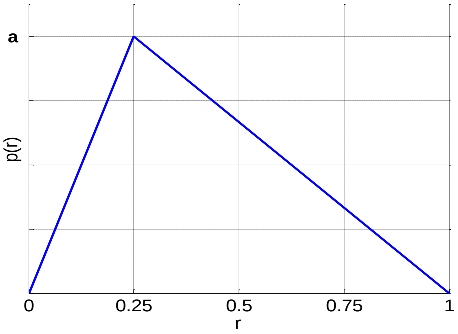
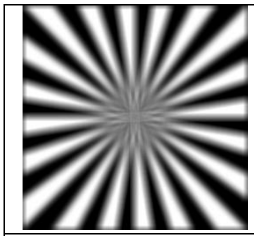
Problem 1

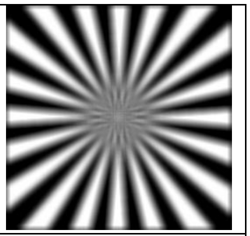
1.1 Consider the image grayscale, r, to be a continuous random variable with the range [0,1] and the probability density function $p(r)$, as in the figure below ($p(0) = 0$, $p(0.25) = a$, $p(1) = 0$). The image is subject to a grayscale transformation $s = T(r)$ so that the probability density function of the output image, $p(s)$, becomes a constant. Find the transformation $T(r)$ and plot it.

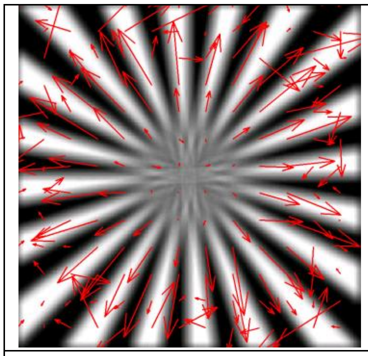
(4p)

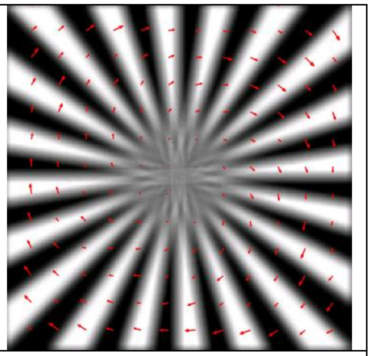


1.2 We are going to estimate the optical flow for the following two images (frame #1 & #2).


Frame #1


Frame #2


(a)


(b)

After applying the Horn-Schunck optical flow method we obtained the result as in Figure (a). However, the true optical flow is close to the velocity field presented in Figure (b). Suggest and motivate a way to improve the optical flow result from (a) to (b).

(2p)

1.3
a) Apply the binary operation **erosion** to the image in Fig. A, using the structuring element as in Fig. B.

(1p)

b) Apply the binary operation **dilation** to the image in Fig. A, using the structuring element as in Fig. B.

(1p)

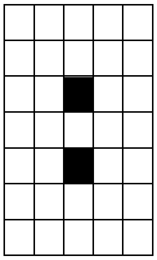


Fig. A: The input image.
(Points belonging to the object
are denoted by black squares).

Fig. B: The structuring element.
The representative point denoted by X.

1.4 Explain the relation/difference between linear diffusion filtering and filtering with Gaussian filter.

(2p)

Problem 2

Below you find one input image, normalized 0..1, and five output images. There is also a list of ten image analysis procedures. The task is to link each output image to a specific procedure (1-a, 2-b, etc.). Each correct link will give one point. For each link, an additional point will be given, if you can clearly justify your choice.

(10 p)


(input image)


(1)


(2)


(3)


(4)


(5)

(a) bilateraral filtering
(b) added Gaussian noise with zero mean and variance =0.01
(c) added salt and peppar noise
(d) Fourier phase
(e) reconstruction using Fourier phase only
(f) Canny edge detector
(g) mean shift filtering
(h) cubic transformation
(i) non-linear diffusion filtering
(j) added Gaussian noise with mean=0.2 and zero variance

Problem 3

3.1 What is the size of GLCM (Gray Level Cooccurrence Matrix) for 3-bits image of size $16x16$ pixels, for distance $d = 1$, and direction $\varphi = 0$ degrees ?

(1p)

3.2 Calculate the textural entropy (see definition below) for a uniform random grayscale image of size $64x64$, with number of gray levels $Ng = 4$, for distance $d = 1$, and direction $\varphi = 0$ degrees.

