Problem 1

Consider an image I as below:

$$I = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Apply Canny edge detection to the image I by steps **1a)** and **1b)**:

1a) Compute the edge strength (magnitude) image of the image I.
Use the following Sobel filter kernels:

$$h_1 = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} \qquad h_3 = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

(4p)

1b) Apply the *hysteresis* thresholding to the magnitude image from **(1a),** use the threshold values $\mathbf{T_{low}=12}$ and $\mathbf{T_{high} = 20}$. Neighboring pixels are the eight pixels surrounding the center pixels.

(4p)

1c) In what sense is the Canny filter optimal ?

(1p)

1d) Give some examples of *non-linear* filters for image smoothing.

(1p)

Problem 2

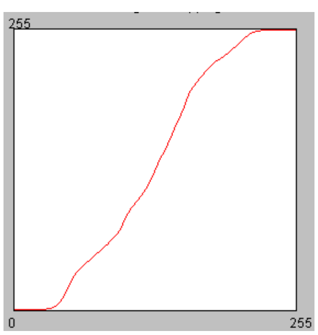
We have applied a histogram equalization to the Image 1 (left) and the result is presented in the Image 2 (right).

Image 1

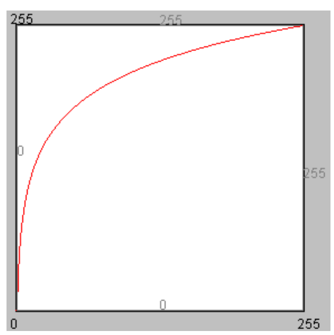
Image 2

2a) What histogram-mapping has been used, A, B or C ? Justify your answer.

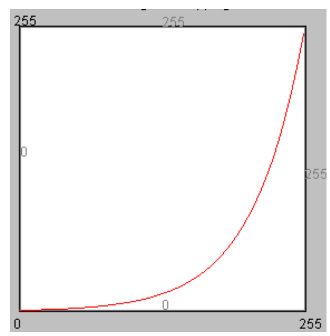
(2p)



A)



B)



C)

2b) In the image below the grayscale has been reduced to two levels: black and white. Explain how the original grayscale can be recovered by histogram equalization.

(2p)

2c) Consider the image grayscale, *r*, to be a continuous random variable with the range [0,1] and the probability density function $p_r(r)$ as:

$$p_r(r) = A \cdot exp(-a \cdot r)$$

where A and a are some constants.

The image is subject to a grayscale transformation $s=T(r)$ so that the probability density function of the output image, $p_s(s)$, becomes a constant (histogram equalization).

Find the transformation s=T(r).

(4p)

Assume that $A=\frac{e}{e-1}$, $a=1$, and verify that probability density function $p_s(s) = 1$ for $0 \leq s \leq 1$.

(2p)

Problem 3

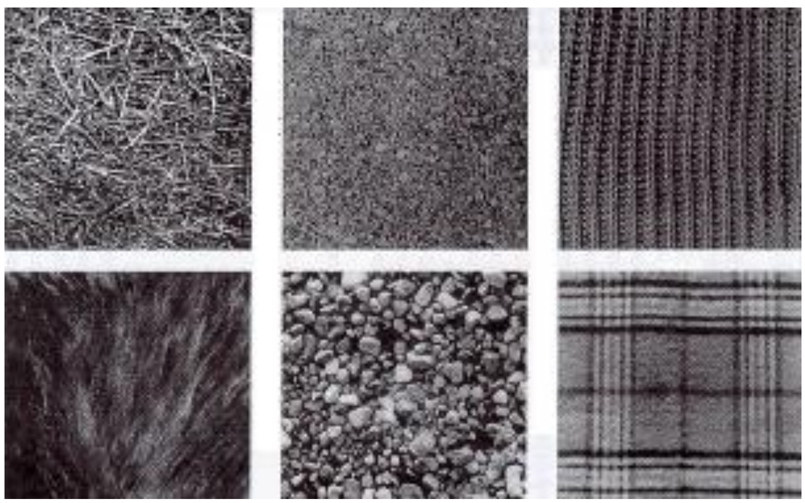
3a) Explain what is meant by "Hu's moment invariants" and give an example of their application.

(2p)

3b) Suggest a method for distinguishing mirror images of otherwise identical images.

(2p)

3c) Consider a texture database consisting of images of grass, cork, knitted fabric, dog fur, river pebbles and checkered textile (see the six image examples below).



Which of the images in (3c) seem to have the lowest texture entropy ? Why ?

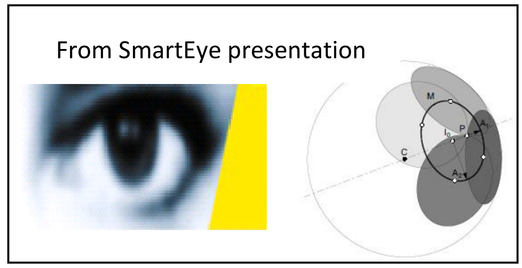
(2p)

3d) Give some examples of applications using Optical Flow technique.

(2p)

3e) Mention some methods for solving the Active Contour Models (snakes), i.e. for finding the position, where the snake reaches the minimum of its energy.

