Problem 1

1.1 Consider the image grayscale, r, to be a continuous random variable with the range [0,1] and the probability density function $p(r)$, as in the Figure 1 below:

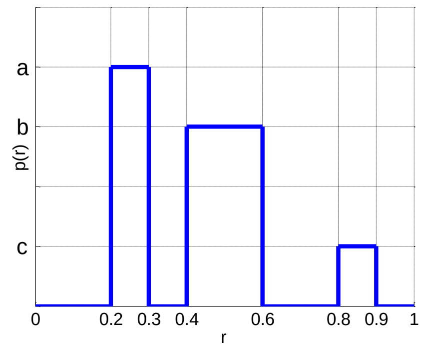


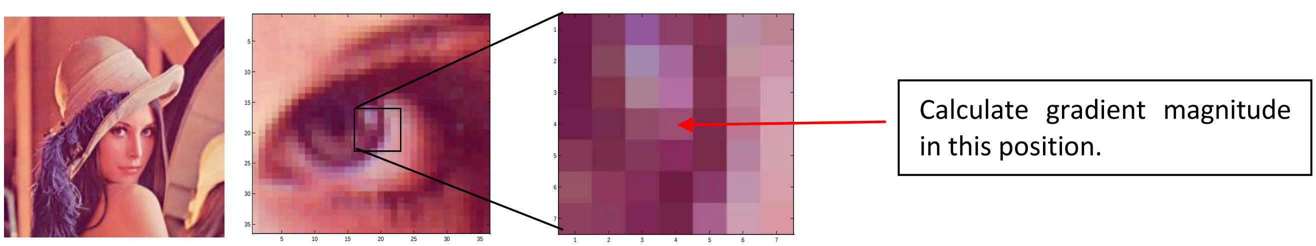
Fig.1. Probability density function $p(r)$ of a grayscale r, for some image.

$$
\begin{aligned}
p(r) &= a \quad \text{when } 0.2 \le r \le 0.3 \\
p(r) &= b \quad \text{when } 0.4 \le r \le 0.6 \\
p(r) &= c \quad \text{when } 0.8 \le r \le 0.9 \\
p(r) &= 0 \quad \text{otherwise}
\end{aligned}
$$

The image is subject to a grayscale transformation $s = T(r)$ so that the probability density function of the output image, $p(s)$, becomes a constant.

→ Find the transformation $T(r)$, for $r \in [0,1]$, and **plot** it. (3p)

1.2 Consider the color image below consisting of the three bands: RED, GREEN and BLUE and a small part of this image shown in the figure below (rightmost). Calculate the gradient magnitude, using **Prewitt** filter, of this color image in the position of the underlined pixel. (4p)



RED=	11	13	15	14	13	18	19
	11	13	16	17	12	19	20
	11	13	17	18	13	19	21
	11	11	14	**16**	12	19	21
	14	12	13	14	12	18	21
	15	14	13	11	14	19	22
	14	14	13	12	17	21	22

GREEN=	3	6	9	7	5	14	13
	2	7	14	10	4	15	14
	3	5	13	11	5	13	16
	3	4	8	**8**	4	12	16
	6	4	6	4	4	13	16
	8	6	4	3	6	15	16
	6	6	4	4	10	16	15

BLUE=	7	9	16	10	8	16	15
	7	9	17	15	7	16	17
	8	7	15	16	8	15	18
	7	8	10	**12**	8	15	18
	9	8	9	9	7	16	18
	10	9	9	7	10	17	17
	9	10	8	7	14	18	16

1.3 Calculate the operation erosion applied to the binary image X (Fig.1.3 left) using the structuring element B (Fig.1.3 right). Use the set definition of erosion operation as in Sonka, def. (13.15), page 662 (see Appendix – **page 2** in this exam).

→ Erosion result: $X \ominus B = ?$

(3p)

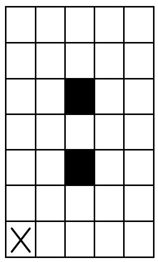


Fig. 1.3 (left): The input image $X = \{(2,2),(2,4)\}$ (points belonging to the object are denoted by black squares).

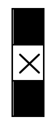


Fig. 1.3 (right).
The structuring element $B = \{(0,-1),(0,1)\}$.
The representative point denoted by ☒

Problem 2

2.1 In mammography imaging we use nowadays grayscale images represented by 12 or 16 bits. Using all of the bits would result in a huge GLCMs (Gray Level Cooccurrence Matrices). To reduce the size of GLCM we have to quantize the 12-bits image to a lower number of bits. How many bits will we use for obtaining GLCMs of size 64 x 64 ?

(1p)

2.2 Calculate the textural feature **contrast** for the image below, for displacement $\vec{d} = (+1,-3)$, i.e. $dx = +1, dy = -3$.

y ↑

4	5	4
2	4	2
4	5	4
2	4	2
4	5	4

→ x

(3p)

2.3 For pattern recognition applications we usually extract image features that are invariant with respect to translation, size and rotation. Explain shortly, why we prefer the **invariant** features.

(2p)

2.4 Which of the image and imaging techniques/applications below have been presented on the guest lectures this year (2010) ?

