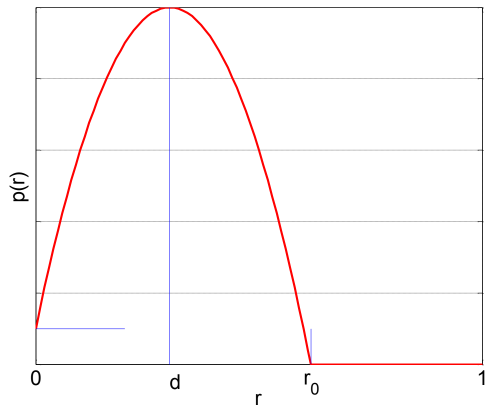
Problem 1

1.1 Consider the image grayscale, r, to be a continuous random variable taking values in the interval [0,1], defined by the probability density function $p(r)$, shown in Figure 1 below:



$$p(r) = \begin{cases} -a \cdot (r-d)^2 + c, & 0 \le r \le r_0 \\ 0, & r_0 \le r \le 1 \end{cases}$$

Fig.1. Probability density function $p(r)$ of grayscale r, for some image.

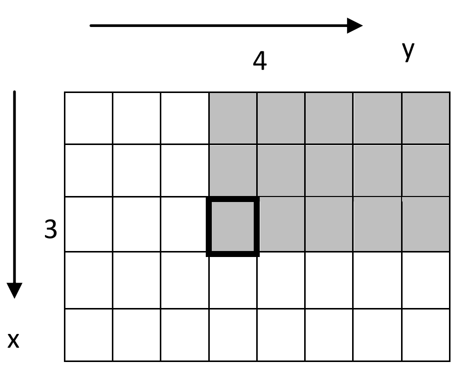
The image is subject to a grayscale transformation $s = T(r)$ so that the probability density function of the output image, $p(s)$, becomes a constant.

→ Find the **transformation $T(r)$**, for $r \in [0,1]$, and **plot** it. (4p)

1.2 Consider the image of size 5x8 in the figure below. The pixels in gray have value 0 and the pixels in white have value 1.

→ Calculate the gradient magnitude and the gradient direction in the pixel with coordinates (x, y) = (3, 4). Use the Prewitt gradient operator defined with 3x3 kernels. (3p)

→ Draw (illustrate) on the image the gradient vector and the corresponding edge direction. (1p)



<=>

	1	1	0	0	0	0	0
1	1	1	0	0	0	0	0
1	1	1	0	0	0	0	0
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1

1.3 Apply the binary mathematical morphology operation **dilate** to the image below left using the structuring element below right. (**Note:** only the result is required).

(2p)

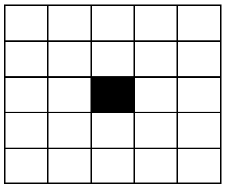


Fig.1.3 (left): The input image (points belonging to the object are denoted by black squares).

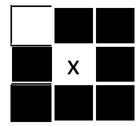


Fig.1.3 (right): The structuring element. The representative point (origin) is denoted by x .

Problem 2

2.1 Optical Flow – demonstrate the optical flow calculation using the Lucas-Kanade method in the pixel position (x, y) = (3, 3), marked with the **bold** border in the image frames below. Use a 3x3 pixel neighborhood.

Frame #1

1	2	3	4	5
1	2	3	4	5
1	2	**3**	4	5
1	2	3	4	5
1	2	3	4	5

Frame #2

0	1	2	3	4
0	1	2	3	4
0	1	**2**	3	4
0	1	2	3	4
0	1	2	3	4

Note: The exact numerical solution is not necessary, but you should set up the proper equation(s) with correct input numerical data and describe how to calculate the final result.

(5p)

2.2 Segment the small gray-scale image below of size 3x3 pixels, into **two regions** using a segmentation method of your choice (except simple thresholding). Clearly specify all of the steps and demonstrate numerical calculations for at least one step of your segmentation method.

0	0	5
1	1	5
0	7	6

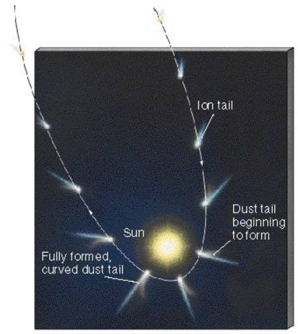
(5p)

Problem 3

We are interested in studying the comet trajectories in digitized images. In particular we are interested in those that follow **hyperbolic** trajectories.

A **hyperbola** with tranverse axis aligned with the *x*-axis of a Cartesian coordinate system and centered on the origin, can be written as:

$$-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \text{(Eq. A)}$$



(a) → Write a **Hough Transform based algorithm** (using pseudo-code or Matlab notation), that investigates if the detected points/pixels (in Fig 3-right) belong to a hyperbola defined by equation (A).

