THE HOUGH TRANSFORM

Image Analysis Group
Chalmers University of Technology

Summary

- Introduction
- Theory of the Hough Transform
- Examples
- Advantages and Disadvantages
- Practical Issues
- Generalizations

Introduction

• The Hough transform (HT) is named after Paul Hough who patented the method in 1962.

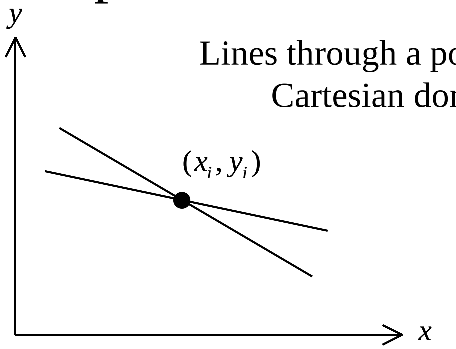
• It is a powerful global method for detecting parameterized boundaries or curves.

• It transforms between the Cartesian space and a parameter space in which a straight line (or any parameterized curve) can be defined.

Theory

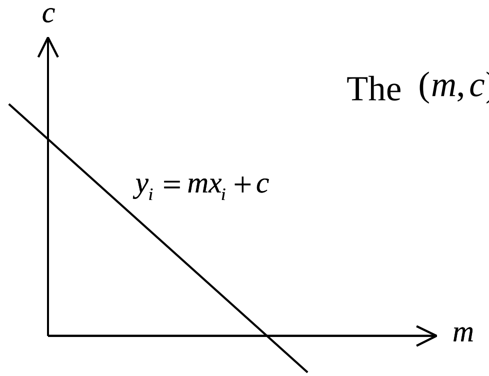
Consider the case where we have straight lines in an image. Note that for every point (x_i, y_i) in that image, all the straight lines passing through that point satisfy Equation 1 for varying values of line slope and intercept.

$$y_i = mx_i + c \qquad \text{.................... Equation 1}$$



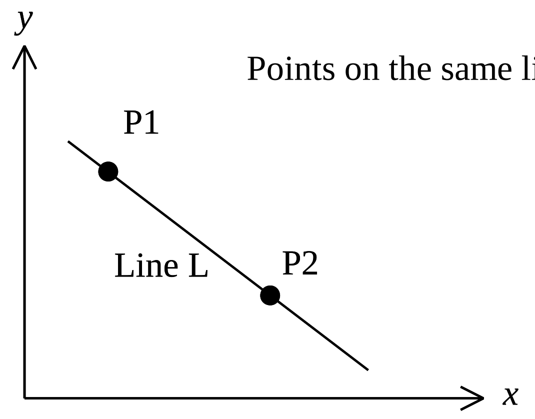
Now if we reverse our variables and look instead at the values of (m,c) as a function of the image point coordinates (x_i, y_i), then Equation 1 becomes Equation 2 which describes a straight line.

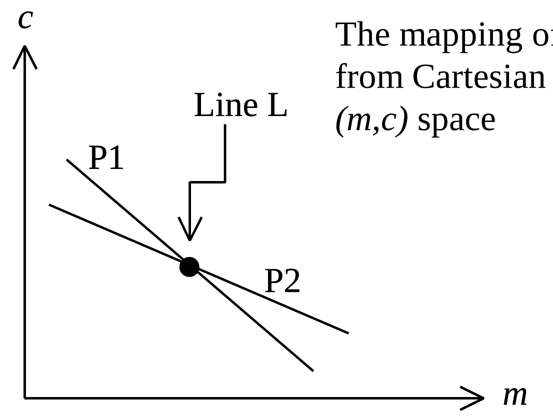
$$c = y_i - mx_i \qquad \text{.................... Equation 2}$$



Theory _{cont}

• Consider two pixels *P1* and *P2*, which lie on the same line in the *(x,y)* space.
• For each pixel, we can represent all the possible lines through it by a single line in the *(m,c)* space.
• Thus a line in the *(x,y)* space that passes through both pixels must lie on the intersection of the two lines in the *(m,c)* space representing the two pixels.
• This means that all pixels which lie on the same line in the *(x,y)* space are represented by lines which all pass through a single point in the *(m,c)* space.





