P1

1a) X-8, Y-1, Z-2

1b)

$$p_r(r) = \begin{cases} -ar + a & 0 \le r \le 1 \\ 0 & otherwise \end{cases}$$

$$s = T(r) = \int_0^r p_r(w)dw \quad 0 \le r \le 1$$

$$s = T(r) = \int_0^r (-aw + a)dw \;=\; -\frac{aw^2}{2} + aw\Big| = -\frac{ar^2}{2} + ar \quad \Rightarrow$$

$$-ar2 + ar - s = 0 \quad \Rightarrow$$

$$r = 1 - \sqrt{1 - \frac{2s}{a}} = \; T^{-1}(s)$$

P2

Hough transform**:** see the textbook.

P3

$gradient\ magnitude = \sqrt{G_x^2 + G_y^2}$, where G_x is gradient in x direction, G_y in y-direction

3a) Applying Sobel filters and computing the magnitude of the gradient lead to :

0	0	0	0	0
0	7	10	7	0
0	10	0	10	0
0	7	10	7	0
0	0	0	0	0

3b) The pixels with value 10 survive T_{high}=10, then the pixels with value 7 survive T_{low}=6, and they are linked to the previous pixels. The final edge map is:

0	0	0	0	0
0	1	1	1	0
0	1	0	1	0
0	1	1	1	0
0	0	0	0	0

Observe, that the input image consisted of just one pixel and the Canny Edge detector found the edge as pixels surrounding the input pixel, which seem to be reasonable.

3c) Optimal for step edges corrupted by white noise, with additional criteria for:
– Detection (no missing edges, no spurious edges)
– Localization (minimal distance between true and detected edge position)
– One response (minimize multiple responses to single edges)

P4 Texture

a. See Lab3, Part 2
b. Largest textural entropy : cork (most resembles a random image).
   (Entropy, in general, is a measure of disorder. Large disorder == large entropy)
c. A-3, B-4, C-1, D-2

P5

5a)

$m_{0,0} = 1 + a$

$m_{0,1} = 2a + 3$

$\bar{y} = \frac{m_{0,1}}{m_{0,0}} = \frac{2a+3}{a+1}$

$$\mu_{0,2} = a(2-\bar{y})^2 + (3-\bar{y})^2 = a\left(2 - \frac{2a+3}{a+1}\right)^2 + \left(3 - \frac{2a+3}{a+1}\right)^2 = B$$

$$\rightarrow \qquad a = \frac{B}{1-B}$$

5b) Moment invariants $\Phi_0 \ldots \Phi_6$ are known as Hu's moment invariants. They are scale, translation and rotation invariant and can be used for shape discrimination.

5c) See LectureNotes, page 30.

$X(0) = 1, Y(0) = 1$
$X(1) = 2, Y(1) = 1$
$X(2) = 2, Y(2) = 2$
$X(3) = 1, Y(3) = 2$

$U(n) = X(n) + jY(n) \quad n = 0,1..N \quad N = 3$

$$a(k) = \sum_{n=0}^{3} U(n) e^{-j2\pi kn/3} \qquad k = 0..3$$

$$a(3) = \sum_{n=0}^{3} U(n) e^{-j2\pi 3n/3} \quad = 6 + 6i$$

$$Roughness = \sqrt{\sum_{k=0}^{3} |a(k)|^2}$$

Roughness can be used for discriminating between rough and smooth contours.

P6

a. See the Exercise MA-1, with I_1 and I_2 rotated 90°.
b. A larger value λ results in a more smooth velocity field.
c. If every point of brightness pattern moves independently, then the velocities will be not recovered.