Problem 1

1.1 Consider the image grayscale, r, to be a continuous random variable with the range [0,1] and the probability density function $p(r)$, as in the figure below ($p(0) = 0$, $p(0.2) = a$, $p(0.4) = 0, p(0.6) = 0, p(0.8) = a, p(1) = 0, p(r) = 0 \; for \; 0.4 \le r \le 0.6$). The image is subject to a grayscale transformation $s = T(r)$ so that the probability density function of the output image, $p(s)$, becomes a constant.

→ Find the transformation $T(r)$, for $r \in [0,1]$.

(3p)

→ Plot the resulting $T(r)$ and interpret your result. Is your result feasible as **c**umulative **d**istribution **f**unction (**cdf**) ?

(2p)

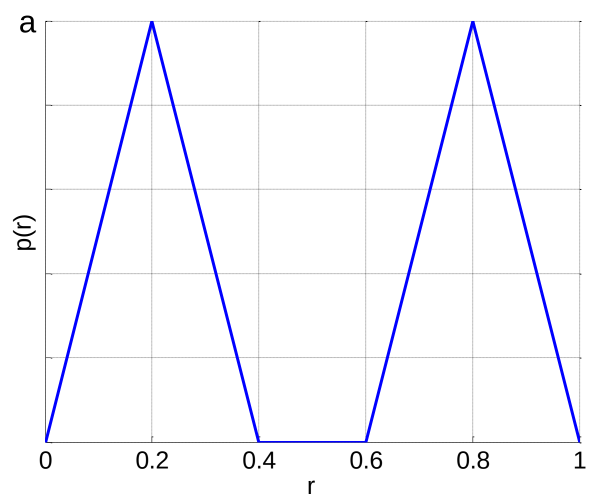


Fig.1. Probability density function $p(r)$ of a grayscale r, for some image.

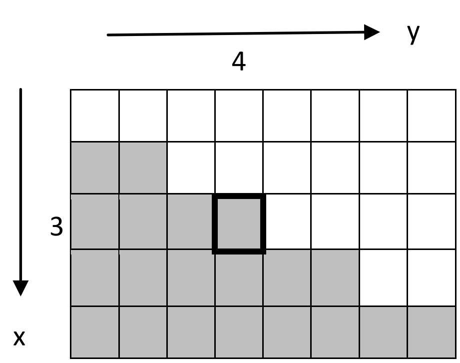
1.2 Consider an image of size 5x8 in the figure below. The pixels in gray have value 0 and the pixels in white have value 1.

→ Calculate gradient magnitude and gradient direction in the pixel with coordinates (3,4). Use the Prewitt gradient operator with 3x3 region.

(3p)

→ Draw (illustrate) the gradient vector and the edge direction on the image.

(2p)



<=>

1	1	1	1	1	1	1	1
0	0	1	1	1	1	1	1
0	0	0	0	1	1	1	1
0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	0

Problem 2

Below you find one input image, with grayscale values in the range **1..256**, shown as normalized to grayscale values **0..1**, and five output images. There is also a list of ten image analysis procedures. The output images are the result of the five image analysis procedures applied to the input image (**1..256**), presented as images scaled between **0..1**.

The task is to link each output image to a specific procedure (1-a, 2-b, etc.). Each correct link will give one point.

(1p)

For each link, an additional point will be given, if you can clearly justify your choice.

(1p)

(Totally 10p)

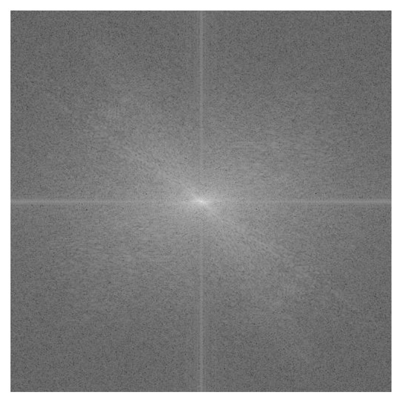
(input image)

(1)

(2)

(3)

(4)



(5)

(a) median filtering
(b) added Gaussian noise with zero mean and variance=0.01
(c) histogram equalization
(d) Fourier phase
(e) Fourier amplitude
(f) filtering with gaussian filter
(g) logarithmic transformation
(h) cubic transformation
(i) non-linear diffusion filtering
(j) added salt and peppar noise

Problem 3

3.1 What is the size of GLCM (Gray Level Cooccurrence Matrix) for 6-bits image of size $1024 \; x \; 1024$ pixels, for distance $d = 1$, and direction $\varphi = 90$ degrees ?

