Problem 1

1.1 Consider the image grayscale, r, to be a continuous random variable with the range [0,1] and the probability density function $p(r)$, as in the Figure 1 below:

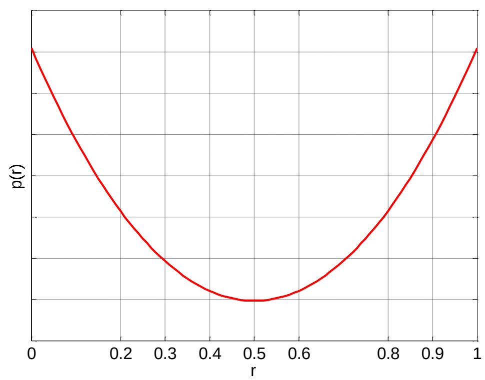


Fig.1. Probability density function $p(r)$ of a grayscale r, for some image.

$$p(r) = a \cdot \left[e^{-(r-0.5)} + e^{(r-0.5)}\right]$$

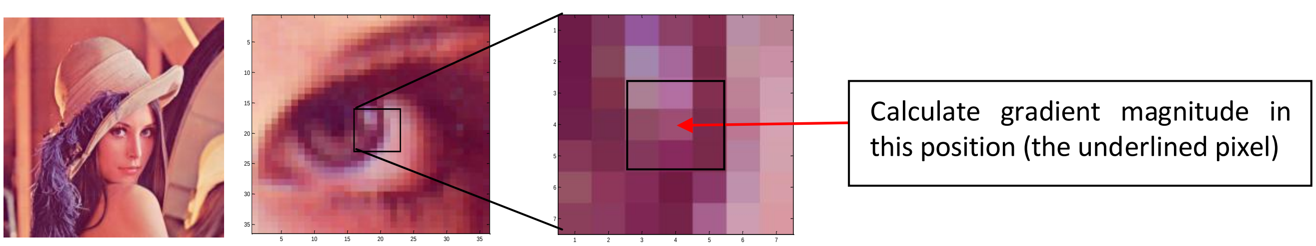
The image is subject to a grayscale transformation $s = T(r)$ so that the probability density function of the output image, $p(s)$, becomes a constant.

→ Find the transformation $T(r)$, for $r \in [0,1]$, and **plot** it. (3p)

1.2 Consider the color image below consisting of the three bands: RED, GREEN and BLUE and a small part of this image shown in the figure below (rightmost). Describe (the steps) how to calculate the gradient magnitude and gradient orientation using **vector field** approach.

Note: the exact numerical solution is not necessary, however you should demonstrate how the input data (R, G and B pixel values) are used in the algorithm.

(3p)



$$\text{RED} = \begin{matrix} 17 & 18 & 13 \\ 14 & \underline{16} & 12 \\ 13 & 14 & 12 \end{matrix} \qquad \text{GREEN} = \begin{matrix} 13 & 11 & 5 \\ 8 & \underline{8} & 4 \\ 6 & 4 & 4 \end{matrix} \qquad \text{BLUE} = \begin{matrix} 15 & 16 & 8 \\ 10 & \underline{12} & 9 \\ 9 & 9 & 9 \end{matrix}$$

1.3 (a) Sketch the result of operation erosion applied to the binary image X (Fig.1.3 left) using the structuring element B (Fig.1.3 right). Only the result is required.

(2p)

(b) Show that the point (3, 3) is a part of the result, using the set definition of erosion operation as in Sonka, def. (13.15), page 662 (see Appendix – **page 2** in this exam)

(1p)

(c) Show that the point (4, 2) is **NOT** a part of the result, using the set definition of erosion operation as in Sonka, def. (13.15), page 662 (see Appendix – **page 2** in this exam)

(1p)

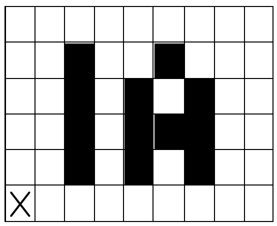


Fig. 1.3(left): The input image X (points belonging to the object are denoted by black squares).