(7p)

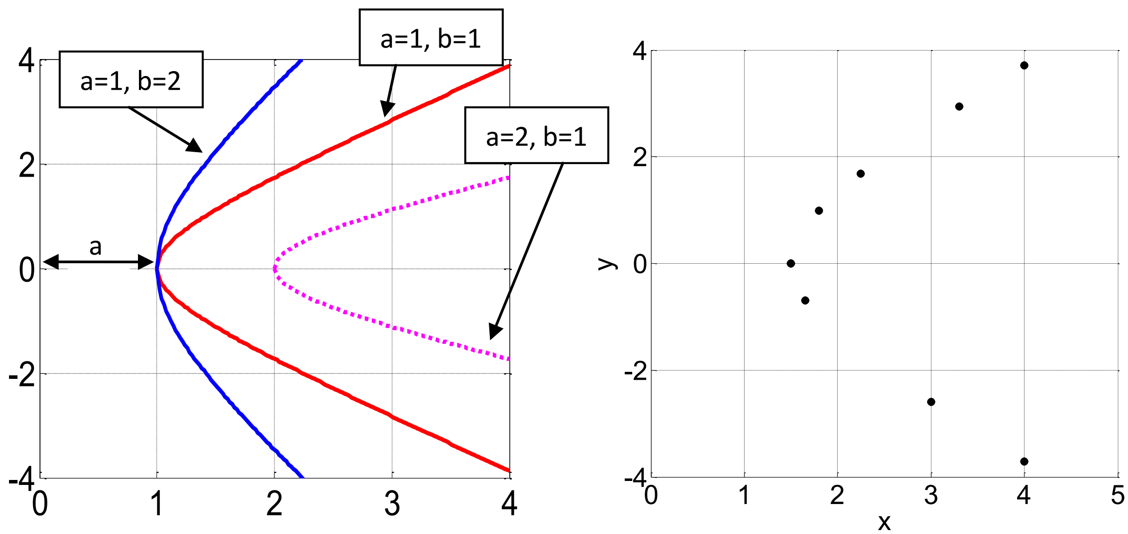


Fig 3 (left) Example of some hyperbolas. (right) The data points to be investigated. What is the equation of the most likely hyperbola that the points belong to?

(b)→ Demonstrate numerically how some data point, e.g. (x, y)=(3, -2.6), contributes to your Hough Transform voting matrix.

(3p)

Problem 4

One input image X of size 512x512 (with grayscale values x in the interval [0, 1]) and five output images are shown below. A list of ten image analysis procedures is also shown below. The task is to link each output image to a specific procedure (1-a, 2-b, etc.). Each correct link will give one point.

(1p)

For each link, an additional point will be given, if you can clearly justify your choice.

(1p)

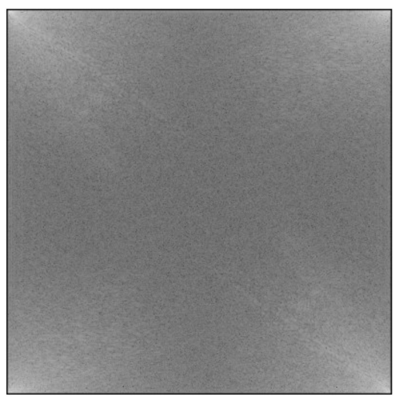
(Totally 10p)

(input image)

(1)

(2)

(3)



(4)

(5)

(a) maximum filter with 7 x 7 mask
(b) reconstruction from Fourier magnitude
(c) Fourier magnitude spectrum (non-centered)
(d) median filter 7 x 7
(e) Fourier magnitude spectrum (centered)
(f) $x^{\ln(e)}$
(g) x^{e}
(h) $|x - 0.5|$
(i) $1 - 2 \cdot |x - 0.5|$
(j) $x^{1/2}$

Problem 5

5.1

(a) Find the mimimum cost path(s) for the cost matrix C_1 shown in Figure 5.1, using the **dynamic programming** approach. The starting layer is the bottom row and the last layer is the upper row. Assume that the smoothness cost is $C_2(p_{i-1}, p_i) = |\Delta x|^2$, where Δx is a change in the x-coordinate. The cost at pixel p_i is $C(p_i) = C_1(p_i) + C_2(p_{i-1}, p_i)$, where p_i= pixel from layer i.

The solution should consist of the accumulation matrix and the minimum cost path pointers.

Last layer (i=3)

0	7	6	1	
3	1	0	4	0
1	2	4	1	

First layer (i=1)

x

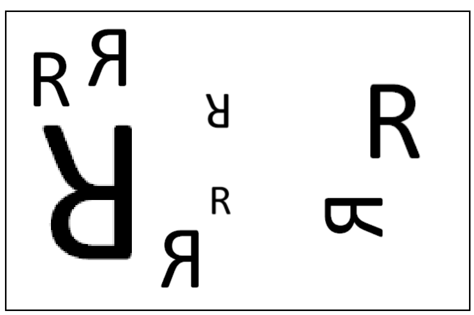
Figure 5.1 The cost matrix.

(4p)

(b) → How many paths do we need to investigate in the case of **exhaustive search**?

(1p)

5.2 The image below shows R-letter shapes and mirror image R-letter shapes.



(a) → Describe how you would use the **moment invariants** to discriminate between these objects, i.e. between R and its mirror image Я , regardless of size, position and orientation.

(2p)

(b) → Describe how you would use the **Fourier descriptors** (FDs) to discriminate between these objects.