(1p)

3.2 Calculate the textural entropy (see definition below) for a grayscale image of size $2000 \; x \; 4000$ pixels (see Fig. 3), with number of gray levels $Ng = 4$, for distance $d = 1$, and direction $\varphi = 0$ degrees.

(4p)

$$Textural\ entropy = -\sum_{i=1}^{Ng}\sum_{j=1}^{Ng} p_{\varphi,d}(i,j) \cdot \log_2\left(p_{\varphi,d}(i,j)\right)$$

where i, j are gray levels and $p_{\varphi,d}(i,j)$ are the corresponding probabilities.

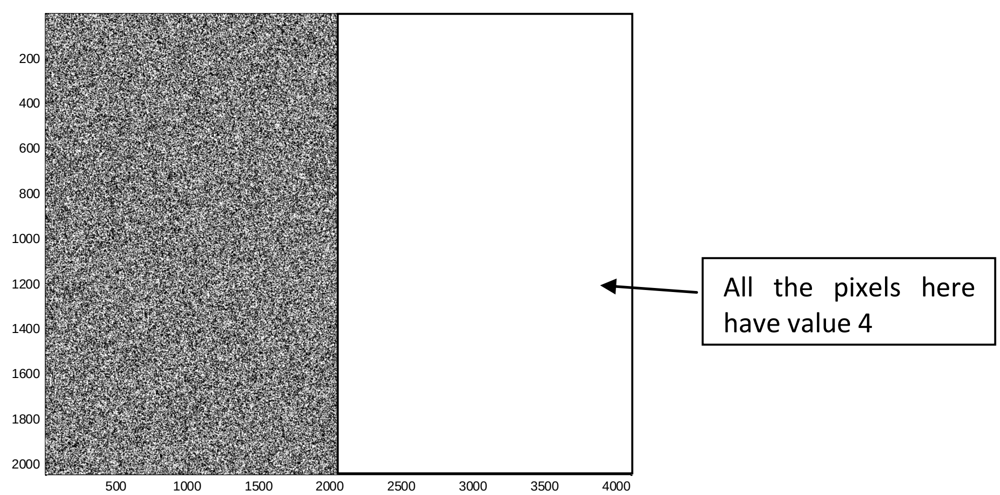
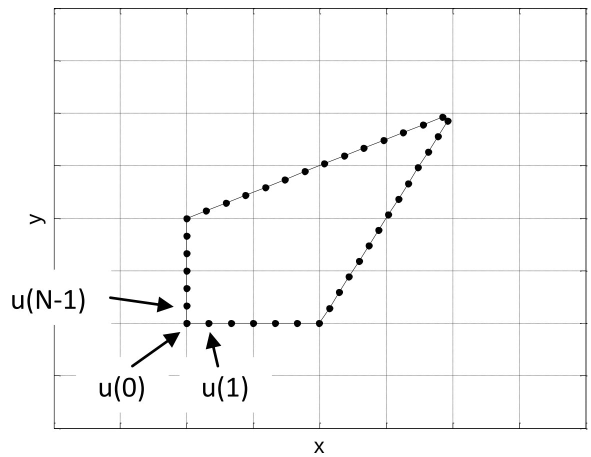


Fig 3. An image with grayscale values between 1..4. The left part (half-image) consists of uniform random texture with pixel values between 1..4, the right part is composed of pixels with value 4 only.

3.3 Consider an object represented by the the boundary as in the Figure below. We sample a number of N points on the boundary and calculate the Fourier descriptors. We represent the points using the complex representation $u(n) = x(n) + j \cdot y(n)$:



$$u(n) = x(n) + j \cdot y(n), \quad n = 0..N-1, j = \sqrt{-1}$$

The complex Fourier descriptors are defined as:

$$a(k) = \sum_{n=0}^{N-1} u(n) \exp\left(\frac{-j2\pi kn}{N}\right)$$

(a) Show that the Fourier descriptor $\boldsymbol{a}(\boldsymbol{0})$ is **NOT translation invariant**.