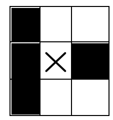


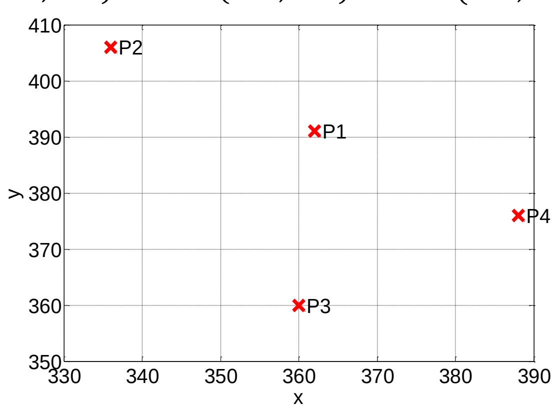
Fig.1.3(right): The structuring element $B = \{(-1,-1),(-1,0),(-1,1),(1,0)\}$. The representative point denoted by

$$X = \{(2,1),(2,2),(2,3),(2,4),(4,1),(4,2),(4,3),(5,2),(5,4),(6,1),(6,2),(6,3)\}$$

Problem 2

2.1a After edge detection on an image we have found a number of pixels (P1 ... P4) potentially belonging to the straight egde(s). The pixel coordinates are (see Figure below):

$P1 = (362, 391)$ $P2 = (336, 406)$ $P3 = (360, 360)$ $P4 = (388, 376)$



Your task is to find the strongest line in the image using the Hough Transform and (s, θ) line representation: $s = x\cos(\theta) + y\sin(\theta)$. Use the following quantized values for s and θ:

s : integer values

θ: [0 60 120] degrees

You should show the voting matrix and the final detected line equation should be presented as $y = kx + q$, (i.e. you should specify k and q values)

(5p)

2.1b Mention some disadvantages with Hough transform, when number of parameters is high.

(2p)

2.2 Explain the terms **voxel**, **snaxel**, and **texel**. (3p)

Problem 3

Below you find one input image X of size 512x512 (with grayscale values x between 0..1) and five output images. There is also a list of ten image analysis procedures. The task is to link each output image to a specific procedure (1-a, 2-b, etc.). Each correct link will give one point.

(1p)

For each link, an additional point will be given, if you can clearly justify your choice.

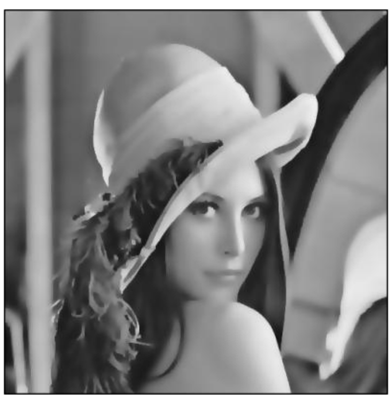
(1p)

(Totally 10p)

(input image)

(1)

(2)



(3)

(4)

(5)

(a) negative transformation
(b) $x^{5/2}$
(c) mean-shift filtering
(d) Gaussian filter of size 33x33 pixels with $\sigma = 1$ pixel
(e) Gaussian filter of size 33x33 pixels with $\sigma = 8$ pixels
(f) $x^{2/15}$
(g) $x^{2/5}$
(h) nonlinear diffusion filtering with diffusivity $c = 1/(1 + (|\nabla X|/k)^2),\ k = 10$
(i) $2 \cdot |x - 0.5|$
(j) $1 - x$

Problem 4

The four images below represent multispectral images from LANDSAT5 satellite.

(a) Calculate (show the first iteration) the convergence point for the pixel with value (9, 2, 1, 8), using mean-shift filtering with Epanechnikov kernel with bandwidth **h=9**, and 4-D datapoints from 3x3 area as in Fig.4.1b. Assume filtering in the range domain only.

The kernel profile $k_E(x)$ for the Epanechnikov kernel is given by:

$$k_E(x) = \begin{cases} 1 - x & for \quad 0 \leq x \leq 1 \\ 0 & for \; x > 1 \end{cases}$$

(8p)

(b) How many regions (clusters) will be generated when using mean-shift segmentation with very large bandwidth (e.g. $h = 100$) ?

(1p)

(c) How many regions (clusters) will be generated when bandwidth $h = 1$? Motivate your answer.

