Solutions 041214

1a

Determine a:

$$P(1) = 1 \Rightarrow \int_0^1 p(r)\mathrm{d}r = 1 \Rightarrow$$

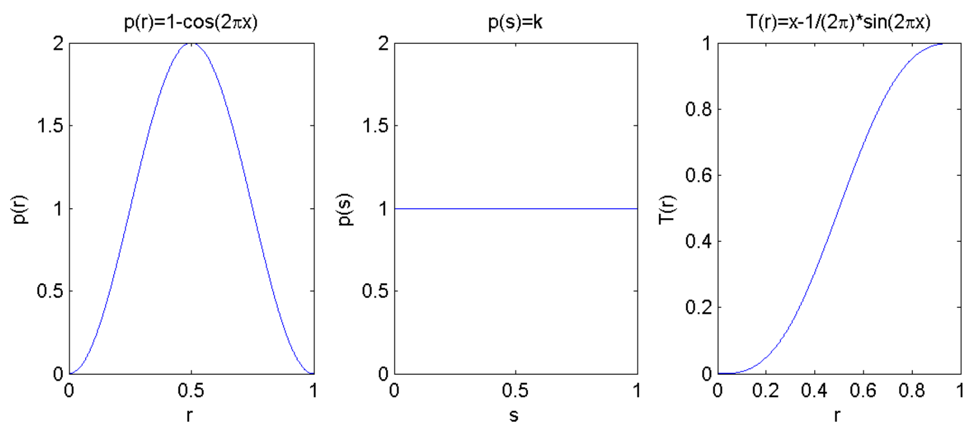
$$a\int_0^1 1 - \cos(2\pi\, r)\mathrm{d}r = a\left[r - \frac{\sin(2\pi\, r)}{2\pi}\right]_0^1 = a\left(1 - 0 - 0 + 0\right) = a = 1$$

$$\because p(r) = 1 - \cos(2\pi\, r)$$

Histogram equalization:

s = T(r) =

$$\int_0^{0.3} p(r)\mathrm{d}r = \int_0^{0.3} 1 - \cos(2\pi\, r)\mathrm{d}r = a\left[r - \frac{\sin(2\pi\, r)}{2\pi}\right]_0^{0.3} = a\left(0.3 - \frac{\sin(0.6\pi\,)}{2\pi} - 0 + 0\right) = 0.1486$$



b)

i) [0.0, 0.2] and [0.8, 1.0]
ii) [0.2, 0.4]
iii) [0.4, 0.8]

c)

[0.00, 0.25]: T(r) = 0
[0.25, 0.50]: T(r) = 1/3
[0.50, 0.75]: T(r) = 2/3
[0.75, 1.00]: T(r) = 1

Solution 2

a) Signal-to-noise ration equals zero means that $(I_c - I_b)^2 \;/\; \sigma_n^2 = 1 \Rightarrow \sigma_n^2 = (I_c - I_b)^2 =$ 30^2. Then, $\frac{\sigma_n^2}{\sigma^2(x,y)} = 30^2/90^2 = 1/9$.

In a 5 x 5 unweighted mean filter, 1/25 of the information comes from the center pixel itself and the remaining, i.e. 24/25, from the surrounding pixels, f_s. This gives

$g = f - 1/9\,(f - 1/25\, f - 24/25\, f_s) = 151/175\, f + 24/175\, f_s$. The fraction becomes 151 / (151 + 24) = 0.86, i.e. 86%.

If $\sigma^2(x,y)$ equals 30^2, then $\frac{\sigma_n^2}{\sigma^2(x,y)} = 30^2/30^2 = 1$. This gives

$g = f - (f - 1/25\, f - 24/25\, f_s) = 1/25\, f + 24\, f_s$. The fraction becomes 1 / 25 = 0.04, i.e. 4%.

b) If the local image variance is much larger than the overall image variance, then the information comes mainly from the processed pixel itself. If the local image variance is of the order of the overall image variance, then the information comes mainly from surrounding pixels, i.e. the image is being smoothed.

c) The median removes corners.

d) The minimum size is (N-1) x (N-1) for any N larger than 3. If N equals 3, then the size of the square needs to be at least 3 x 3.

e) It thickens the structures.

f) It shrinks the structures.