(1p)

(b) Show that the Fourier descriptors $\boldsymbol{a}(\boldsymbol{i}), i = 1..N-1$ are **translation invariant**.

(2p)

(c) Show that the magnitudes of the Fourier descriptors $|\boldsymbol{a}(i)|$, $i = 1..N-1$, are **rotationaly invariant**.

(2p)

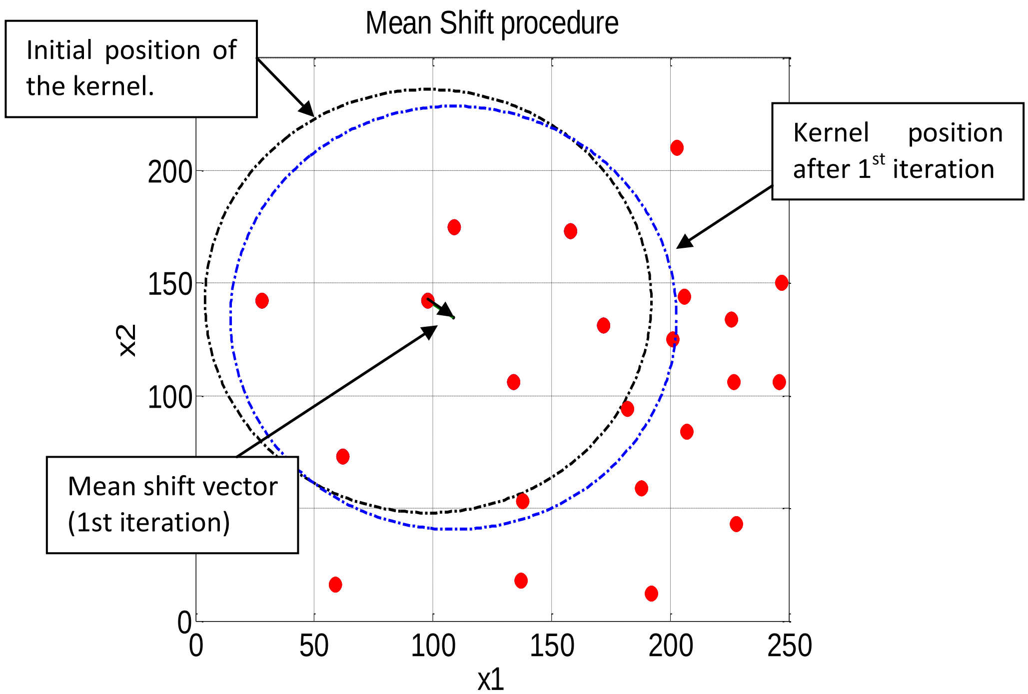
Hint: Boundary rotation by angle θ corresponds to multiplication of coordinates by $exp(j\theta)$.

Problem 4

Consider the following data points (x1,x2) as in the Table4 and plotted in Figure below. The figure shows a circle with the starting position of the kernel and the starting point P=(98,142), and the kernel position after first iteration in the Mean Shift procedure. The first iteration mean-shift vector is [10.7, -7.4].

Table 4:

x1	x2
28	142
59	16
62	73
98	142
109	175
134	106
137	18
138	53
158	173
172	131
182	94
188	59
192	12
201	125
203	210
206	144
207	84
226	134
227	106
228	43
246	106
247	150



(a) Show, using the result of the first iteration, that the Mean Shift algorithm is the gradient ascent method (at least at the first iteration).

(2p)

(b) The Mean Shift procedure is regarded as adaptive gradient ascent method. But adaptive gradient ascent of what? what kind function are we climbing?

(2p)

(c) Demonstrate/describe how to segment a part of the image below using Mean-Shift algorithm. Use the R(red) and G(green) color bands for the segmentation (Note: It is not necessary to report exact numerical values, just demonstrate all the steps and how you use the R and G pixel values).

