NON-LINEAR DIFFUSION FILTERING

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Summary

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Introduction

• A whole field in image processing and computer vision is based on Partial Differential equations (PDEs).

• The main application of PDE-based methods in this area is perhaps smoothing and restoration of images.

• Linear Diffusion is a traditional way to smooth an image in a controlled way is to convolve it with a Gaussian kernel.

$$G_\sigma(x) = \frac{1}{2\pi.\sigma^2}\exp\left(-\frac{\left|x^2\right|}{2\sigma^2}\right)$$

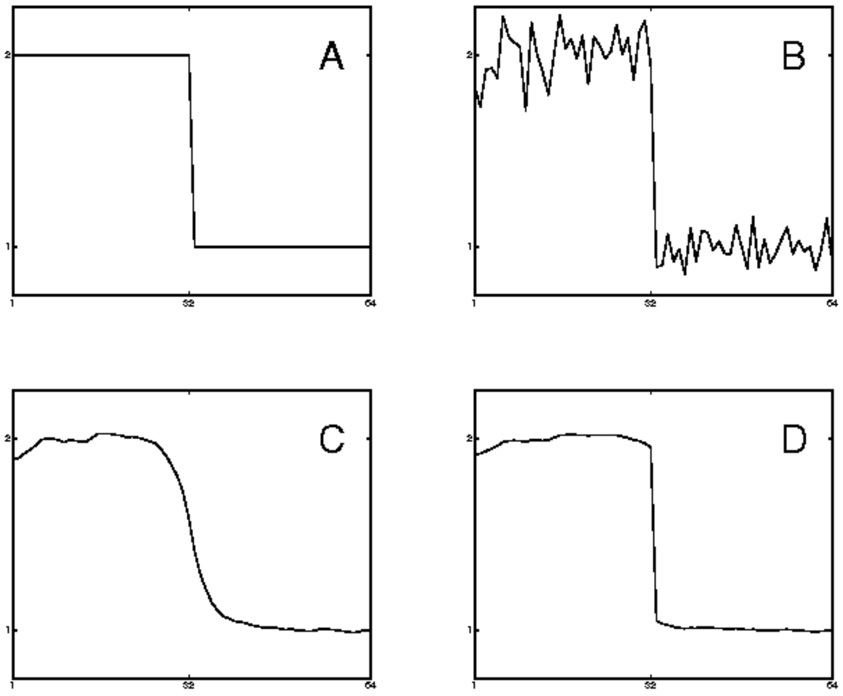
• Non-Linear Diffusion

  • Reduces noise and enhances contours in images.

  • The diffusion coefficient is locally adapted, becoming negligible as object boundaries are approached.

  • Noise is efficiently removed and object contours are strongly enhanced.

