Problem 1

1.1 Consider the image grayscale, r, to be a continuous random variable with the range [0,1] and the probability density function $p(r)$, as in the figure below:

$$p(r) = a * \sin(10 \cdot r) \quad \text{when } a * \sin(10 \cdot r) \geq 0$$
$$p(r) = 0 \quad \text{when } a * \sin(10 \cdot r) < 0$$

The image is subject to a grayscale transformation $s = T(r)$ so that the probability density function of the output image, $p(s)$, becomes a constant.

→ Find the transformation $T(r)$, for $r \in [0,1]$.

(3p)

→ Plot the resulting $T(r)$ and interpret your result.

(1p)

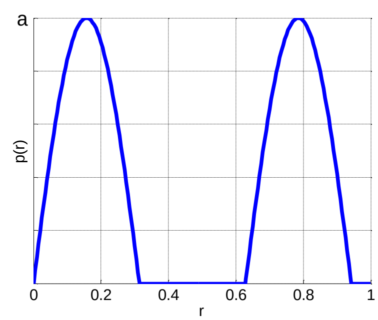


Fig.1. Probability density function $p(r)$ of a grayscale r, for some image.

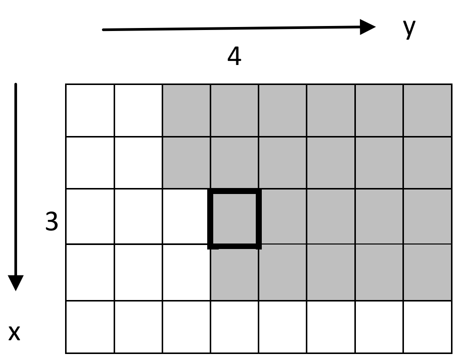
1.2 Consider an image of size 5x8 in the figure below. The pixels in gray have value 0 and the pixels in white have value 1.

→ Calculate gradient magnitude and gradient direction in the pixel with coordinates (3,4). Use (1) Prewitt gradient operator and (2) Sobel gradient operator with 3x3 region.

(3p)

→ Draw (illustrate) on the image the gradient vectors (Prewitt and Sobel) and the corresponding edge directions. Is there any difference between these two gradients (Prewitt and Sobel) calculated for this particular image?

(3p)



<=>

1	1	0	0	0	0	0	0
1	1	0	0	0	0	0	0
1	1	1	0	0	0	0	0
1	1	1	0	0	0	0	0
1	1	1	1	1	1	1	1

Problem 2

Below you find one input image (with grayscale values between 0..1) and five output images. There is also a list of ten image analysis procedures. The task is to link each output image to a specific procedure (1-a, 2-b, etc.). Each correct link will give one point. For each link, an additional point will be given, if you can clearly justify your choice. The task is to link each output image to a specific procedure (1-a, 2-b, etc.). Each correct link will give one point.

(1p)

For each link, an additional point will be given, if you can clearly justify your choice.

(1p)

(Totally 10p)

(input image)

(1)

(2)

(3)

(4)