(4p)

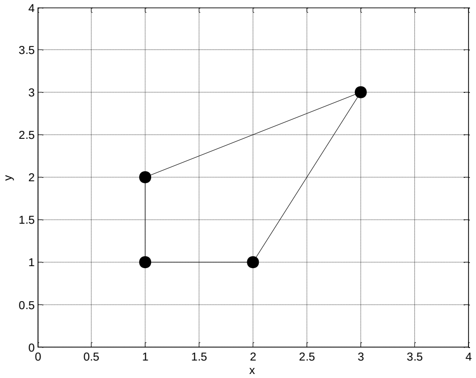
$$Textural\ entropy = -\sum_{i=1}^{Ng}\sum_{j=1}^{Ng} p_{\varphi,d}(i,j) \cdot \log_2 (p_{\varphi,d}(i,j))$$

where i, j are gray levels and $p_{\varphi,d}(i,j)$ are the corresponding probabilities.

Note:
$Ng = 4$, uniform random grayscale ↔ the random pixel values obey a uniform distribution between $1...4$

3.3 Consider an object represented by the four data points, $P1...P4$, in the figure below, where $P_i = (x_i, y_i)$: $P1 = (1,1), P2 = (2,1), P3 = (3,3), P4 = (1,2)$. We represent the points using the complex representation $z_i = x_i + j \cdot y_i$, and after performing the Fourier Transform we obtain the following Fourier descriptors (FDs):

$a(0) = 7 + 7j$
$a(1) = -3 - 3j$
$a(2) = 1 + j$
$a(3) = -1 - j$



Compute at least three translation and rotation invariant Fourier descriptors, from the above presented original FDs, $a(i)$, $i = 0..3$.

(3p)

3.4 Mention at least two image processing/analysis applications presented on the guest lectures.

(2p)

Problem 4

4.1 The Mean Shift procedure used in the Mean Shift Segmentation method can be regarded as:
- (a) adaptive gradient ascent method
- (b) adaptive gradient descent method
- (c) adaptive zero-crossing method

Choose (a), (b) and/or (c) and motivate your answer.

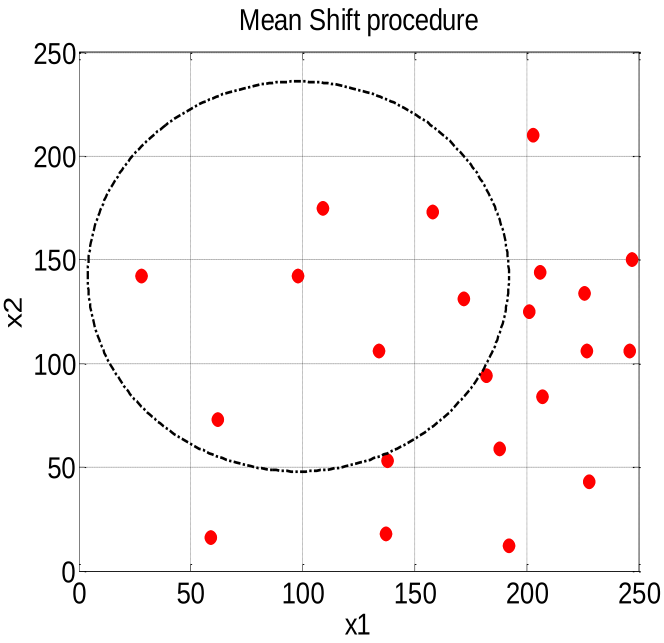
(2p)

4.2 Find the Mean Shift vector for the first step iteration for the following (x1, x2) data points (Table4). The circle in the figure below shows the starting position of the kernel and the starting point P=(98,142).

(2p)

Table 4.

x1	x2
28	142
59	16
62	73
98	142
109	175
134	106
137	18
138	53
158	173
172	131
182	94
188	59
192	12
201	125
203	210
206	144
207	84
226	134
227	106
228	43
246	106
247	150



4.3 The figure below shows an image f(x,y) with grayscale values between 1 and 6. Segment this image into two regions using **Mean Shift Segmentation** algorithm and the gray level value of the pixels as the feature. Use a free kernel type (e.g. Epanechnikov, Gaussian or other).

(6p)

f(x,y):

1	1	5	5
1	1	5	4
1	2	5	5
1	1	5	6

Problem 5

5.1 The image below shows blood vessels (the bright lines/segments) in the eye. The enlarged square is a part of the original image and the values represent the graylevels of the pixels. The task is to find the vessel position using Dynamic Programming.