(1p)

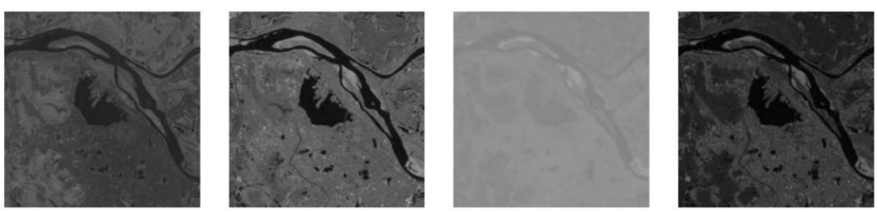


Figure 4.1a: LANDSAT5 images of Red River in China: bands 4, 5, 6 and 7.

3	3	3
9	9	3
9	9	3

Band 4

2	2	2
8	2	2
8	8	2

Band 5

5	5	5
1	1	5
1	1	5

Band 6

3	3	3
8	8	3
8	8	3

Band 7

Figure 4.1b: Intensity values in some 3x3 area, four bands.

Problem 5

5.1 A 6 x 3 image is shown below. As can be seen, the image consists of "concentric" layers. For each position (x, y) in the image there is an associated cost c, e.g. c(2, 1) = 7. The problem is to find all the paths from the **inner** layer to the **outer** layer (two travel steps are required for each path) so that the cumulative cost is minimized. Each layer may only be visited once. The allowed travel directions are **north, west, and north-west**. There is a penalty term **p = 1** associated with the travel directions north and west. For the travel direction north-west **p = 0**. The result should present the optimal path(s) (may be more than just one path), e.g. by specifying the three coordinate pairs of this path. It should also present the values of the cumulated travel costs in the form of a 6 x 3 cumulative matrix with clearly indicated back tracing pointers.

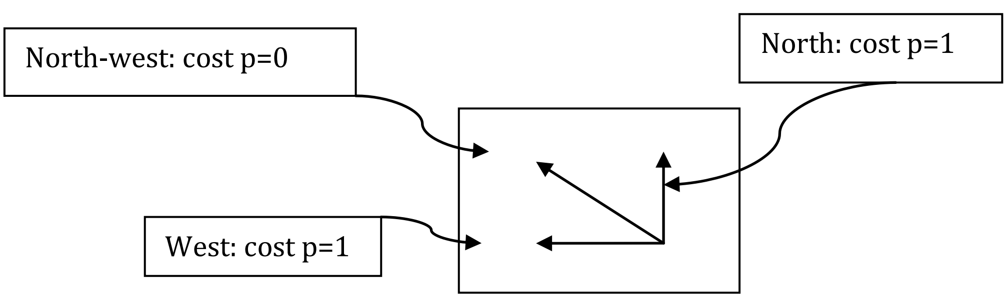
(5p)

	1	**2**	**3**
1	7	7	8
2	5	5	3
3	6	6	4
4	6	6	7
5	4	*3*	*6*
6	3	5	7

← x coordinate

y coordinate

Inner layer consists of pixels: (1,6), (2,6), (3,6), (3,5) ,(3,4),(3,3) ← (x,y)
Middle layer: (1,5), (2,5), (2,4), (2,3), (2,2) and (3,2)
Outer layer: (1,4), (1,3), (1,2), (1,1), (2,1) and (3,1)



5.2 Consider the image f(x,y) below and the problem of finding the optimal path (minimum cost path) from left to right. Assume that brightness cost $C_1(p_i) = \mathrm{f}(\mathrm{x},\mathrm{y})$ and smoothness cost $C_2(p_{i-1}, p_i) = 0$, for all pixels. How many minimum paths do exist for this problem?

f(x,y):

1	2	3	1	5	6	7	8	9	9	11
1	1	3	4	1	6	7	8	9	10	11
1	2	3	4	1	6	7	8	9	10	11

first layer last layer

(2p)

5.3 Describe what is meant by "**affine moment invariants**" and sketch some figures/objects that can be detected using such invariants.