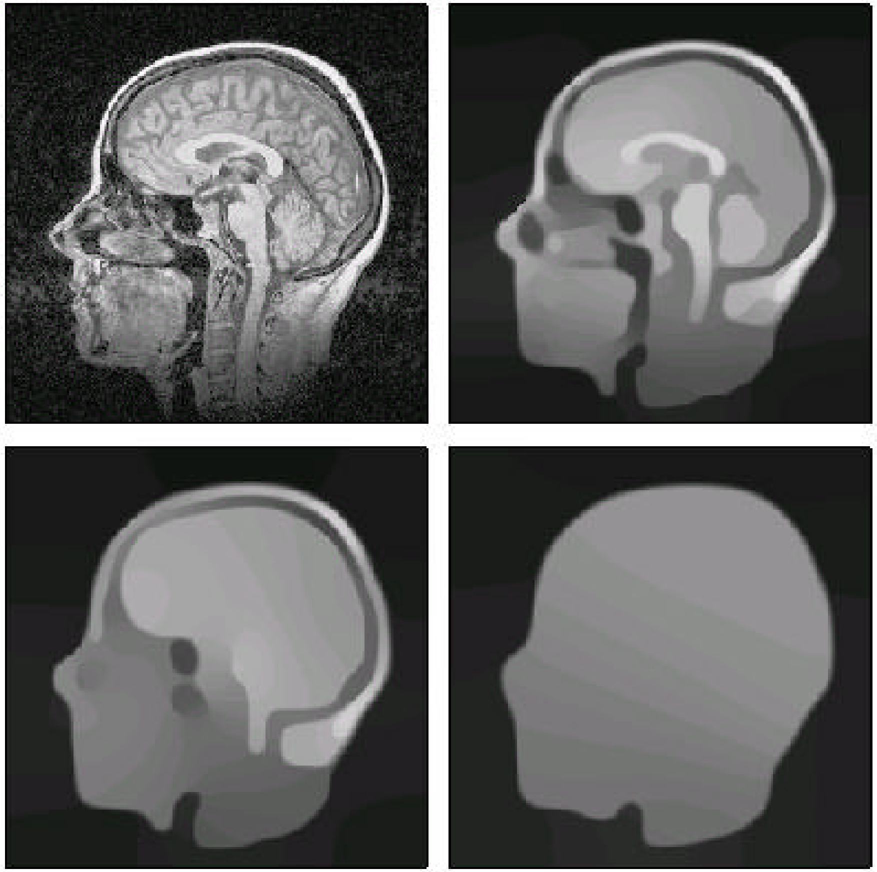
Linear vs Nonlinear Diffusion



Comparison of low pass filtering to NLDF. a) Original step. b) Original step with white noise superimposed. c) Result of simple low pass spatial frequency filtering. d) Result of NLDF (edge-preserving noise reduction).

Linear vs Nonlinear Diffusion cont

Scale space behaviour of **linear** Diffusion filtering



Scale space behaviour of **Nonlinear** Diffusion filtering

Courtesy, Joachim Weikert

Non-Linear Diffusion

• NLDF was originally formulated by Perona and Malik (1987).

• It has several advantages:

  • Noise is smoothed locally, ’within’ regions defined by object boundaries whereas little or no smoothing occurs between image objects.

  • Local edges are enhanced since discontinuities, such as boundaries, are amplified.

• Mathematically one treats the problem like a diffusion process, where the diffusion coefficient is adapted locally to the effect that diffusion stops as soon as an object boundary is reached.

Theory

Diffusion can be thought of as the physical process that equilibrates concentration differences without creating or destroying mass. Mathematically, this is described by Fick's law:

$$j = -D \cdot \nabla u$$

where the flux j is generated to compensate for the concentration gradient ∇u. D is a tensor that describes the relation between them.

Now, using the Continuity Equation (Conservation of mass):

$$\partial_t (u) = -div(j)$$

We get:

$$\partial_t (u) = div(D \cdot \nabla u)$$

Theory cont

The solution of the linear diffusion equation with a scalar diffusivity d

$$\partial_t u = div\left(d \nabla u\right)$$

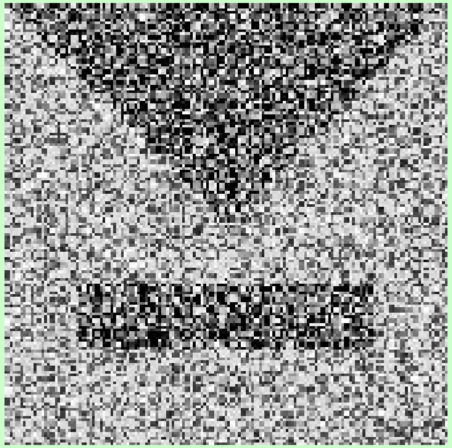
is exactly the same operation as convolving the image u with a Gaussian kernel of width $\sqrt{2t}$.