(a) sausage packaging (b) fire detection (c) fluorescence imaging
(d) moon landing (e) gaze detection (f) liver volume segmentation

(2p)

2.5 Specify what kind of algorithm/method (within the scope of the SSY095 Image Analysis course) was developed by B. Lucas and T. Kanade. How many pixels are used in Lucas-Kanade method for estimation of the output in a (x, y) position of some pixel? (2p)

Problem 3

Below you find one input image of size 512x512 (with grayscale values I between 0..1) and five output images. There is also a list of ten image analysis procedures. The task is to link each output image to a specific procedure (1-a, 2-b, etc.). Each correct link will give one point.

(1p)

For each link, an additional point will be given, if you can clearly justify your choice.

(1p)

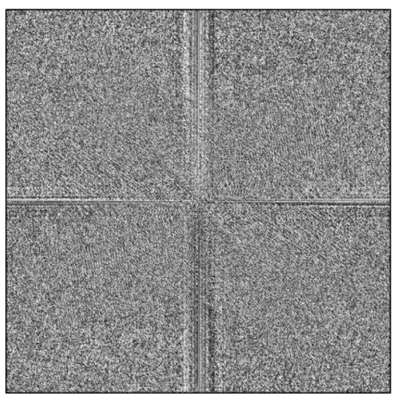
(Totally 10p)

(input image)

(1)

(2)

(3)



(4)

(5)

(a) homomorphic filtering
(b) cube root ($\sqrt[3]{x}$)
(c) reconstruction using Fourier amplitude
(d) Gaussian filter 33x33 with $\sigma = 1$ pixel
(e) Gaussian filter 33x33 with $\sigma = 8$ pixels
(f) negative transformation
(g) vignetting
(h) nonlinear diffusion filtering with diffusivity $c = 1/(1 + (|\nabla I|/k)^2),\ k = 10$
(i) nonlinear difussion filtering with diffusivity $c = 1/(1 + (|\nabla I|/k)^2),\ k = 30$
(j) Fourier phase

Problem 4

4.1 The four images below represent multispectral images from LANDSAT5 satellite. Demonstrate how to segment a part (3x3 pixels area) of the images below using the Mean-Shift algorithm in the 4-D feature space. For segmentation use as the features the intensity values from all of the bands (4, 5, 6 and 7), i.e. your input data points will be **four-dimensional**. You should use the numerical values below and the exact numerical segmentation is necessary. The choice of the kernel is free, however you should choose kernel(s) that lead to final segmentation with two clusters (=two regions).

(8p)

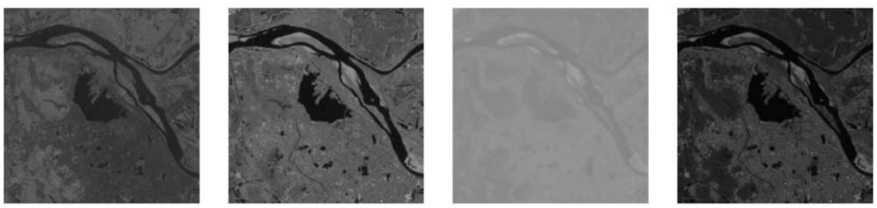


Figure 4.1a: LANDSAT5 images of Red River in China: bands 4, 5, 6 and 7.

3	3	3
9	9	3
9	9	3

Band 4

2	2	2
8	2	2
8	8	2

Band 5

5	5	5
1	1	5
1	1	5

Band 6

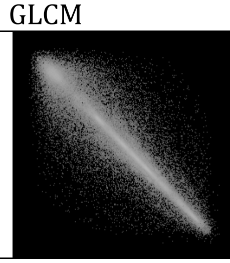
3	3	3
8	8	3
8	8	3

Band 7

Figure 4.1b: Intensity values in some 3x3 area, four bands.

4.2 In texture analysis we often present the Gray Level Coocurence Matrices as images, for example, the GLCM $(\varphi = 0°, d = 1)$ of image "Lena" is shown in Figure 4 (upper right). Draw the corresponding image of GLCM $(\varphi = 90°, d = 1)$ for the uniform random image of size 512x512, represented by 3 bits as in Figure 4 (lower-left), and explain your drawing.

(2p)

Image	GLCM
	
	?

Problem 5

5.1 The image below shows three pixels we know belong to a circle with radius $R = 10$ pixels. The pixel coordinates are: P1 =(X,Y) = (20,45), P2=(10,35), P3=(30,35).
Calculate the circle's center point (xc, yc), using circular Hough Transform. Use the circle representation, $x = xc + R \cdot cos(\varphi)$, $y = yc + R \cdot sin(\varphi)$, and investigate $\varphi = 0, \pi/2, \pi, 3\pi/2$. You should demonstrate the voting procedure and the solution should contain the voting matrix and the center point (xc, yc) coordinates of the detected circle.

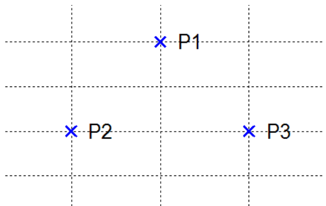


Figure 5: Three points belonging to a circle with R=10.

