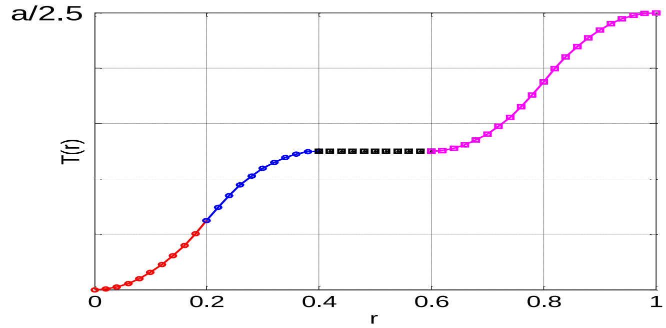


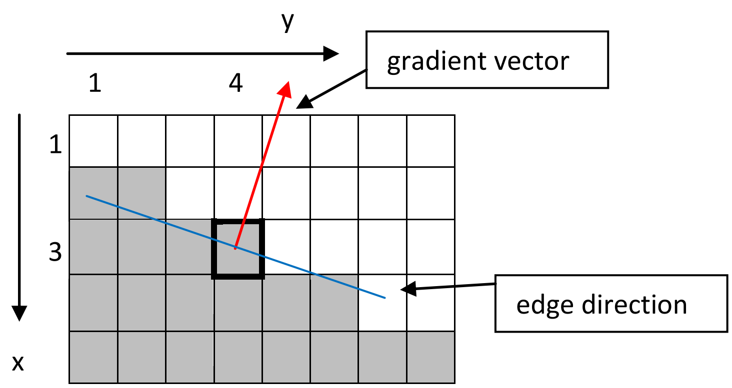
1.1

$$p(r) = \begin{cases} 5ar & 0 \le r \le 0.2 \\ -5ar + 2a & 0.2 \le r \le 0.4 \\ 0 & 0.4 \le r \le 0.6 \\ & .. \\ & .. \end{cases}$$

$$s = T(r) = \int_0^r 5awdw = \left(\frac{5}{2}\right) aw^2 \Big|_0^r = 2.5ar^2 \qquad 0 \le r \le 0.2$$

..
..
..

1.2 Gradient vector = $[g_x \; g_y] = [-3 \; 1]$, gradient magnitude = $\sqrt{10}$



2. 1-g, 2-j, 3-c, 4-h, 5-e

3.1 3-bits: GLCM: 8 x 8, 6 bits: GLCM size = 2^6 x 2^6 = 64 x 64
3.2 Cooccurrence matrices:
Left half image≈

$$\begin{bmatrix} \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\ \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\ \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\ \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \end{bmatrix}$$

Right half-image: $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Cooc for the whole image≈ $\begin{bmatrix} \frac{1}{32} & \frac{1}{32} & \frac{1}{32} & \frac{1}{32} \\ \frac{1}{32} & \frac{1}{32} & \frac{1}{32} & \frac{1}{32} \\ \frac{1}{32} & \frac{1}{32} & \frac{1}{32} & \frac{1}{32} \\ \frac{1}{32} & \frac{1}{32} & \frac{1}{32} & \frac{17}{32} \end{bmatrix}$

Textural entropy ≈ 2.83 `% -p*log2(p))`

3.3 The complex Fourier descriptors $a(k)$ are defined as:

$$a(k) = \sum_{n=0}^{N-1} u(n) \exp\left(\frac{-j2pkn}{N}\right)$$

$u(n) = X(n) + jY(n)$, where u(n) are the boundary points.

Translation invariancy of a(0):

$$a(0) = \sum_{n=0}^{N-1} u(n) \exp\left(\frac{-j2\pi \cdot 0 \cdot n}{N}\right) = \sum_{n=0}^{\cdot N-1} u(n)$$

Let's translate the object by vector (c, d). The new coordinates $u_{new}(n)$ will become

$$u_{new}(n) = X(n) + jY(n) + c + jd$$

The new FDs will be:

$$a_{new}(0) = \sum_{n=0}^{N-1} [u(n) + \mathrm{c} + \mathrm{jd}] \exp\left(\frac{-j2\pi \cdot 0 \cdot n}{N}\right) = N \cdot (c + jd) + \sum_{n=0}^{N-1} u(n) = N \cdot (c + jd) + a(0)$$

$a_{new}(0)$ is not equal the original $a(0) \rightarrow$ a(0) is **NOT** translation invariant

Translation invariancy of a(k), k>0:

$$a(k) = \sum_{n=0}^{N-1} u(n) \exp\left(\frac{-j2\pi \cdot k \cdot n}{N}\right)$$

Let's translate the object by vector (c, d). The new coordinates $u_{new}(n)$ will become

$$u_{new}(n) = X(n) + jY(n) + c + jd$$

The new FDs will be:

$$a_{new}(k) = \sum_{n=0}^{N-1} [u(n) + \mathrm{c} + \mathrm{jd}] \exp\left(\frac{-j2\pi \cdot k \cdot n}{N}\right) = a(k) + (c + jd) \cdot \underbrace{\sum_{n=0}^{N-1} \exp\left(\frac{-j2\pi \cdot k \cdot n}{N}\right)} = a(k)$$

This sum is equal zero.