Perona and Malik proposed to exchange the scalar diffusion constant d with a scalar-valued function g of the gradient ∇u of the grey levels in the image. The diffusion equation then reads:

$$\partial_t u = div\left(g\left(\left|\nabla u\right|\right) \nabla u\right)$$

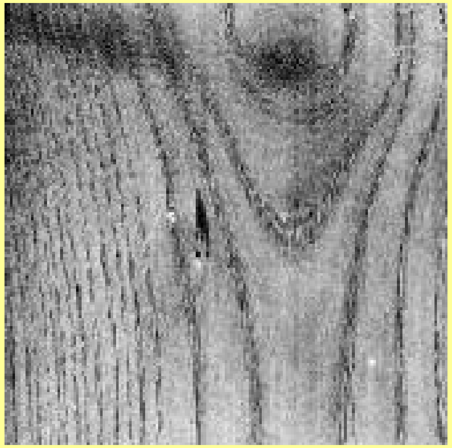
The length of the gradient $\left|\nabla u\right|$ is a good measure of the edge strength of the current location which is dependent on the differential structure of the image. This dependence makes the diffusion process nonlinear.

Applications



Denoising

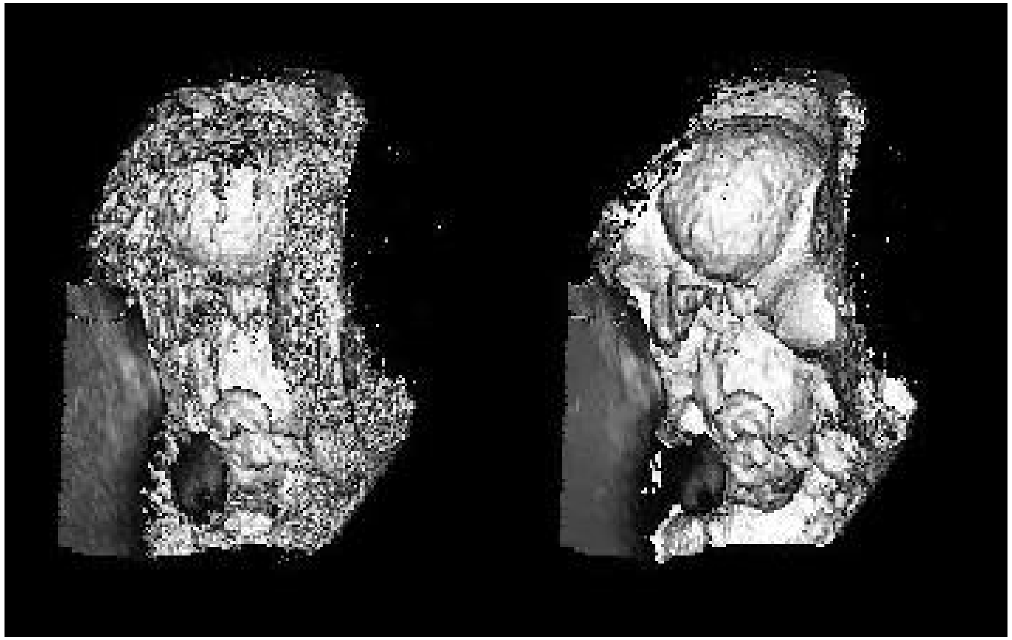
Gap completion



Computer aided quality control

Courtesy, Joachim Weikert

Applications cont



Denoising in medical imaging

design of Christmas postcards

Courtesy, Joachim Weikert

Implemenation

- A MATALB implementation of Non-Linear Diffusion Filtering is available.

- peronaexp3.m:
  - Perona-Malik's nonlinear isotropic diffusion model for segmentation.
  - function u2 = peronaexp3(inimage,lambda,k,n,sigma) with Gaussian smoothing and exponential flow function.
  - inimage = inputimage.
  - lambda = at lambda equal to 1 we have maximum diffusion.
  - k= time intgration step (choose according to problem) set k<0.2 for a 2D problem with 4 point neighbours (Euler approximation).
  - n= number of integration steps to be done.
  - sigma = Gaussian smoothing kernel.

References

• P. Perona and J. Malik. Scale-space and edge detection using anisotropic diffusion. IEEE Trans. on Patt. Analysis and Machine Intelligence, 12(7):629–639, 1990.

• J. Weickert. Anisotropic diffusion in image processing, ECMI Series, Teubner, Stuttgart, 1998. ISBN 3-519-02606-6.