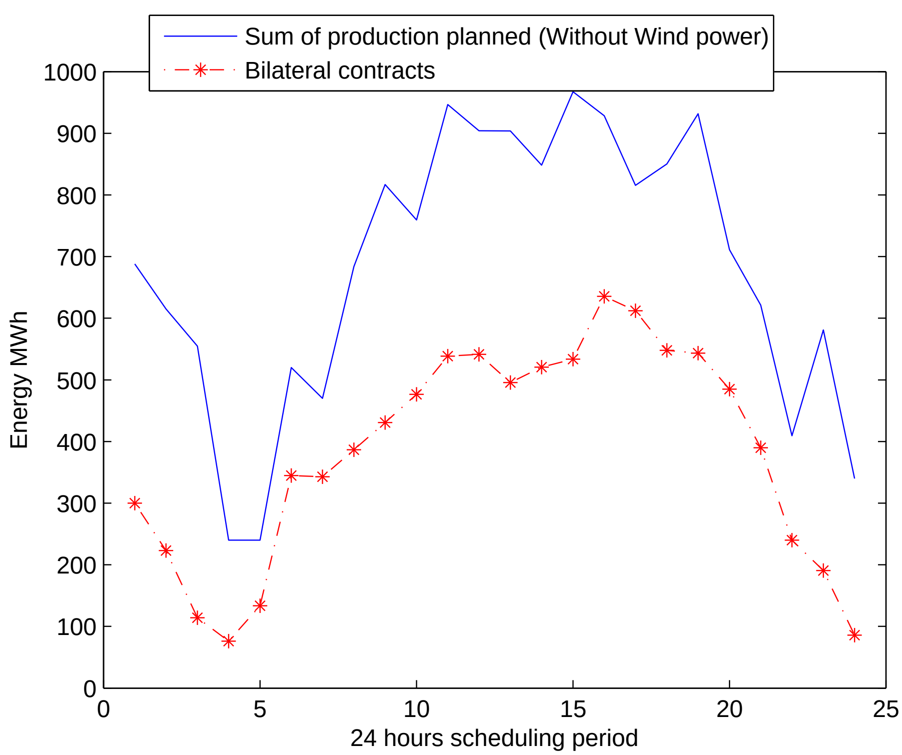


Figure 4.6: Sum of the production planned for the day ahead for 25% of wind penetration