$a_{new}(k)$ is equal the original $a(k) \rightarrow$ a(k), k>0, is translation invariant

Rotation invariancy of $|a(i)|$:

$u_{new}(n) = [X(n) + jY(n)] \exp(j\theta)$

$$a_{new}(k) = \sum_{n=0}^{N-1} u(n) \cdot \exp(\mathrm{j}\theta) \cdot \exp\left(\frac{-j2\pi k \cdot n}{N}\right) = \exp(j\theta) \cdot a(k)$$

$$|a_{new}(k)| = |\exp(j\theta) \cdot a(k)| = |\exp(j\theta)| \cdot |a(k)| = |a(k)|$$

as $|\exp(j\theta)|=1$ for any θ:

$$|\exp(j\theta)| = |\cos(\theta) + jsin(\theta)| = (\cos^2(\theta) + \sin^2(\theta))^{1/2} = 1$$

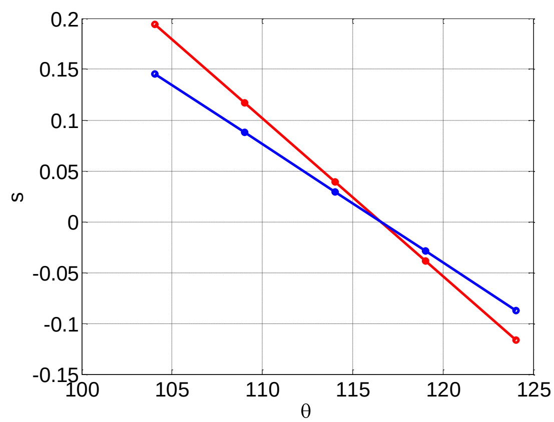
$|a_{new}(k)| = |a(k)| \rightarrow$ magnitude of FDs is rotationally invariant feature

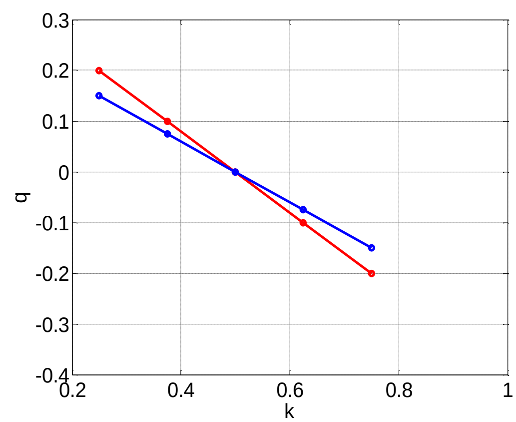
4.1 After the first iteration we have three more samples inside the kernel--> the density (assuming uniform kernel) in this region will be higher than in the starting position of the kernel--> we are moving toward a more dense region (=ascending pdf)
4.2 The (estimate) of the joint probability density function of image attributes.
4.3 See the textbook, e.g. Sonka et al., Pages 259-263.

5.1

	1	**2**	**3**
1	20	21	20
2	12	12	15
3	6	12	14
4	6	10	15
5	4	8	11
6	4	5	7

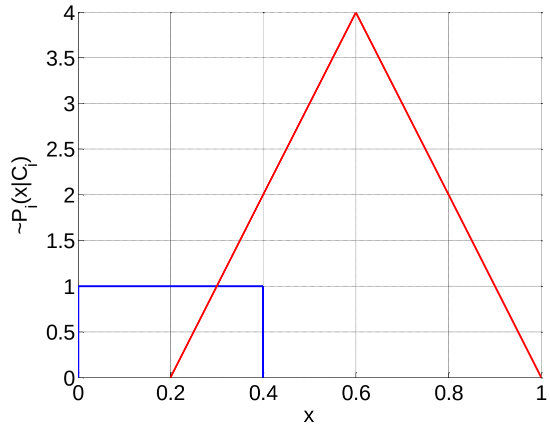
5.2 To detect the line, the curves should cross each other (otherwise the line will not be detected).





6.1

$10(x-0.2)=1 \Longrightarrow$
(a) $x=0.3$



Total area A = B + R (Blue+Red)
R= $0.4 \cdot 4 = 1.6$ $B=0.4 \cdot 1 = 0.4$ A=B+R=2.0
Positive == Object
Negative == Background

E1 = Type I Error (False Positive = False Object): $(0.3-0.2) \cdot \frac{1}{2} = 0.05$
E2 = Type II Error II (False Negative = False Background): $(0.4-0.3) \cdot 1 = 0.1$

(b) Minimum error (for T=0.3) = (Error I + Error II)/A = (0.05+0.1)/2.0=0.075
(c) Sensitivity (T=0.3) = 0.3/0.4 = 0.75
Specificity(T=0.3) = 1- 0.05/1.6=1-0.013= 0.9688
(d) Sensitivity = $T/0.4$ for $T <= 0.4, 1$ for $T > 0.4$
Specificity = $[1.6 - 10(T-0.2)(T-0.2)/2]/1.6$ for $0.2 \leq T \leq 0.6$
(e) See the plot(s) below.
(f) $T >= 0.4$
(g) $T <= 0.2$