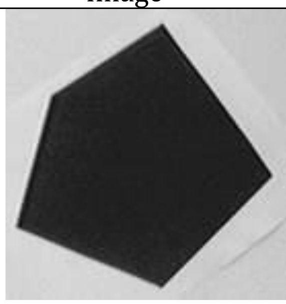
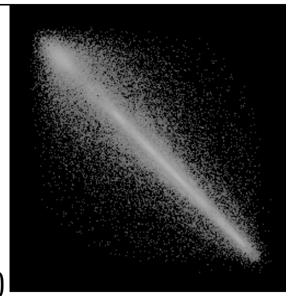
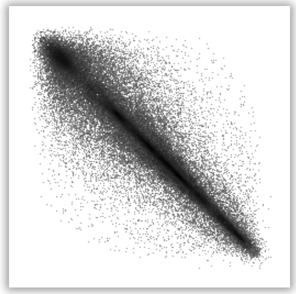
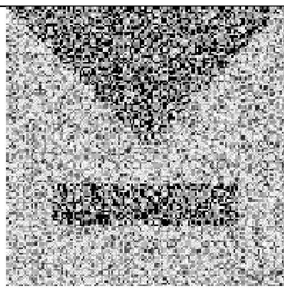
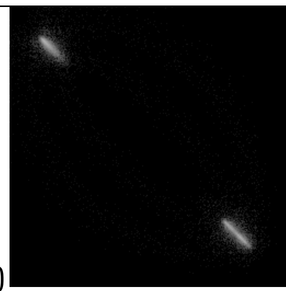
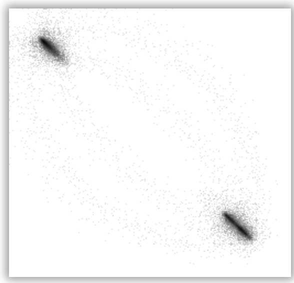
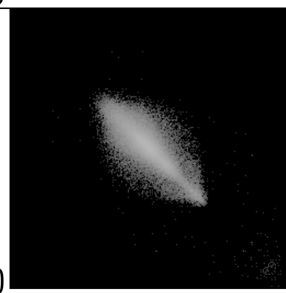
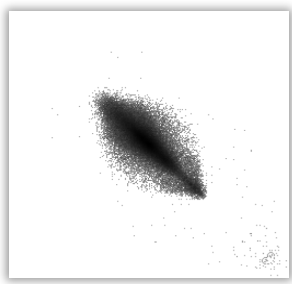
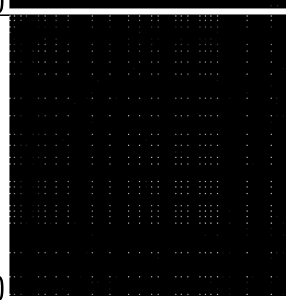
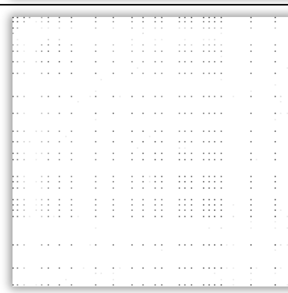
(5)

(a) ideal highpass filtering
(b) minimum 5x5 filter
(c) reconstruction using Fourier phase and magnitude
(d) binarization and morphological opening
(e) binarization and morphological closing
(f) maximium 5x5 filter
(g) ideal lowpass filtering
(h) square transformation
(i) $sin(2 \cdot I)$, where I is input image, sin = sinus
(j) binarization by thresholding with T=0.5

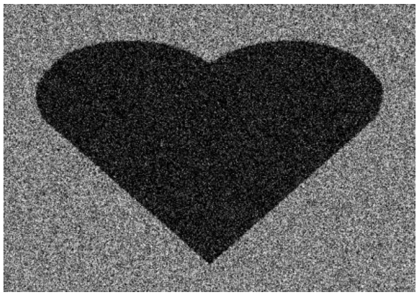
Problem 3

3.1 The figure below shows some grayscale images (column 1) and their corresponding Gray Level Cooccurrence Matrices GLCM (d=1, 0 degrees) in column 2. (Column 3 is inverted column 2, just for making it more visible).
Your task is to mach the images with the GLCMs, e.g. (A-1, B-2, …) and justify your choices.

(4 p)

Image	GLCM	GLCM inverted
(A) 	(1) 	
(B) 	(2) 	
(C) 	(3) 	
(D) 	(4) 	

3.2 The image belows shows a heart corrupted by gaussian noise. Describe how to find the borders of the heart using **Dynamic Programming.** You should specify
(1) your image preprocessing steps, if any
(2) how you construct the cost matrix (i.e how do the cost terms included in the cost matrix look like?).
Note: It is not necessary to present any numerical results.



(6p)

Problem 4

Demonstrate how to segment a part (5x5 pixels) of the image below using the Mean-Shift algorithm in the RGB color space. Use all of the the R(red), G(green) and B(blue) color bands for the segmentation, i.e. your input data points will be three-dimensional. You should use the numerical values below and the exact numerical segmentation is necessary.