The mapping of P1 and P2 from Cartesian space to the *(m,c)* space

Theory cont

We can now describe an algorithm for detecting lines in images:

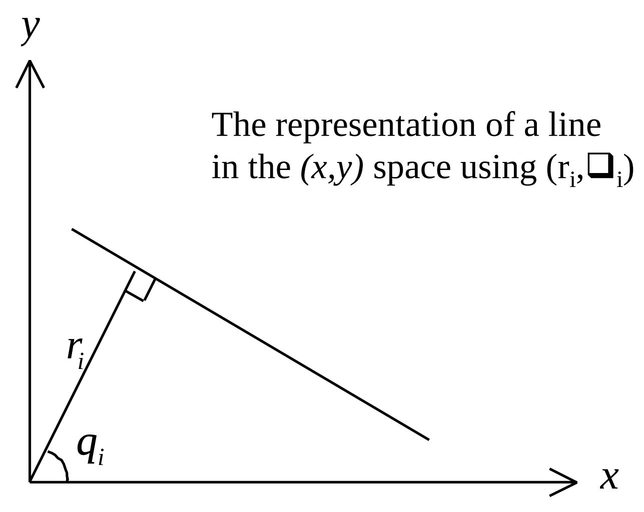
- Find all edge points in the image using any suitable edge detection scheme.
- Quantize the (m,c) space into a two-dimensional matrix H with appropriate quantization levels.
- Initialize the matrix H to zero.
- Increment by 1 each element of $H(m_i,c_i)$ which corresponds to an edge point.
- The result is a histogram or a 'vote matrix' showing the frequency of edge points corresponding to certain (m,c) values (i.e. points lying on a common line).
- H is thresholded such as only the large valued elements are taken. These elements correspond to "strong" lines in the original image.

Practical Issues

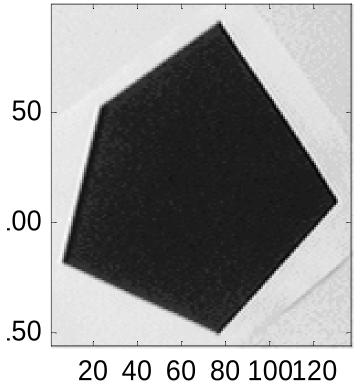
To avoid the problem of infinite *m* values which occurs when vertical lines exist in the image, the alternative formulation shown in Equation 3 can be used to describe a line.

$$x\cos q + y\sin q = r \qquad \text{.................... Equation 3}$$

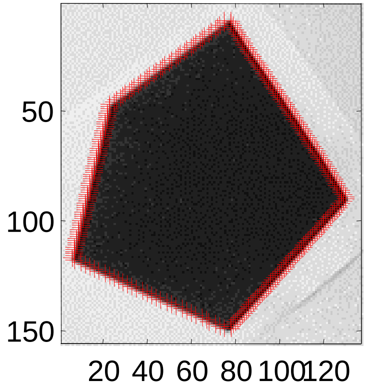
This means that a point in the *(x,y)* space is now represented by a curve in *(r, θ)* space rather than a straight line.



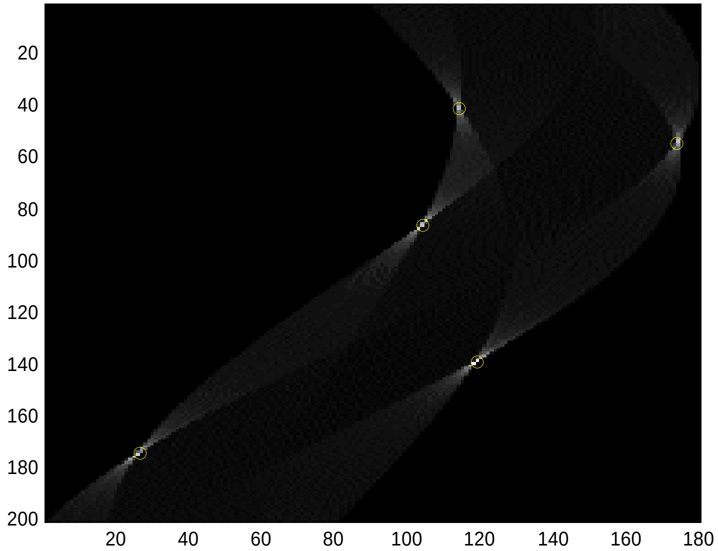
Examples



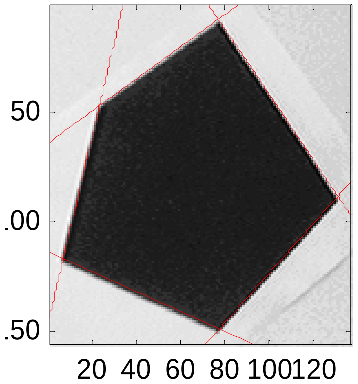
Original Image



Detected Edges



The vote histogram with the detected lines marked with 'o'