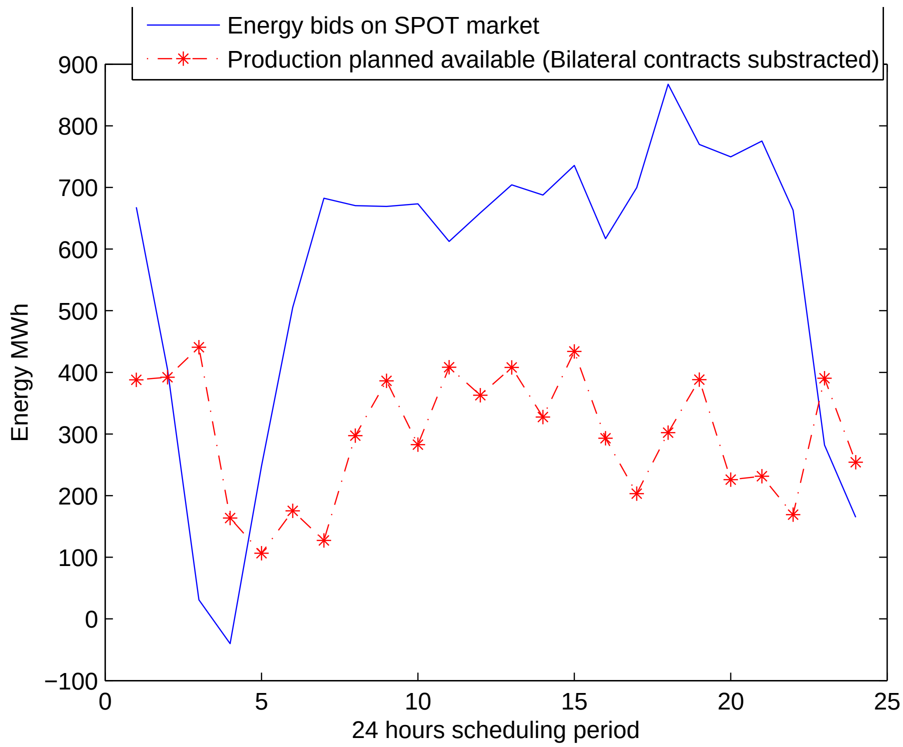


Figure 4.7: Energy bids on the spot market for 25% of wind penetration 1/2

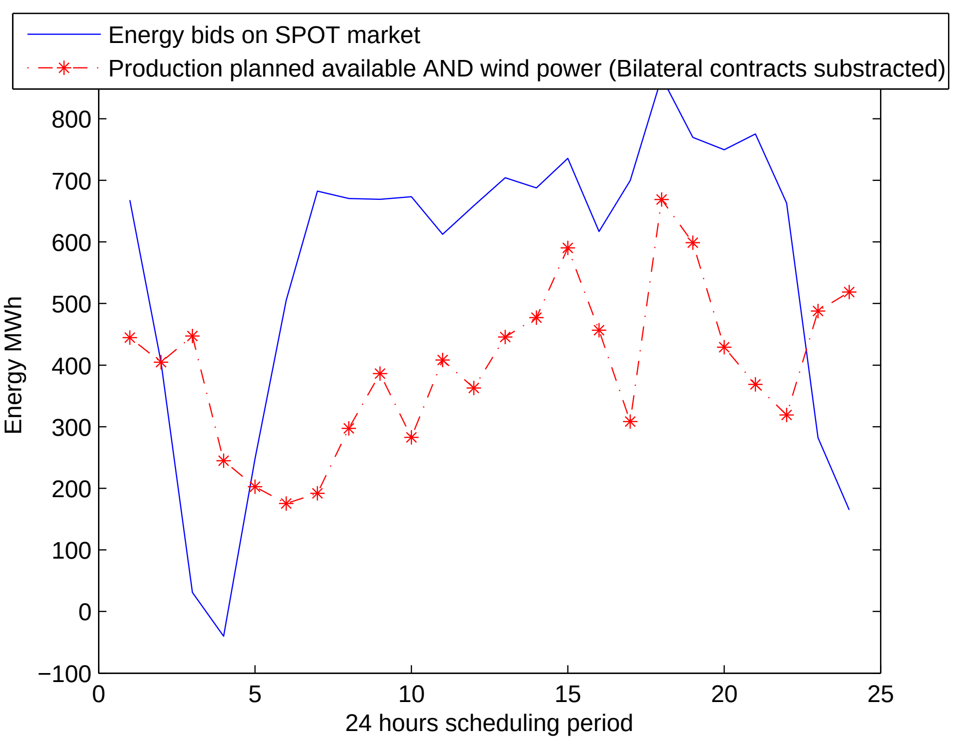


Figure 4.8: Energy bids on the spot market for 25% of wind penetration 2/2

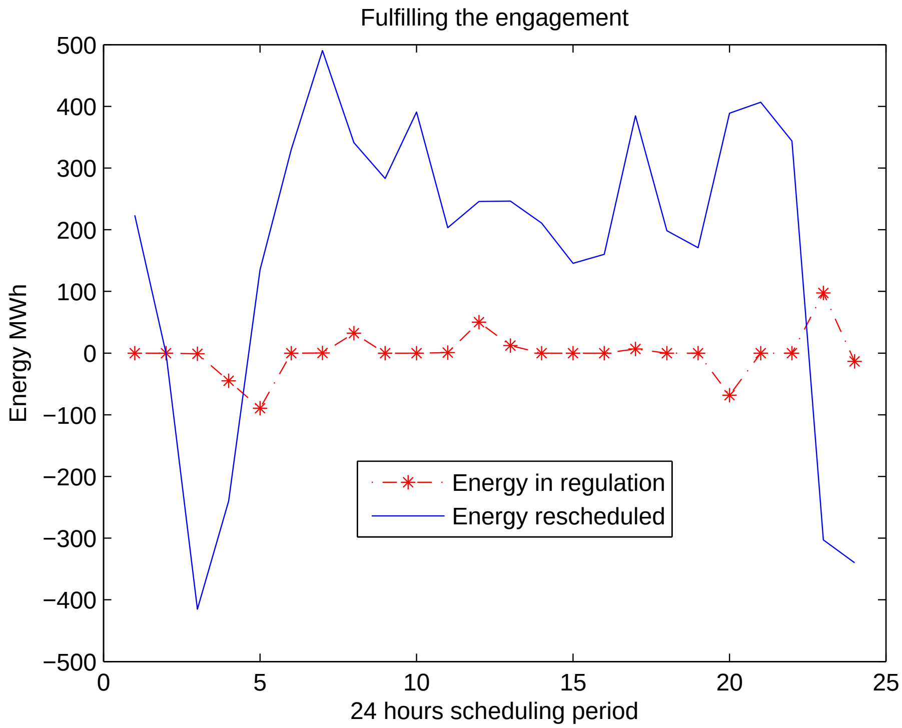