(5p)

5.2 Apply Dynamic Programming for computing the optimal (minimum cost) path from the left to the right border of the image, $f(x, y)$, below.

The brightness cost is $C_1(p_i) = f(p_i)$ and the smoothness cost $C_2(p_{i-1}, p_i) = |\Delta y|$.
The cost to go from pixel p_{i-1} to pixel p_i is $C(p_i) = w_1 C_1(p_i) + w_2 C_2(p_{i-1}, p_i)$,
where the weights $w_1 = 2$ and $w_2 = 1$.

p_{i-1} is a pixel in layer $i - 1$, p_i is a pixel in layer i

The answer should include the cost accumulation matrix with back tracing pointers of the optimal path(s).
Note: You should compute **all** of the minimum paths (there can be more than just one).

f(x,y):

5	1	2
2	3	4
3	2	1
2	2	3

(4p)

5.3 Consider the image f(x,y) below and the problem of finding the optimal path (minimum cost path) from left to right. Assume that brightness cost $C_1(p_i) = f(x, y)$ and smoothness cost $C_2(p_{i-1}, p_i) = 0$, for all pixels. How many minimum paths do exist for this problem?

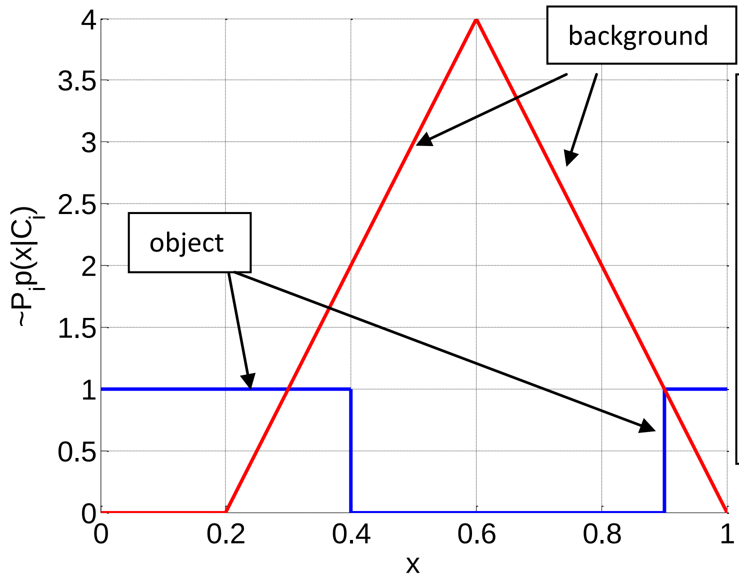
f(x,y):

1	2	3	4	5	6	7	8	9	9	9
1	2	3	4	5	6	7	8	9	10	11
1	2	3	4	5	6	7	8	9	10	11

(1p)

first layer (arrow to first column) — last layer (arrow to last column)

Problem 6
A microscope image that presents cells against background should be segmented by thresholding. The thresholding should be carried out so that the total number of misclassified pixels is minimized. The diagram below shows the estimated histograms for the object (cell) and background pixels. The background histogram (see the Figure below, triangular) is proportional to $P_2 \cdot p(x|C_2)$, while the object histogram (left & right, uniform) is proportional to $P_1 \cdot p(x|C_1)$, where C_1= object, C_2=background, P_1, P_2 - *a priori* probabilities of object and background, respectively, $p(x|C_i), i = 1,2$ are class-conditional probability density functions, x is a grayscale value.



$$P_1 p(x|C_1) = \begin{cases} 1 & for\ 0 \le x \le 0.4 \\ 0 & for\ 0.4 < x < 0.9 \\ 1 & for\ 0.9 \le x \le 1 \end{cases}$$

$$P_2 p(x|C_2) = \\ = \begin{cases} 0 & for\ 0 \le x \le 0.2 \\ 10(x-0.2) & for\ 0.2 < x < 0.6 \\ -10(x-1) & for\ 0.6 \le x \le 1 \end{cases}$$

As the class-wise histograms intersect in more than one point we use two thresholds, T1 and T2, $(T1 < T2)$ and the following classification rule:

IF $(T1 \le\ pixel\ value\ x \le T2)$ THEN the pixel belongs to the background **(Rule 1)**
OTHERWISE the pixel belongs to the object.

T1 and T2 are the x values for which the distributions intersect each other, see the figure.

(a) → Calculate the threshold values T1 and T2 (= x values for intersections). (1p)
(b) → Calculate the missclassification error corresponding T1 and T2 above, when using Rule 1. (3p)
(c) → Keep the threshold T2 constant (as found in (a)) and express the **sensitivity** and **specificity** as a function of threshold T $(0 \le T \le T2)$, when using the Rule 2 below:

IF $(T \le\ pixel\ value\ x \le T2)$ THEN the pixel belongs to the background
OTHERWISE the pixel belongs to the object.

(Rule2)

(4p)

(d) → For which values of T (using Rule 2) will all the object pixels be correctly classified ? (1p)

(e) → For which values of T (using Rule 2) will all the background pixels be correctly classified ? (1p)

(Totally 10p)