Problem 1

a) Consider the image grayscale, r, to be a continuous random variable with the range [0,1] and the probability density function $p(r) = a \cdot (1 - \cos(2\pi r))$. The image is subject to a grayscale transformation, T(r), so that the probability density function of the output image, p(s), becomes a constant. Determine the value of the output grayscale variable, s, if the value of the input variable, r, equals 0.3.

(5p)

b) The input image in a) is subject to a piece-wise linear grayscale transformation, T(r), uniquely defined by

T(0) = 0.0
T(0.2) = 0.2
T(0.4) = 0.6
T(0.8) = 0.8
T(1) = 1.0

Specify the ranges of r in which T(r) results in an image output contrast that is

i) not changing
(1p)
ii) increasing
(1p)
iii) decreasing
(1p)

c) Construct a grayscale transformation, T(r), so that the output grayscale variable, s, keeps the input range, is quantized in equal steps over the range, and can be represented by 2 bits.

(2p)

Problem 2

a) An input image depicts a cell against a background. The average intensities of the cell, I_c, and background, I_b , are 80 and 50, respectively. The signal-to-noise ratio, defined as of the ratio between the squared cell and background intensity difference, i.e. $(I_c - I_b)^2$ and the overall image noise variance, σ_n^2, has been estimated to zero (expressed in decibel). An adaptive filter computes the output as:

$$g(x,y) = f(x,y) - \frac{\sigma_n^2}{\sigma^2(x,y)}\left[f(x,y) - \overline{f}(x,y)\right]$$

where $\sigma^2(x,y)$ is the local image variance computed in a 5 x 5 neighborhood centered on (x,y,) and $\overline{f}(x,y)$ is a local unweighted image mean of that neighborhood. Determine the fraction of information in g(x,y) (in percentage) that comes from f(x,y) given that the local image variance, $\sigma^2(x,y)$, equals 90^2. Determine the same fraction if $\sigma^2(x,y)$ equals 30^2.

(4p)

b) Interpret the results.
(2p)

c) What is the drawback of applying non-linear square shaped median filters to images depicting square shaped objects?
(1p)

d) What is the minimal size of a square shaped object that can survive an N x N symmetric median filtering?
(1p)

e) What is the effect of the minimum filter as being applied to an image consisting of long thin black structures on a bright background?
(1p)

f) What is the effect of the maximum filter as being applied to an image with the same characteristics?

Problem 3

a) Show by computing the GLCM feature Contrast that the two images, I_1 and I_2, have the same texture as considered for the highest frequency in the horizontal direction.(4p)

I_1:

0 1 0 1
2 3 2 3
0 2 0 2
1 3 1 3

I_2:

2 0 2 0 2 0
0 1 0 1 0 1
1 3 1 3 1 3
2 3 2 3 2 3
3 1 3 1 3 1
1 0 1 0 1 0

b) Consider the GCLM to represent the second order joint probabilities p(i,j). Compute p(2,3) for I_1 and I_2.
(2p)

c) Consider the first order difference statistics p(i). Compute p(1) for I_1 and I_2.
(2p)

d) Comment and interpret the results in a), b), and c).

Problem 4

Below you find one input image and five output images. There is also a list of ten image analysis procedures. The task is to link each output image to a specific procedure (1-a, 2-b, etc.). Each correct link will give one point. For each link, an additional point will be given if you can clearly motivate your choice. Any incorrect motivation will lead to subtraction by one point. (10p)


Input image


1


2


3


4


5

a) Canny edge detection b) Sobel edge detection c) Histogram equalization

d) Ideal low-pass filtering e) Butterworth low-pass-filtering f) Fourier phase
g) Fourier magnitude h) Reconstruction using Fourier magnitude
i) Reconstruction using Fourier phase j) Ideal high-pass filtering