Apply Dynamic Programming for computing the optimal path from the bottom to the upper layer of the image, f, below. The total cost at pixel p_i: $C(p_i) = C_1(p_i) + C_2(p_{i-1}, p_i)$
where

$C_1(p_i)$	is a brightness cost (to be specified)
$C_2(p_{i-1}, p_i) = \|\Delta y\|$	is a smoothness cost (transition cost)

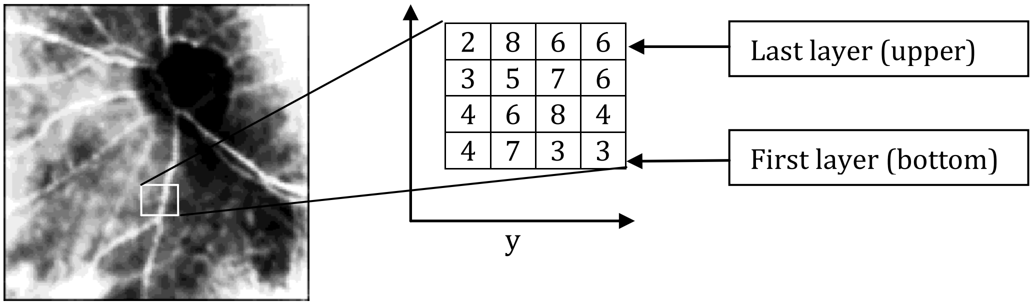
p_i– pixel belonging to layer i

You should choose a brightness cost, suitable for detection of the bright structures (vessels).

The answer should include the specification of the brightness cost, the cost accumulation matrix with back tracing pointers and the coordinates of the optimal path (or paths).

(7p)

Image f(x,y):



2	8	6	6
3	5	7	6
4	6	8	4
4	7	3	3

5.2 We are looking for a circular object in some image. The egde detection procedure resulted in a egde image as in Fig. A. **Sketch** the corresponding Hough transform (as an image) applied to the edge pixels. Assume that the radius of the object is known and equals R.

(3p)

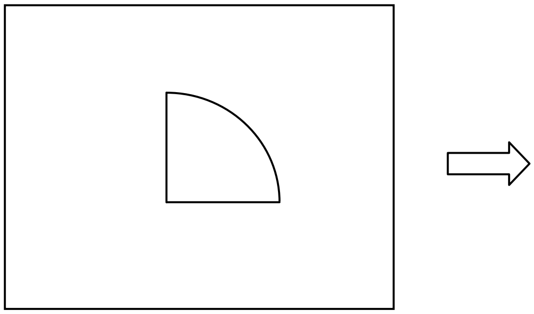


Fig. A. An edge image.

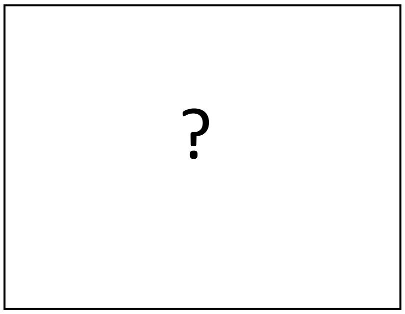


Fig.B The Hough transform of Fig.A

Problem 6

6.1 We are searching for lines in an image. The Hough Tranform (using $s = x \cdot cos(\theta) + y \cdot sin(\theta)$ representation) of the edge image looks like in Fig. C, with maxima in points P1 = (99, 102°), P2 = (28, 147°) and P3=(-99, 282°). How many lines have been detected ? Motivate your answer.

(2p)

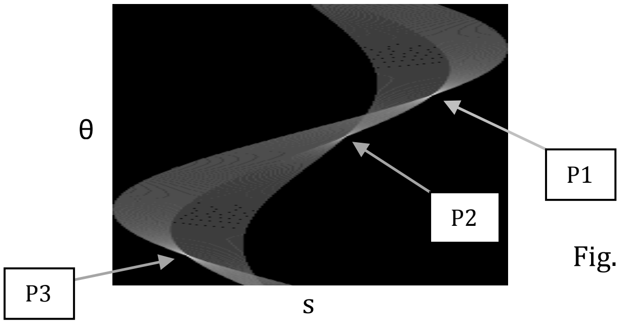


Fig. C. The Hough Transform in (θ, s) space.

6.2 See the next page !

6.2 A microscope image that presents cells against background should be segmented by thresholding. The thresholding should be carried out so that the total number of misclassified pixels is minimized. The diagram below shows the estimated histograms (weighted by the a priori probabilities) for the object (cell) and background pixels.

The object histogram (left) is proportional to $P_1 \cdot p(x|C_1)$, while the background histogram (right) is proportional to $P_2 \cdot p(x|C_2)$, where C_1= object, C_2= background, P_1, P_2 - *a priori* probabilities of object and background, respectively.

The histograms approximate the Laplace distribution: $p(x) = \frac{1}{2b} \exp\left(-\frac{|x-m|}{b}\right)$, where m and b are parameters:

$$P_1 \cdot p_1(x) \sim exp(-5|x-0.2|)$$
$$P_2 \cdot p_2(x) \sim 5 \cdot exp(-10|x-0.5|) \qquad x = 0..1$$

→ Find the optimal threshold value $\boldsymbol{T_{opt}}$ for this segmentation problem and the corresponding value *of* ***minimum error***.

(4p)

→ Calculate the **sensitivity** and **specificity** corresponding to the optimal threshold T_{opt}

(4p)