(6p)

R(red) band:

137	120	81	58
120	92	56	56
107	60	43	43
80	56	43	51

G(green) band:

188	174	124	89
172	150	94	84
152	105	82	82
107	66	76	76

Problem 5

5.1 A 3 x 6 image is shown below. As can be seen, the image consists of "concentric" layers. The coordinates of the inner layer are (1, 3), (1, 4), (1, 5), (1, 6), (2,6) and (3,6). For each position (x, y) in the image there is an associated cost c, e.g. c(2, 1) = 8. The problem is to find all the paths from the **inner** layer to the **outer** layer (two travel steps are required for each path) so that the cumulative cost is minimized. Each layer may only be visited once. The allowed travel directions are **north, east, and north-east**. There is a penalty term **p = 1** associated with the travel directions north and east. For the travel direction north-east **p = 0**. The result should present the optimal path(s), e.g. by specifying the three coordinate pairs of this path. It should also present the values of the cumulated travel costs in the form of a 3 x 6 cumulative matrix with clearly indicated back tracing pointers.

(6p)

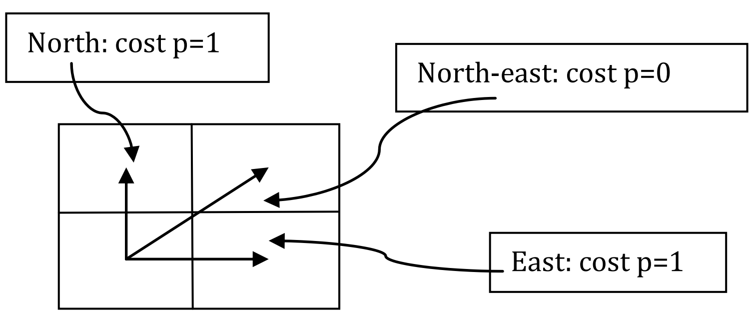
	1	**2**	**3**
1	7	8	8
2	5	5	3
3	6	6	4
4	6	6	7
5	4	*4*	*6*
6	4	5	7

← image x coordinate

image y coordinate

Inner layer consists of pixels: (1,3), (1,4), (1,5) ,(1,6),(2,6) and (3,6) ← (x,y)
Middle layer: (1,2), (2,2), (2,3), (2,4), (2,5) and (3,5)
Outer layer: (1,1), (2,1), (3,1), (3,2), (3,3) and (3,4)

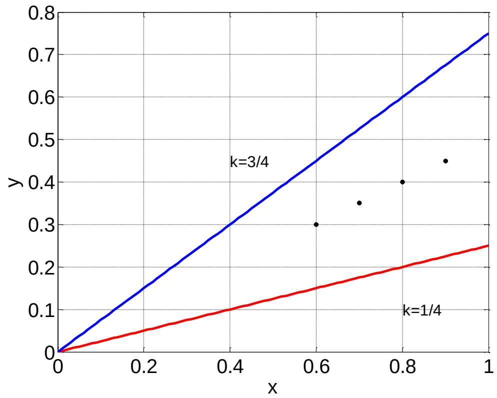
Allowed travel directions:

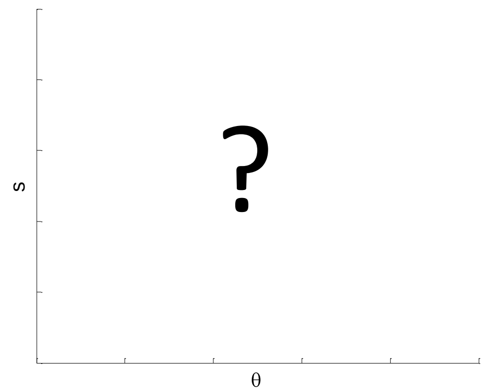


5.2 We are looking for straight lines, represented as $y = k \cdot x + q$, in an image using the Hough Transform. We know that the slope coefficient k of the lines is between ¼ and ¾. The image below show just some pixels (black dots) belonging to the egde pixels we investigate.

(a) → Sketch the Hough Transform of the pixels (x,y)= P1=(0.8, 0.4) and P2=(0.6, 0.3) using the Hough Transform in (s, θ) space, i.e. $s = x \cdot cos(\theta) + y \cdot sin(\theta)$, assuming that the pixels belong to some line with $0.25 \leq k \leq 0.75$.