(3p)

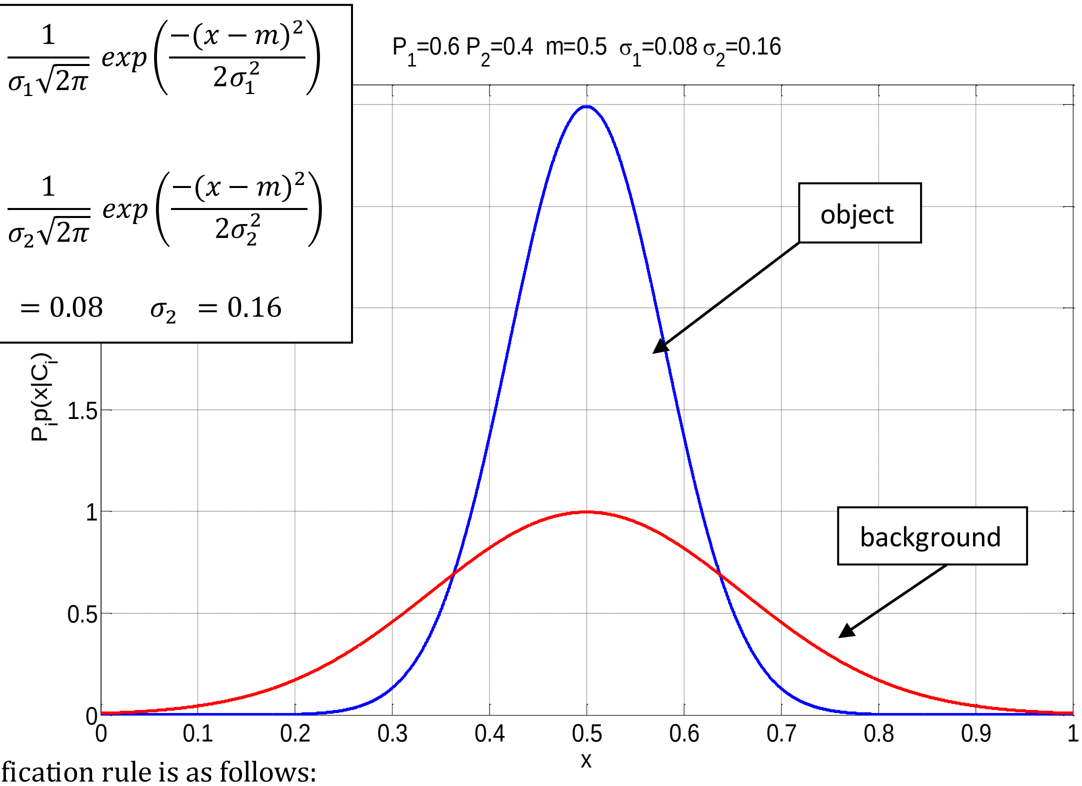
Problem 6

A microscope image that presents cells against background should be segmented by thresholding. The thresholding should be carried out so that the total number of misclassified pixels is minimized. The diagram below shows the estimated histograms for the object (cell) and background pixels. The background histogram (see the Figure below, Gaussian (m, σ_2)) is proportional to $P_2 \cdot p(x|C_2)$, while the object histogram (Gaussian (m, σ_1)) is proportional to $P_1 \cdot p(x|C_1)$, where C_1= object, C_2=background, P_1, P_2 - *a priori* probabilities of object and background, respectively, $p(x|C_i), i = 1,2$ are class-conditional probability density functions, x is a grayscale value. $P_1 = 0.6$, $P_2 = 0.4$

$$P_1 p(x|C_1) = P_1 \cdot \frac{1}{\sigma_1\sqrt{2\pi}} \, exp\left(\frac{-(x-m)^2}{2\sigma_1^2}\right)$$

$$P_2 p(x|C_2) = P_2 \cdot \frac{1}{\sigma_2\sqrt{2\pi}} \, exp\left(\frac{-(x-m)^2}{2\sigma_2^2}\right)$$

$$m = 0.5 \qquad \sigma_1 = 0.08 \qquad \sigma_2 = 0.16$$



The optimal classification rule is as follows:

if $(T1 < x < T2)$ **then**

pixel belongs to the object

else

pixel belongs to the background

$(a) \rightarrow$ Calculate the optimal threshold values T1 and T2 (= x values for intersections). (4p)

$(b) \rightarrow$ Calculate the missclassification error corresponding T1 and T2 above (a). (2p)

$(c) \rightarrow$ Calculate sensitivity and specificity for the optimal T1 and T2. (2p)

$(d) \rightarrow$ For which values of T1, T2 will all the object pixels be correctly classified ? (1p)

$(e) \rightarrow$ For which values of T1, T2 will all the background pixels be correctly classified ? (1p)

Note: In (b) and (c) it is more important to specify the correct integrals. The numerical values are less important.

(Totally 10p)