Figure 4.9: Energy rescheduled in a given scenario for 25% of wind penetration

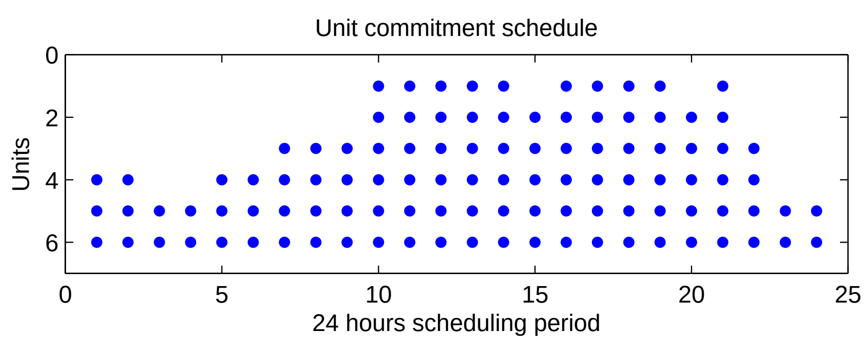


Figure 4.10: Stochastic unit commitment schedule for 45% of wind penetration

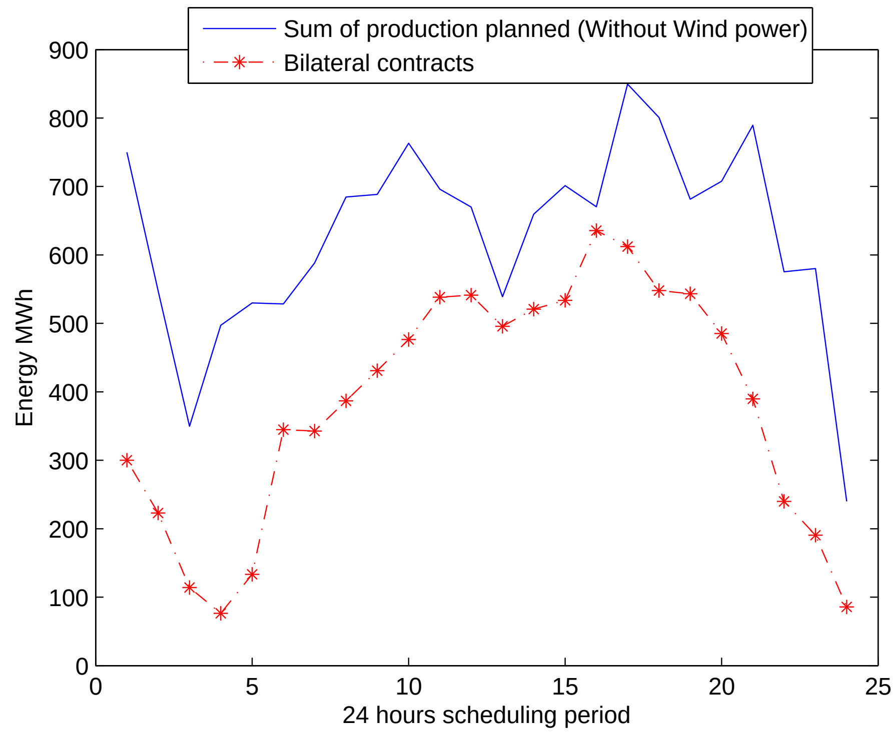


Figure 4.11: Sum of the production planned for the day ahead for 45% of wind penetration