(10p)

RED=

157	157	245	245	245
157	157	245	245	245
157	157	245	255	245
157	157	245	255	255
157	157	245	255	255

GREEN =

7	7	174	174	174
7	7	174	174	174
7	7	174	208	174
7	7	174	208	208
7	7	174	208	208

BLUE =

78	78	1	1	1
78	78	1	1	1
78	78	1	0	1
78	78	1	0	0
78	78	1	0	0

Problem 5

The Figure below contains a number of circular arcs and we are interested in detecting the arcs <u>within the first quadrant</u>, using the circular **Hough transform**. Assume that the input image is the image after some egde detection procedure.
Write an algorithm (using pseudo-code or Matlab-like notation) that detects these arcs and estimates the arcs' center, radius and counts the total number of arcs. Observe that in the input image there are some arcs not belonging to the first quadrant.
The output from your algorithm should be like (where Xi, Yi and Ri are the estimated values):

```
The detected arcs:

1:  Center x= X1,  y=  Y1, Radius=R1
2:  Center x= X2,  y=  Y2, Radius=R2
.....
4:  Center x= X4,  y=  Y4, Radius=R4
Total number of detected arcs within the 1st quadrant: 4 (for example)
```

(10p)

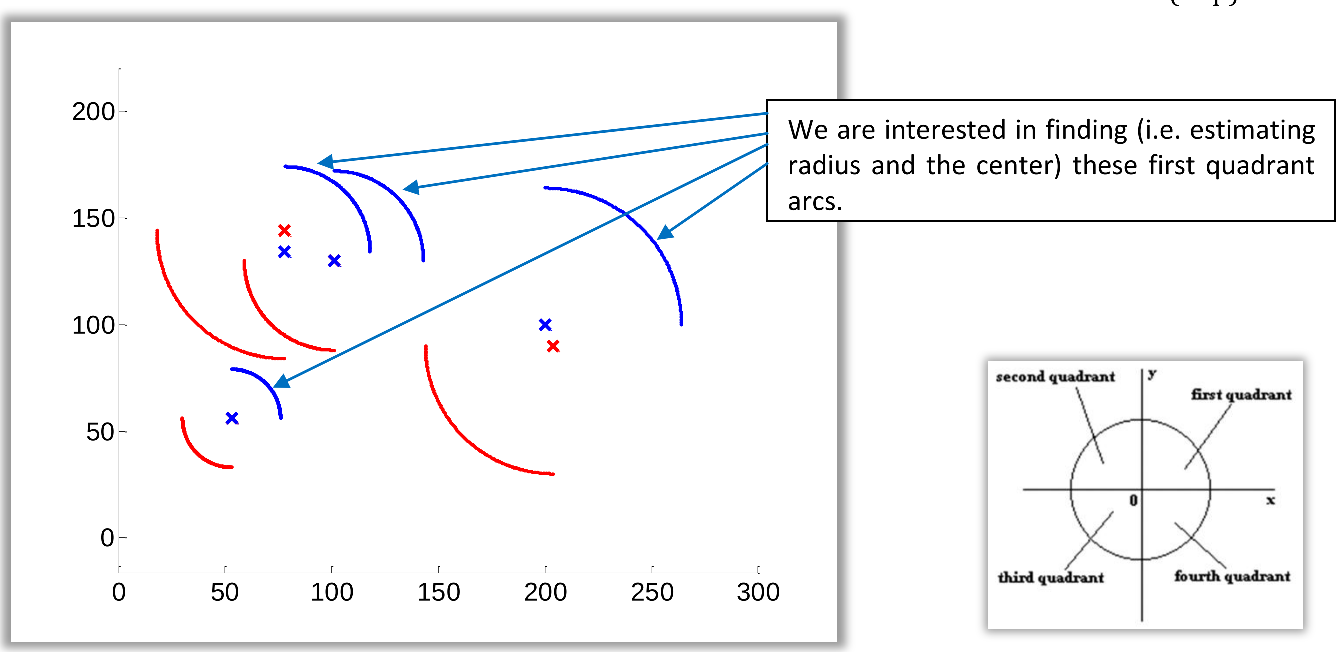
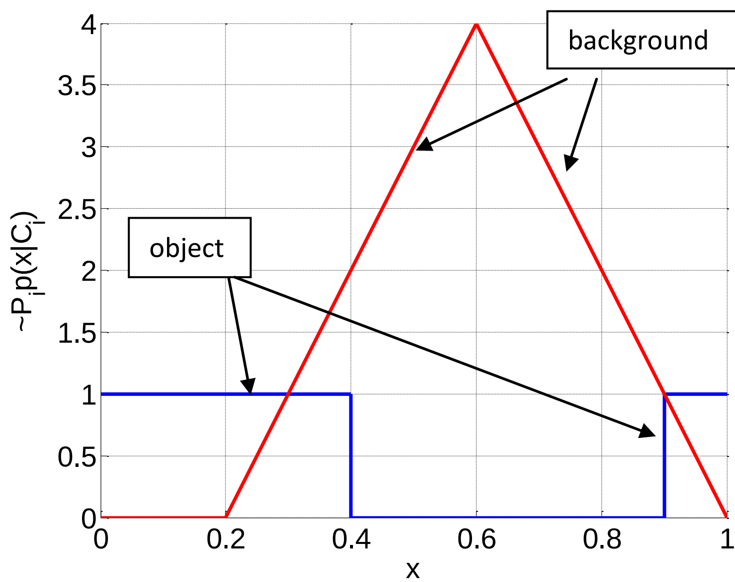