Problem 5

Apply Dynamic Programming for computing the optimal path from the left to the right border of the image, f, below. The brightness cost $C_1(p_i) = \max(f) - f(p_i)$ and the smoothness cost $C_2(p_{i-1}, p_i) = \Delta y$. The total cost $C(p_i) = w_1 * C_1(p_i) + w_2 * C_2(p_{i-1}, p_i)$, where the weights $w_1 = 2$ and $w_2 = 3$. The answer should include the cost accumulation matrix with back tracing pointers and the coordinates of the optimal path. (10p)

f(x, y):

2	0	2	2
2	1	4	5
3	4	3	1
2	3	1	1

Problem 6

Fig. X and Y show an image at two different times: t_1 and t_2. The local minima are underlined.

a) Segment the objects in both images using the watershed algorithm. (5p)

b) Use the auction algorithm to associate the three objects in Fig. X with the three objects in Fig. Y. To calculate the assignment weights use:

$$a(i, j) = a_{dist}(i, j) + a_{area}(i, j)$$

	$d(i,j)<0.5$	$0.5\le d(i,j)<2$	$2\le d(i,j)<4$	$4\le d(i,j)<6$	$6\le d(i,j)<10$	$10\le d(i,j)<17$
$a_{dist}(i, j)$	10	6	4	3	2	1

where *d(i,j)* is the Euclidean distance between object *i*'s center point in image t_1 and object *j*'s center point in image t_2.

n(i,j)	0	1	2	3	4
$a_{area}(i, j)$	9	5	3	2	1

where *n(i,j)* is the difference in area (=number of pixels) between object *i* in image t_1 and object *j* in image t_2.

Initial prices are zero, and ε=0.1. (5p)

9	9	9	8	8	7	7	6	6	6	6	6
9	15	15	15	16	8	6	6	6	6	6	6
9	14	13	12	12	8	6	6	6	6	6	6
8	14	13	$\underline{10}$	11	8	7	12	12	12	12	5
8	14	12	11	11	8	7	11	10	10	11	5
8	14	14	13	13	9	7	11	10	$\underline{9}$	11	5
8	7	7	7	7	7	7	11	10	12	12	5
8	13	13	14	13	8	8	8	7	7	6	5
8	12	10	10	14	8	8	8	5	5	4	4
8	12	$\underline{9}$	11	14	9	9	6	5	4	4	4
8	10	10	14	14	9	9	6	5	4	3	$\underline{2}$

Figure X. Image at t_1.

9	9	9	9	9	9	9	10	10	10	10	10
9	9	9	10	11	12	12	12	12	12	10	10
7	8	11	10	8	8	10	9	9	11	8	8
7	8	12	6	12	12	11	7	7	13	4	5
7	14	13	13	13	13	11	6	5	13	4	5
6	14	10	11	15	7	11	12	12	13	4	4
6	14	10	9	15	5	5	4	4	4	3	3
6	14	11	13	15	5	5	4	4	4	3	2
6	13	13	13	5	5	5	4	4	4	3	3
6	6	6	5	5	4	4	4	4	4	3	3
6	6	6	5	5	4	4	4	4	4	3	3

Figure Y. Image at t_2.

Solution 1

Determine a:

$$P(1)=1 \Rightarrow \int_0^1 p(r)\mathrm{d}r = 1 \Rightarrow$$

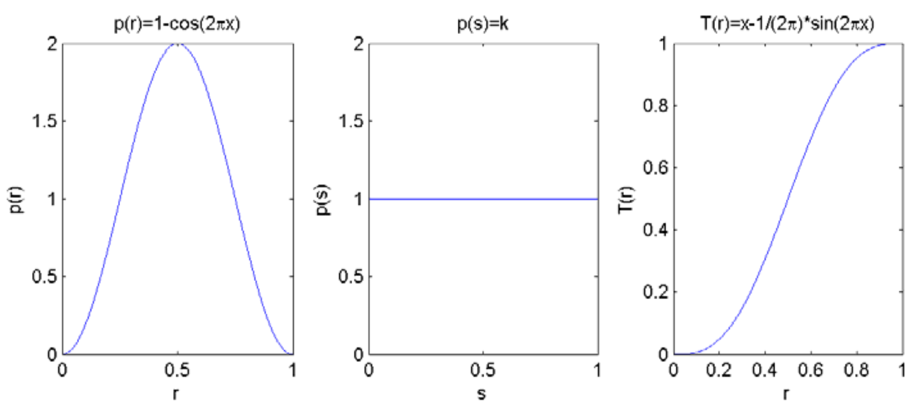
$$a\int_0^1 1-\cos(2\pi r)\mathrm{d}r = a\left[r-\frac{\sin(2\pi r)}{2\pi}\right]_0^1 = a\left(1-0-0+0\right) = a = 1$$

$$\because p(r)=1-\cos(2\pi r)$$

Histogram equalization:

s = T(r) =

$$\int_0^{0.3} p(r)\mathrm{d}r = \int_0^{0.3} 1-\cos(2\pi r)\mathrm{d}r = a\left[r-\frac{\sin(2\pi r)}{2\pi}\right]_0^{0.3} = a\left(0.3-\frac{\sin(0.6\pi)}{2\pi}-0+0\right) = 0.1486$$



b)

i) [0.0, 0.2] and [0.8, 1.0]
ii) [0.2, 0.4]
iii) [0.4, 0.8]

c)

[0.00, 0.25]: T(r) = 0
[0.25, 0.50]: T(r) = 1/3
[0.50, 0.75]: T(r) = 2/3
[0.75, 1.00]: T(r) = 1

Solution 2

a) Signal-to-noise ration equals zero means that $(I_c - I_b)^2 \ / \ \sigma_n^2 = 1 \Rightarrow \sigma_n^2 = (I_c - I_b)^2 =$ 30^2. Then, $\dfrac{\sigma_n^2}{\sigma^2(x,y)} = 30^2/90^2 = 1/9$.

In a 5 x 5 unweighted mean filter, 1/25 of the information comes from the center pixel itself and the remaining, i.e. 24/25, from the surrounding pixels, f_s. This gives

$g = f - 1/9\ (f - 1/25\ f - 24/25\ f_s) = 151/175\ f + 24/175\ f_s$. The fraction becomes 151 / (151 + 24) = 0.86, i.e. 86%.

If $\sigma^2(x,y)$ equals 30^2, then $\dfrac{\sigma_n^2}{\sigma^2(x,y)} = 30^2/30^2 = 1$. This gives

$g = f - (f - 1/25\ f - 24/25\ f_s) = 1/25\ f + 24\ f_s$. The fraction becomes 1 / 25 = 0.04, i.e. 4%.

b) If the local image variance is much larger than the overall image variance, then the information comes mainly from the processed pixel itself. If the local image variance is of the order of the overall image variance, then the information comes mainly from surrounding pixels, i.e. the image is being smoothed.

c) The median removes corners.

d) The minimum size is (N-1) x (N-1) for any N larger than 3. If N equals 3, then the size of the square needs to be at least 3 x 3.

e) It thickens the structures.

f) It shrinks the structures.

Solution 3

a) The GLCM should be computed for dx = +/- 1 and dy =0. Notice, because the images are of different size, we need to normalize the GLCM.

I_1

	0	1	2	3	
0	0	3	3	0	
1	3	0	0	3	/ 24
2	3	0	0	3	
3	0	3	3	0	

Contrast = $12/24 + 2^2\ 12/24 = 2.5$

I_2

	0	1	2	3	
0	0	10	5	0	
1	10	0	0	10	/ 60
2	5	0	0	5	
3	0	10	5	0	

Contrast = $30/60 + 2^2\ 30/60 = 2.5$

b)

$I_1 : p(2,3) = 3/24 = 1/8$

$I_2 : p(2,3) = 5/60 = 1/12$

c)

$I_1 : p(1) = 12/24 = 1/2$

$I_2 : p(1) = 30/60 = 1/2$.

d) The images present the same frequency characteristics in the horizontal direction. In I_1, all the non-zero second-order joint probabilities p(i,j) are the same. This is not the case for I_2. This is why, for example, p(2,3) becomes different for the two images. The difference is not detected by the feature Contrast and not by the first-order difference statistics. Second-order joint probabilities may be a more powerful tool for discriminating between two images than other texture analysis methods based on first-order statistics.