Solution 3

a) The GLCM should be computed for dx = +/- 1 and dy =0. Notice, because the images are of different size, we need to normalize the GLCM.

I_1

	0	1	2	3	
0	0	3	3	0	
1	3	0	0	3	/ 24
2	3	0	0	3	
3	0	3	3	0	

Contrast = $12/24 + 2^2\ 12/24 = 2.5$

I_2

	0	1	2	3	
0	0	10	5	0	
1	10	0	0	10	/ 60
2	5	0	0	5	
3	0	10	5	0	

Contrast = $30/60 + 2^2\ 30/60 = 2.5$

b)

I_1 : $p(2,3) = 3/24 = 1/8$

I_2 : $p(2,3) = 5/60 = 1/12$

c)

I_1 : $p(1) = 12/24 = 1/2$

I_2 : $p(1) = 30/60 = 1/2$.

d) The images present the same frequency characteristics in the horizontal direction. In I_1, all the non-zero second-order joint probabilities p(i,j) are the same. This is not the case for I_2. This is why, for example, p(2,3) becomes different for the two images. The difference is not detected by the feature Contrast and not by the first-order difference statistics. Second-order joint probabilities may be a more powerful tool for discriminating between two images than other texture analysis methods based on first-order statistics.

Solution 4

1 – d (Ideal low-pass filtering).
The image is smoothed but the ringing effect typical for ideal lp-filtering is clearly demonstrated.

2 – e (Butterworth low-pass filtering)
The image is smoothed but without any ringing pattern.

3 – a (Canny edge detection)
Although weak, the edges are thin and linked together. Also, it seems clear that the gradients have been thresholded.

4 - i (Reconstruction using Fourier phase)
The image shows the position of image intensity shifts but has not got the characteristics of either Canny nor Sobel filtering.

5 – b (Sobel edge detection)
The image presents thick and signed edges in both directions. The image has clearly been subject to an additive offset so pixels with negative sign becomes zero or higher. Hence, homogenous regions with zero-response become gray because of this offset.

Solution 5

	1	2	3	4
1	6	16	18	20
2	6	14	11	11
3	4	6	10	18
4	6	10	17	21

Solution 6:

a)

Watershed segmentation result t_1:

1	1	1	1	1	1	1	1	1	1	1	1
1	0	0	0	0	1	1	1	1	1	1	1
1	0	4	4	0	1	1	1	1	1	1	1
1	0	4	4	0	1	1	0	0	0	0	1
1	0	4	4	0	1	1	0	3	3	0	1
1	0	0	0	0	1	1	0	3	3	0	1
1	1	1	1	1	1	1	0	0	0	0	1
1	0	0	0	0	1	1	1	1	1	1	1
1	0	2	2	0	1	1	1	1	1	1	1
1	0	2	2	0	1	1	1	1	1	1	1
1	0	2	2	0	1	1	1	1	1	1	1

0= watershed lines, 1= background, 2=object A, 3=object B, 4=object C

Watershed segmentation result t_2:

1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	0	0	0	1	1
1	1	0	0	3	3	0	2	2	0	1	1
1	1	0	3	3	3	0	2	2	0	1	1
1	0	0	0	0	0	0	2	2	0	1	1
1	0	4	4	0	1	0	0	0	0	1	1
1	0	4	4	0	1	1	1	1	1	1	1
1	0	4	0	0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1

0= watershed lines, 1= background, 2=object I, 3=object II, 4=object III

b)

a(i,j):

	I	II	III
A	2+9=11	2+5=7	4+5=9
B	6+3=9	3+5=8	2+5=7
C	3+9=12	6+5=11	4+5=9

bids:

iteration:	1	2	3
A	2.1- I		
B	1.1- I	1.1- II	0.2- III
C	1.1- I	1.2- II	

prices:

iteration:	0	1	2	3
I	0	2.1	2.1	2.1
II	0	0	1.2	1.2
III	0	0	0	0.2

Final assignment: A-I, B-III, C-II