(3p)

Note: No numerical calculations are necessary. You should specify what moments/Fourier descriptors can be used, and specify the mathematical expressions of your suggested moments/FDs, if any.

Problem 6 An image that presents objects against background should be segmented by thresholding. The thresholding should be carried out so that the total number of misclassified pixels is minimized. The figures below show the image (left) and the corresponding histogram (right). Here, the objects of interest are dark and light spots with circular shape.

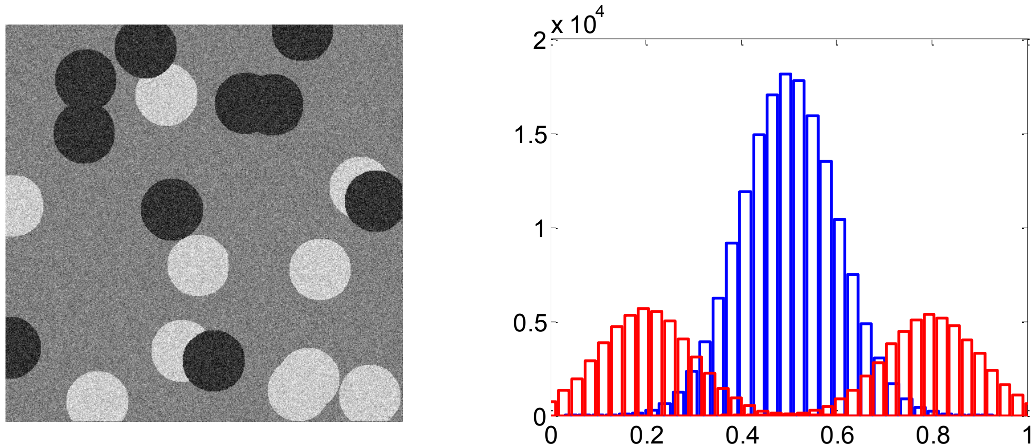
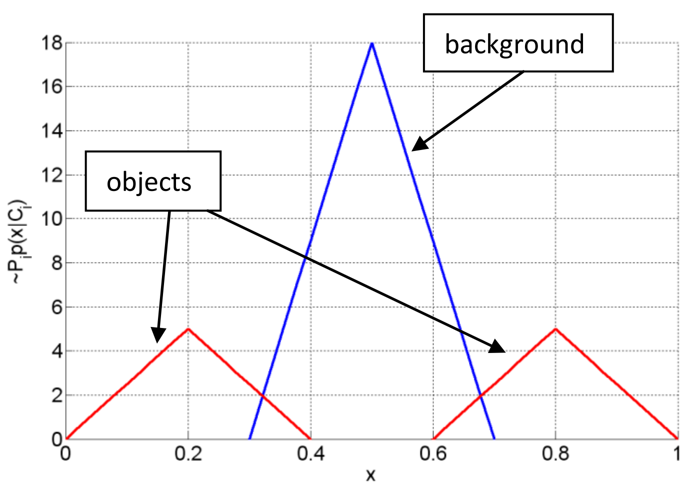


Fig 6. (left) An image with light and dark objects on gray background.
(right) The corresponding histograms of the image, red color = objects' bars, blue = background's bars, x-axis = pixel gray values, y-axis – frequency.

We approximate the histograms in Fig. 6 (right) with triangles as in Fig 7 below. The background histogram is proportional to $P_2 \cdot p(x|C_2)$, while the objects' histograms are proportional to $P_1 \cdot p(x|C_1)$ and $P_3 \cdot p(x|C_3)$, where C_1= light object, C_2=background, C_3= dark object, P_1, P_2, P_3 - *a priori* probabilities of objects and background, respectively, $p(x|C_i), i = 1, 2, 3$ are class-conditional probability density functions, and x is a grayscale value.



$$P_1 p(x|C_1) \sim \quad 5 - 25 \cdot |x - 0.2|$$

$$P_2 p(x|C_2) \sim \quad 18 - 90 \cdot |x - 0.5|$$

$$P_3 p(x|C_3) \sim \quad 5 - 25 \cdot |x - 0.8|$$

Fig. 7: The weighted class-conditional histograms from Fig. 6, approximated with triangular distributions.

The task is to discriminate the objects (light or dark) from the background (two-class problem).

(a) → We use the following classification **rule,** using two threshold values as parameters:

if $(T_1 <$ pixel value $x < T_2)$ **then**
pixel belongs to the background (Rule 1)
else
pixel belongs to the object

(b) → Calculate the optimal threshold values T_1 and T_2, and the corresponding **value of minimum error**, using the rule from (a).

(3p)

(c) → Calculate the **sensitivity** and **specificity** at the point of minimum error.

(2p)

(d) → Assume that we use Rule 1 and start with $T_1 = 0$ and $T_2 = 1$, and linearly increase T_1 and decrease T_2 until $T_1 = T_2 = 0.5$. Express the **sensitivity** and **specificity** for such a decision rule as a function of thresholds T_1 and T_2.

(4p)

(e) → Plot the **ROC-curve** (ROC = Receiver Operating Characteristic) using values from (d).

(1p)

(Totally 10p)