Fig. The edges (edge pixels) detected by som edge detection procedure (Note: the crosses are drawn to visualize the potential centers of the arcs, the crosses are NOT result of the edge detection). We are interested in finding the position (center) and radius of the arcs lying in the first quadrant (the blue arcs).

Problem 6

A microscope image that presents cells against background should be segmented by thresholding. The thresholding should be carried out so that the total number of misclassified pixels is minimized. The diagram below shows the estimated histograms (weighted by the *a priori* probabilities) for the object (cell) and background pixels.

The background histogram (see the Figure below, triangular) is proportional to $P_2 \cdot p(x|C_2)$, while the object histogram (left & right, uniform) is proportional to $P_1 \cdot p(x|C_1)$, where C_1= object, C_2=background, P_1, P_2 - *a priori* probabilities of object and background, respectively, $p(x|C_i), i = 1,2$ are class-conditional probability density functions, x is a grayscale value.



$$P_1 p(x|C_1) = \begin{cases} 1 & for\ 0 \le x \le 0.4 \\ 0 & for\ 0.4 < x < 0.9 \\ 1 & for\ 0.9 \le x \le 1 \end{cases}$$

$$P_2 p(x|C_2) = \\ = \begin{cases} 0 & for\ 0 \le x \le 0.2 \\ 10(x - 0.2) & for\ 0.2 < x < 0.6 \\ -10(x - 1) & for\ 0.6 \le x \le 1 \end{cases}$$

But instead of applying the minimum classification rule directly, we investigate the threshold based classification rule:

IF $(\, pixel\ value\ x < T)$ THEN the pixel belongs to the object (Rule 1)
OTHERWISE the pixel belongs to the background.

(a) → Express the sensitivity and specificity as a function of threshold T, when using Rule 1. (4p)

(b) → Plot the ROC-curve (ROC = Receiver Operating Characteristic), when using Rule 1. (2p)

(c) → For which threshold value T (using Rule 1) will the classification error be minimum ? (2p)

(d) → For which value of T (using Rule 1) will all the object pixels be correctly classified ? (1p)

(e) → For which value of T (using Rule 1) will all the background pixels be correctly classified ? (1p)

(Totally 10p)