Detected lines

Examples cont

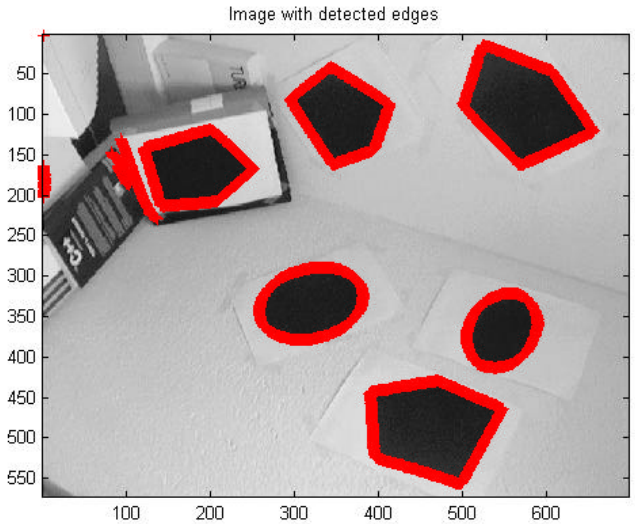
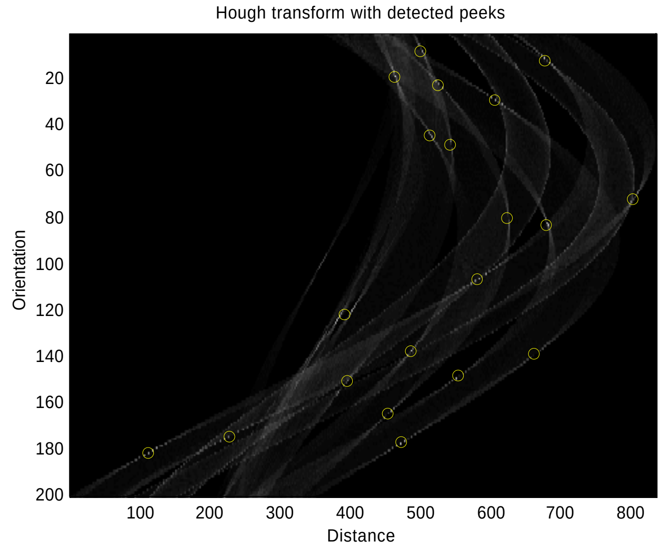


Image with detected edges



The vote histogram with the selected lines

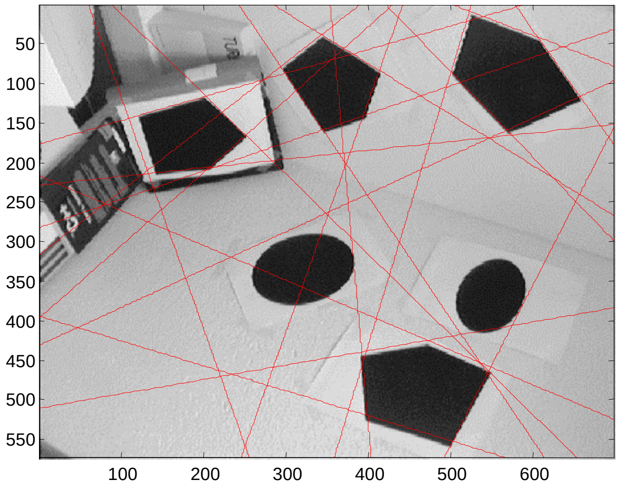


Image with the detected lines

Advantages & Disadvantages

The advantage of the Hough transform is that pixels lying on one line need not all be contiguous. This can be very useful when trying to detect lines with short breaks in them due to noise, or when objects are partially occluded.

The disadvantages of the Hough transform:

- It can give misleading results when objects happen to be aligned by chance.
- Detected lines are infinite lines described by their (m,c) values, rather than finite lines with defined end points.

Generalizations

• The Hough transform can be used to detect shapes in an image other than straight lines such as circles and ellipses or any other parameterized shapes.

• For example, in the case of circles, the parameter space is three dimensional (the radius and the x and y coordinates of the centre).

• Nevertheless, due to increased complexity with increased parameter space dimensionality, it is practically of use only for simple cases.