**Solutions 041215**

1a)

0000xxxxxx
0000x
00xxxxxxxx
000x0
000xxxxxxx

b)

Apply the hit-miss operation using the 3 x 3 structure element below. Remove pixels at a perfect fit.

111
010
111

c) Because it is sensitive to shape noise

d) Because of the fractal effect

e)

0 0 0
1 1 0
1 1 0

**Solution 2**

a)

$$\begin{matrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{matrix}$$

b) 2cos(u+v) + 2cos(u-v)

c) Low-pass

d) To preserve the mean intensity of the image

e) To prevent the filter from giving a non-zero response in a homogenous region

f) It will be shifted one step to the left

g) The Fourier transform of the filter is an exponential. To multiplicate in the Fourier domain with an exponential means shifting in the image domain.

**Solution 3**

a) $C_{f_1 f_2}(\omega) = F[f_1] \cdot \overline{F[f_2]}$

b) $C_{f_1 f_2}(\omega) = F[f_1] \cdot \overline{F[f_2]} = F[f] \cdot \overline{F[f]} \cdot e^{i 2\pi x_0} = \left|F[f]\right|^2 e^{i 2\pi x_0}$

c) In $\arg\left(C_{f_1 f_2}(\omega)\right)$

d) $x_0 = \dfrac{\sum x\delta(x - x_0)}{\sum \delta(x - x_0)} \quad \delta(x - x_0) = F^{-1}\left[e^{i 2\pi x_0}\right] = \left[\dfrac{F[f_1] \cdot \overline{F[f_2]}}{\left|F[f_1] \cdot \overline{F[f_2]}\right|}\right]$

e)

Brightness (or spatio-temporal) constraint:

$$\frac{\partial I}{\partial x} u(x_1) + \frac{\partial I}{\partial y} v(y_1) + \frac{\partial I}{\partial t} = 0$$

This constraint means that the brightness of a particular point in a pattern is constant

Velocity smoothness constraint

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 = 0$$

This constraint means that neighboring points in a pattern have similar velocities

f) A larger value of $\lambda$ results in a more smooth velocity field.

g) If every point of the brightness pattern move independently, there is little hope of recovering the velocities.

**Solution 4**

1 – d (Ideal low-pass filtering).
The image is smoothed but the ringing effect typical for ideal lp-filtering is clearly demonstrated.

2 – c (Histogram equalization)
The image contrast is improved, especially for the darker parts.

3 – f (Fourier phase)
There are no other plausible alternatives.

4 - i (Reconstruction using Fourier phase)
The image shows the position of image intensity shifts but has not got the characteristics of neither Canny nor Sobel filtering.

5 – b (Sobel edge detection)
The image presents thick and signed edges in both directions. The image has clearly been subject to an additive offset so pixels with negative sign becomes zero or higher. Hence, homogenous regions with zero-response become gray because of this offset.

**Solution 5**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 1 | 6 | 11 | 12 | 14 |
| 2 | 4 | 6 | 8 | 12 | 15 |
| 3 | 3 | 7 | 12 | 13 | 13 |
| 4 | 4 | 8 | 11 | 14 | 16 |

**Solution 6:**

a) First use any of the distance transforms:

*Euclidean*:

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 2 | 2 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 3 | 3 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 3 | 3 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 3 | 3 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 2 | 2 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | $\sqrt{2}$ | $\sqrt{2}$ | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 3 | 3 | 3 | 2 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

*Cityblock:*

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 2 | 2 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 3 | 3 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 3 | 3 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 3 | 3 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 2 | 2 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 3 | 3 | 3 | 2 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

*Chessboard:*

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 2 | 2 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 3 | 3 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 3 | 3 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 3 | 3 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 2 | 2 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 3 | 3 | 3 | 2 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Either find local max and let intensity decrease, or use 1/(dist. im. ).

Image after watershed transform:

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 2 | 2 | 2 | 2 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 2 | 2 | 2 | 2 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 2 | 2 | 2 | 2 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 2 | 2 | 2 | 2 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 2 | 2 | 2 | 2 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 3 | 3 | 3 | 3 | 3 | 3 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 3 | 3 | 3 | 3 | 3 | 3 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 3 | 3 | 3 | 3 | 3 | 3 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

1=background, 0=watershed lines, 2=object 1, 3=object 2

b)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 2 | 2 | 2 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 2 | 2 | 2 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

1=background (or object 1), 2= object (or object 2), 0=watershed line

c)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 1 | 2 | 2 | 2 | 2 | 2 | 1 | 2 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

1= background (or object 1), 2= object (object 2) mean of regions for each iteration:

| it: | *1* | *2* | *3* | *4* | *5* | *6* | *7* | *8* | *9* | *10* | *11* | *12* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **b** | 0 | 1 | 1.8 | 1.8 | 1.9 | 1.8 | 1.8 | 1.8 | 1.8 | 1.7 | 1.7 | 1.7 |
| **o** | 9 | 10.5 | 11.8 | 12 | 12.2 | 12.1 | 12.1 | 12.1 | 12.1 | 12.1 | 12.1 | 12.1 |

d) Watershed transform segments based on where the “water level” gets first, not based on intensity on neighboring pixels as region growing. Therefore object area becomes larger when doing region growing since the region with high values is larger than found at watershed segmentation.