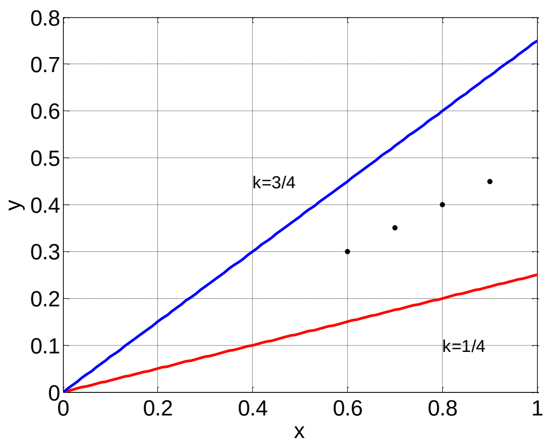
(2p)

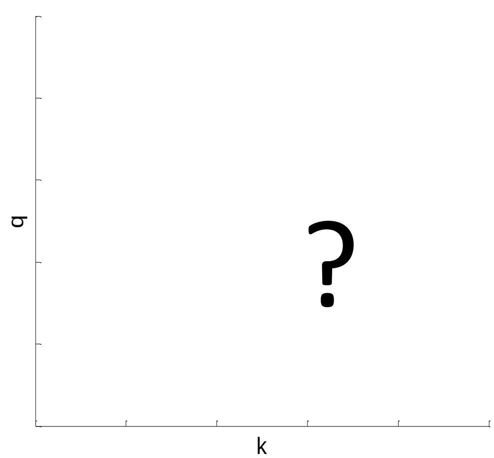




(b) → Sketch the Hough Transform of the pixels (x,y) =P1= (0.8, 0.4) and P2=(0.6, 0.3) using the Hough Transform in (k, q) space, i.e. $y = k \cdot x + q$, assuming that the pixels belong to some line with $0.25 \leq k \leq 0.75$

(2p)

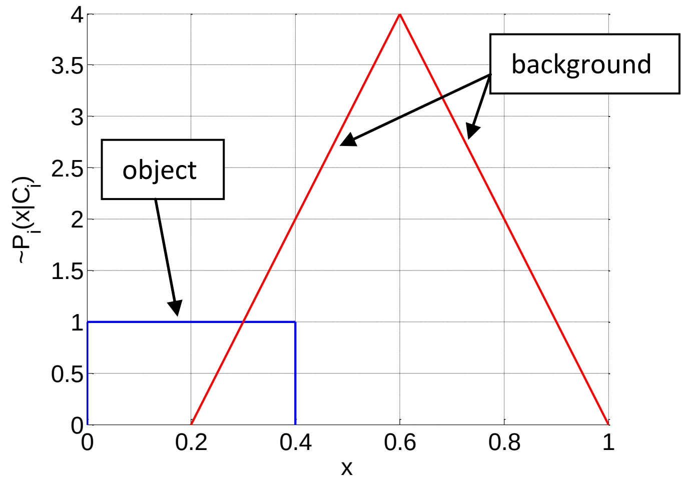




Problem 6

6.1 A microscope image that presents cells against background should be segmented by thresholding. The thresholding should be carried out so that the total number of misclassified pixels is minimized. The diagram below shows the estimated histograms (weighted by the a priori probabilities) for the object (cell) and background pixels.

The background histogram (see the Figure below, right, triangular) is proportional to $P_1 \cdot p(x|C_1)$, while the object histogram (left, uniform) is proportional to $P_2 \cdot p(x|C_2)$, where C_1= background, C_2=object, P_1, P_2 - *a priori* probabilities of background and object, respectively, $p(x|C_i), i = 1{,}2$ are class-conditional probability density functions, x is a grayscale value.



(a) → Calculate the optimal threshold value $\boldsymbol{T_{opt}}$ for this segmentation problem

(1p)

(b) → Calculate the corresponding (when T=T_{opt}) value *of **minimum error***.

(1p)

(c) → Calculate the **sensitivity** and **specificity** corresponding to the optimal threshold T_{opt}

(2p)

(d) → Express the sensitivity and specificity as a function of threshold T

(2p)

(e) → Plot the ROC-curve (ROC = Receiver Operating Characteristic)

(2p)

(f) → For which values of T will all the object pixels be correctly classified ?

(1p)

(g) → For which values of T will all the background pixels be correctly classified ?

(1p)

(Totally 10p)