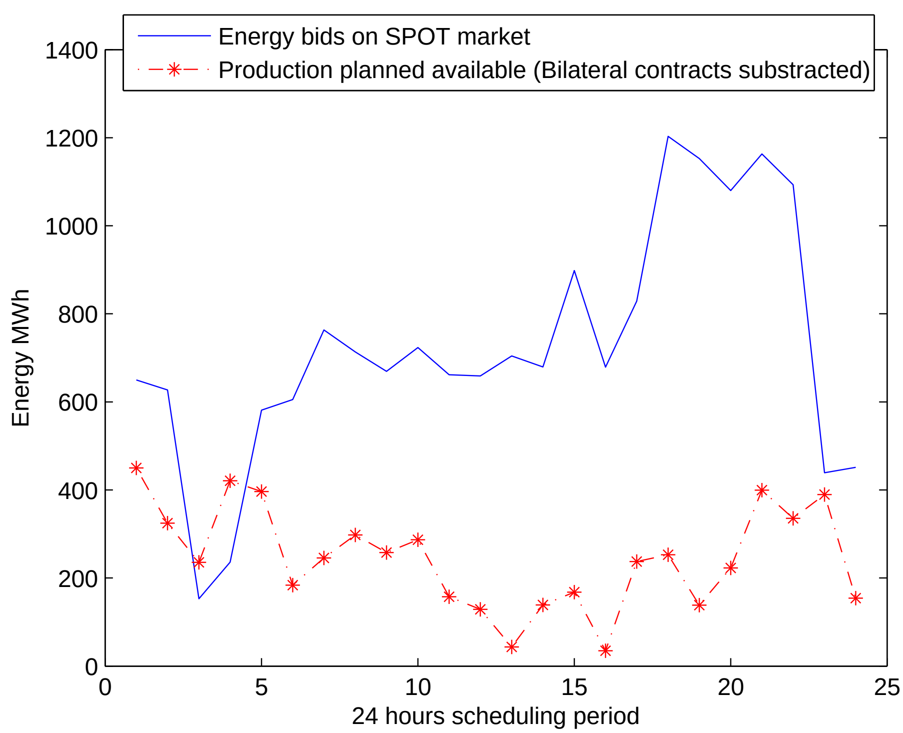


Figure 4.12: Energy bids on the spot market for 45% of wind penetration 1/2

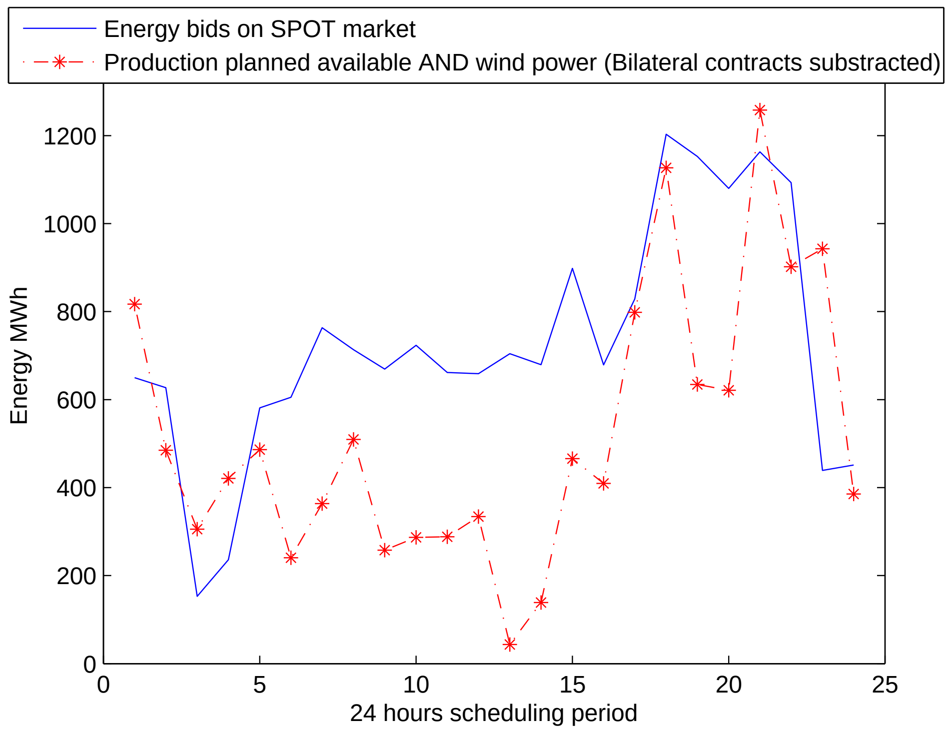


Figure 4.13: Energy bids on the spot market for 45% of wind penetration 2/2

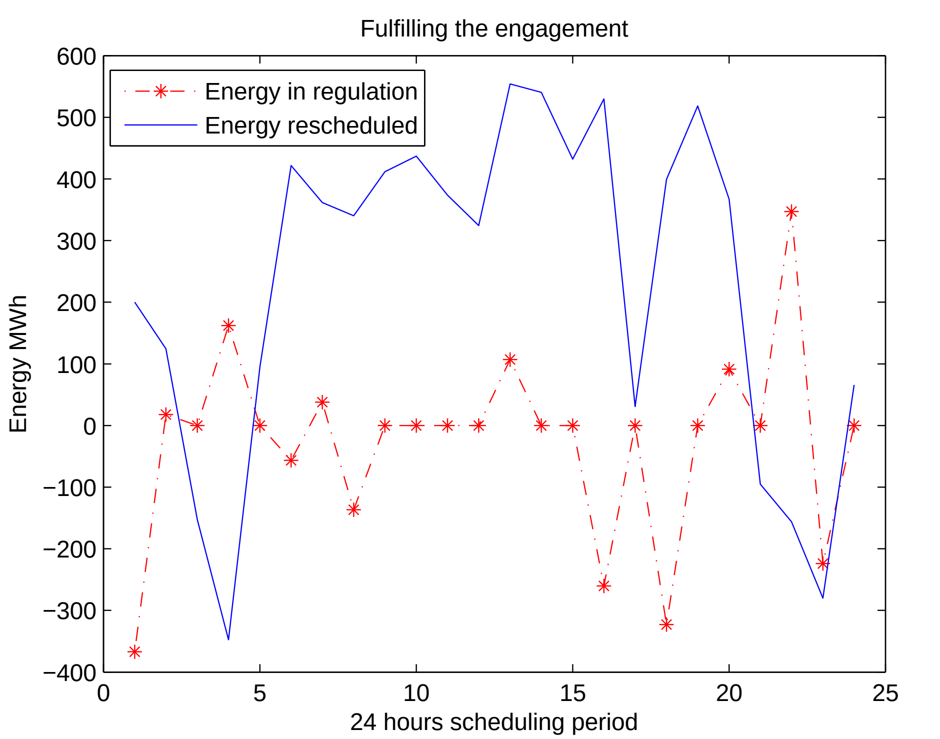


Figure 4.14: Energy rescheduled in a given scenario for 45% of wind penetration

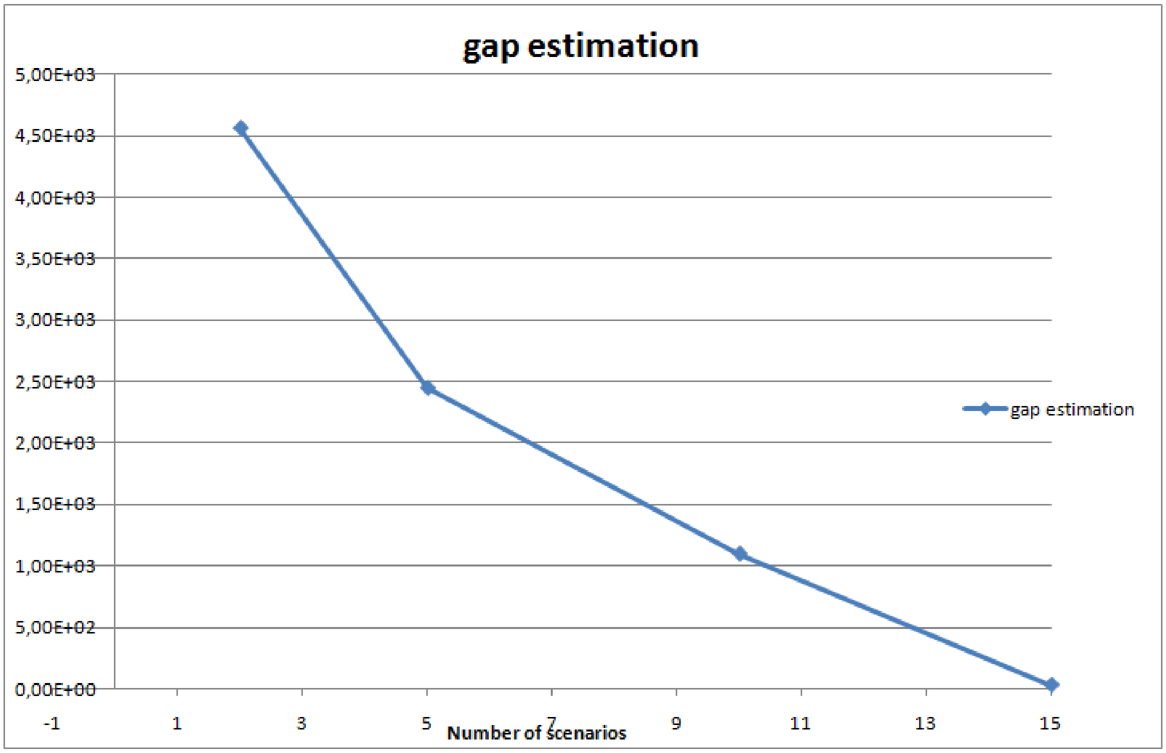


Figure 4.15: Estimated gap

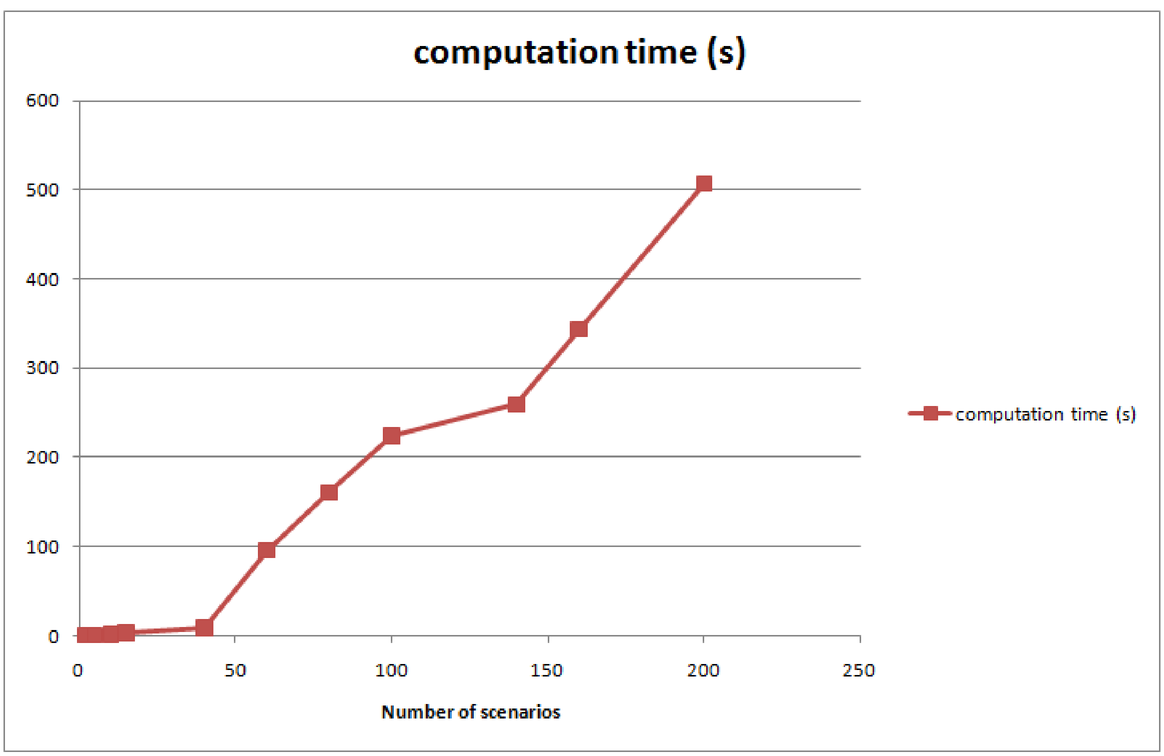


Figure 4.16: Computation time

Chapter 5

Conclusions and future work

5.1 Summaries of the work

In this thesis we have performed different PBUCs and focused on:

- **Deterministic PBUC**:
  - Taking into account the possibility of bidding on the up regulation market (Arbitrage)
- **Stochastic PBUC**:
  - Taking into account the uncertainty in wind farm production forecasts
  - Taking into account the uncertainty in spot prices forecasts
  - Taking into account the uncertainty in the power system imbalance

In the deterministic part of this thesis, we use a formal Lagrangian relaxation technique coupled with a dynamic programming and EPD. This way of solving our optimization problem is very problem specific. In other words, the algorithm cannot be readily used to solve a general problem.

In the stochastic part of this thesis, we use two-stage stochastic programming coupled with mixed integer linear programming, Monte Carlo and statistical methods. The uncertainty in wind farms forecasts and spot market prices bring optimal units commitment plans way different from the deterministic programming ones. It thus seems that, in our programming, stochasticity is necessary to have a theoretical optimal solution the nearest of the real problem optimal solution.

5.2 Conclusions

In this thesis, we have analyzed several cases:

Value of reducing forecasting error

We use the standard deviation of the forecasts (The wind farms production forecasts in our test) to represent the forecasting error. The standard deviation of the forecasts impacts significantly the possible income of the GENCO. Thereby, enhancing the error level of those forecasts by using an advanced wind power forecast model instead of a persistence one, lead to increased profits of nearly $5,6\%$ in our example.

Low and high wind power penetration level

Increasing the percentage of wind farm in the GENCO's production facilities seems to lead to increased profits, even if it increases the costs linked to the uncertainty brought by the wind farm production (regulation costs, when the GENCO can not match its engagement). Indeed, in our example, by increasing the installed production capacity by $\approx 36,3\%$ with wind farm, we increase the net profits by $\approx 113,3\%$. However more studies are needed on this point to confirm our results.