(2p)

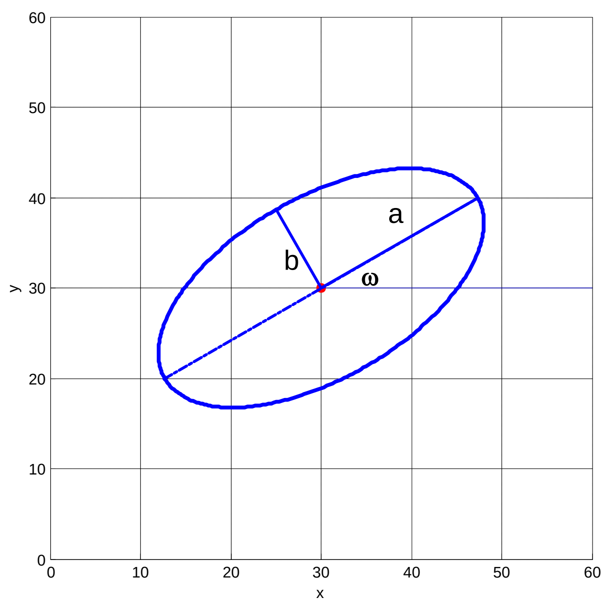


Problem 4

Write an algorithm for the pupil detection using the Hough transform, where the objects (pupils) are assumed to be ellipses. Note that you should develop **your own** Hough transform algorithm (including parameter space investigation and voting).

An ellipse in general position may be expressed parametrically as:

$$x(t) = x_c + \mathrm{a} \cdot cos(t)\, cos(\omega) - b \cdot sin(t)\, sin\,(\omega)$$
$$y(t) = y_c + b \cdot sin(t)\, cos(\omega) + a \cdot cos(t)\, sin\,(\omega)$$
$$t = 0..2\pi$$



where (x_c, y_c) are coordinates of the ellipse center, *a* and *b* are major and minor radius, respectively. ω is the angle between the *X*-axis and the major axis of the ellipse.

4a) Solve the problem, assuming that the center of the eye **(x_c, y_c)** is known, and the pupil is in the zero degree position (i.e. $\boldsymbol{\omega}$**=0**).

(5p)

4b) Solve the problem for the eye in a general position, i.e. **(x_c, y_c), *a*, *b*,** and ω are unknown.

(5p)

Note: The solutions/algorithms should be written in such a way, that they can be easily used for writing a computer program (= implementation).

Problem 5

5a) Apply Dynamic Programming for computing the optimal path from the left to the right border of the image, f, below. The brightness cost $C_1(p_i) = \max(f) - f(p_i)$ and the smoothness cost $C_2(p_{i-1}, p_i) = |\Delta y|$. The total cost $C(p_i) = w_1 * C_1(p_i) + w_2 * C_2(p_{i-1}, p_i)$, where the weights $w_1 = 2$ and $w_2 = 3$. The answer should include the cost accumulation matrix with back tracing pointers and the coordinates of the optimal path. *p_i* – pixel belonging to layer *i*

(6p)

f(x, y):

y

2	0	2	2
2	1	4	0
3	4	3	0
2	3	1	1

x

5b) Next, assume that the image in (5a) has been modified by inserting some additional layers, as in the figure below:

2	0	2	2	1	2	3	4	5
2	1	4	0	1	2	3	4	5
3	4	3	0	1	2	3	4	5
2	3	1	1	1	2	3	4	5

Original layers (as in 5a) Additional layers

and we keep the cost function for the original layers but for the additional layers we have the following modified cost function:

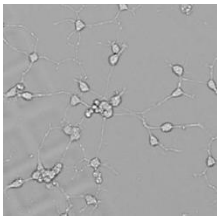
The brightness cost $C_1(p_i) = f(p_i)$ and the smoothness cost $C_2(p_{i-1}, p_i) = 1$, $i=5..9$

5b) What is the value of the total cost for the optimal path in the image (5b) with the additional layers?

(2p)

5c) How many minimum cost paths do exist for the image with additional layers?

(2p)



Problem 6

A microscope image that presents cells against background should be segmented by thresholding. The thresholding should be carried out so that the total number of misclassified pixels is minimized. The diagram below shows the estimated histograms for the background and the cell pixels.

The triangular distribution to the left (see the figure below) represents $P_1 \cdot p(x|background)$, while the rectangular distribution to the right represents $P_2 \cdot p(x|object)$. P_1, P_2 - *a priori* probabilities of background and object, respectively, *p(x|y)* are conditional probability density functions (pdfs).

6a) Find the *a priori probabilities* P_1 and P_2. (Assume that P_i are proportional to the object/background areas, i.e. $P_i = k\, A_i$, where A_i is the total area of the background and the object pixels, respectively, *k* is a proportionality constant),

(2p)

6b) Find the *optimal threshold value* $x = T_{opt}$ for this segmentation problem and the corresponding value of the *minimum error (=error rate)*.

(4p)

6c) Express the *sensitivity* as a function of threshold value T. For what values of T are all the cell pixels correctly classified?

(2p)

6d) Express the *specificity* as a function of threshold value T. For what values of T are all the background pixels correctly classified?

(2p)