Solution 4

1 – d (Ideal low-pass filtering).
The image is smoothed but the ringing effect typical for ideal lp-filtering is clearly demonstrated.

2 – e (Butterworth low-pass filtering)
The image is smoothed but without any ringing pattern.

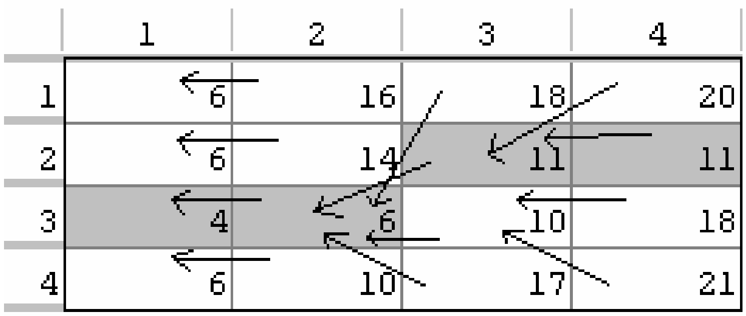
3 – a (Canny edge detection)
Although weak, the edges are thin and linked together. Also, it seems clear that the gradients have been thresholded.

4 - i (Reconstruction using Fourier phase)
The image shows the position of image intensity shifts but has not got the characteristics of either Canny nor Sobel filtering.

5 – b (Sobel edge detection)

The image presents thick and signed edges in both directions. The image has clearly been subject to an additive offset so pixels with negative sign becomes zero or higher. Hence, homogenous regions with zero-response become gray because of this offset.

Solution 5



Solution 6:

a)

Watershed segmentation result t_1:

1	1	1	1	1	1	1	1	1	1	1	1
1	0	0	0	0	1	1	1	1	1	1	1
1	0	4	4	0	1	1	1	1	1	1	1
1	0	4	4	0	1	1	0	0	0	0	1
1	0	4	4	0	1	1	0	3	3	0	1
1	0	0	0	0	1	1	0	3	3	0	1
1	1	1	1	1	1	1	0	0	0	0	1
1	0	0	0	0	1	1	1	1	1	1	1
1	0	2	2	0	1	1	1	1	1	1	1
1	0	2	2	0	1	1	1	1	1	1	1
1	0	2	2	0	1	1	1	1	1	1	1

0= watershed lines, 1= background, 2=object A, 3=object B, 4=object C

Watershed segmentation result t_2:

1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	0	0	0	1	1
1	1	0	0	3	3	0	2	2	0	1	1
1	1	0	3	3	3	0	2	2	0	1	1
1	0	0	0	0	0	0	2	2	0	1	1
1	0	4	4	0	1	0	0	0	0	1	1
1	0	4	4	0	1	1	1	1	1	1	1
1	0	4	0	0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1

0= watershed lines, 1= background, 2=object I, 3=object II, 4=object III b)

a(i,j):

	I	II	III
A	2+9=11	2+5=7	4+5=9
B	6+3=9	3+5=8	2+5=7
C	3+9=12	6+5=11	4+5=9

bids:

iteration:	1	2	3
A	2.1- I		
B	1.1- I	1.1- II	0.2- III
C	1.1- I	1.2- II	

prices:

iteration:	0	1	2	3
I	0	2.1	2.1	2.1
II	0	0	1.2	1.2
III	0	0	0	0.2

Final assignment: A-I, B-III, C-II