Monte Carlo simulation sample size

For our Monte Carlo simulation, a minimum of 15 scenarios are necessary to have a good accuracy. The computational time starts to be non negligible for more than 45 scenarios.

5.3 Future work

The first thing that should be developed is to create a program of scenarios selection in order to reduce the number of scenarios needed to have a good quality solution. Indeed in our case we only use 6 units but if we decide to perform a bigger unit commitment scheduling, the computation time become non-negligible, we can not just increase the number of scenarios until reaching the desired confidence.

The second thing that should be done is to introduce the bidding strategy in our stochastic program. Indeed in our program we assume that all the energy bids we made on the markets are selected, whereas most of the time the probability of not being selected is an important factor. Moreover, we assume that the spot price is independent of our bidding strategies. However, in most of the electricity markets, the spot price depends on the behavior of the market participants. Thus, it is important to take into account bidding on both reserve and spot market. For that purpose, game theory will be useful.

Finally, risk management can be introduced. Indeed maximizing profits in our case comes with an amount of risks linked to the uncertainty in the spot and regulation prices and in the wind power production. For that using value at risk calculation and hedging risk through future market seem necessary.

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Appendix A

Data tables

Table A.1: Feasibility of the different combination of states

Description	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6
a ($)	118,82	118,11	218,34	142,73	176,06	313,91
b ($/MWh)	27,896	24,664	18,1	10,694	10,662	7,612
c ($/MWh^2)	0,0143	0,0126	0,0081	0,0046	0,0014	0,0020
RD	50	50	75	100	175	200
RU	50	50	75	100	175	200
P0	0	0	0	200	350	320
I0	0	0	0	1	1	1
P_{max}	50	50	150	200	350	400
P_{min}	5	10	10	20	140	100
SU	0	0	50	50	500	800
SD	0	0	50	50	500	800
MUT	1	1	5	8	8	8
MDT	-1	-1	-5	-8	-5	-5
TH	-1	-1	-5	8	10	10

Table A.2: Expected value of the spot prices forecast

Period	Expected value of the forecasted Spot prices $
1	15,74
2	11,7
3	5,99
4	4
5	7
6	18,08
7	17,98
8	20,29
9	22,6
10	25
11	28,25
12	28,4
13	26,01
14	27,32
15	28
16	33,34
17	32,12
18	28,75
19	28,51
20	25,45
21	20,45
22	12,59
23	10
24	4,5

Table A.3: Amount of bilateral contracts

Period	Amount of bilateral contracts MWh
1	300
2	223
3	114,17
4	76,24
5	133,42
6	344,6
7	342,69
8	386,72
9	430,75
10	476,49
11	538,44
12	541,3
13	495,74
14	520,71
15	533,67
16	635,45
17	612,2
18	547,97
19	543,39
20	485,07
21	389,77
22	239,96
23	190,6
24	76,24

Table A.4: Expected value of the forecast of the global consumption

Period	Expected value of the forecast of the global consumtion MWh
1	1000
2	1743,33
3	1380,56
4	1254,13
5	1444,73
6	2148,67
7	2142,31
8	2289,07
9	2435,83
10	2588,31
11	2794,79
12	2804,32
13	2652,48
14	2735,71
15	2778,91
16	3118,17
17	3040,66
18	2826,56
19	2811,31
20	2616,9
21	2299,24
22	1799,87
23	1635,32
24	1254,13

Table A.5: Expected value of the forecast of the needed reserve capacity

Period	Expected value of the forecast of the needed reserve capacity MWh
1	90
2	156,9
3	124,25
4	112,87
5	130,03
6	193,38
7	192,81
8	206,02
9	219,22
10	232,95
11	251,53
12	252,39
13	238,72
14	246,21
15	250,1
16	280,64
17	273,66
18	254,39
19	253,02
20	235,52
21	206,93
22	161,99
23	147,18
24	112,87