Problem 2 (totally 5 points)

Problem 2.1 Consider the image grayscale, r, to be a continuous random variable taking values in the interval [0,1], defined by the probability density function $p(r)$, shown in Figure 1 below:

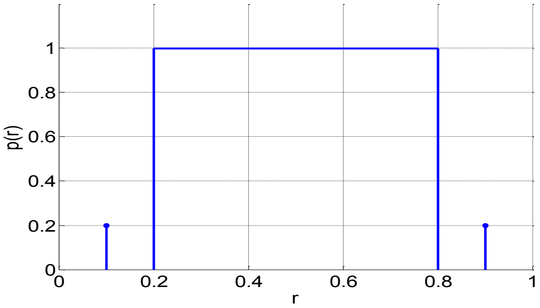


Fig.2a. Probability density function $p(r)$ of grayscale r, for some image.

$$p(r) = \begin{cases} 0.2 & r = 0.1 \\ 1 & r \in \ [0.2, 0.8] \\ 0.2 & r = 0.9 \end{cases}$$

The image is subject to a grayscale transformation $s = T(r)$ so that the probability density function of the output image, $p(s)$, becomes a constant (histogram equalization).

→ Find the **transformation $\boldsymbol{T(r)}$,** for $r \in [0,1]$, and **plot** it.

(2p)

→ For which value(s) of r, $T(r) = 0.4$?

(1p)

Problem 2.2 The color image below, represented by 2 bits per color band, consists of three bands: red, green and blue. Specify the 3D-histogram of this image, i.e. value for every histogram entry. Note: the histogram should be **three**-dimensional.

RED

0	2	3
1	1	3
0	3	3

GREEN

0	2	2
1	1	3
0	3	3

BLUE

0	2	2
1	1	3
0	3	3

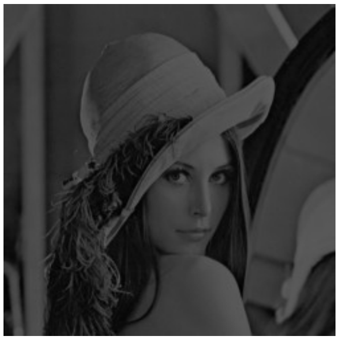
(2p)

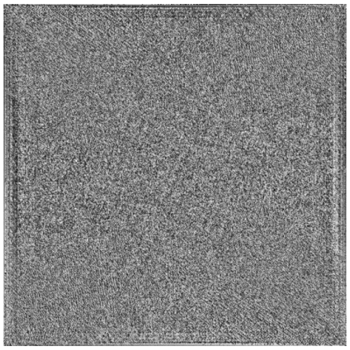
Problem 3 (totally 10 points)

One input image X of size 512x512 (with grayscale values x in the interval [0, 1]) and five output images are shown below. A list of ten image analysis procedures is also shown below. The task is to link each output image to a specific procedure (1-a, 2-b, etc.). Each correct link will give one point.
(1p)

For each link, an additional point will be given, if you can clearly justify your choice.
(1p)


(input image)


(1)


(2)


(3)


(4)

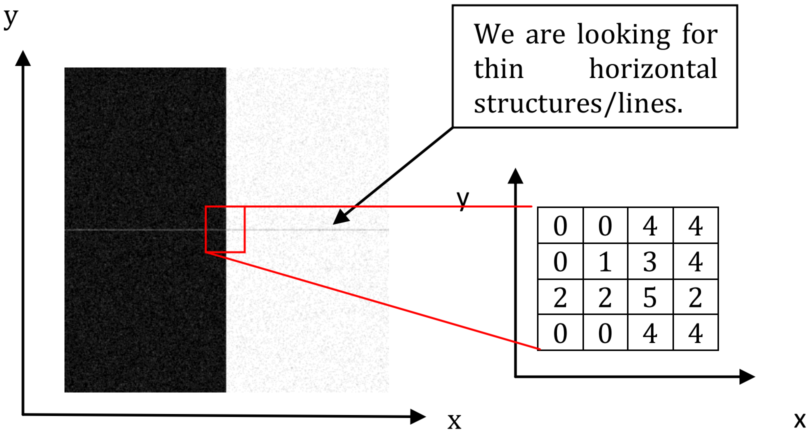

(5)

- (a) $x^{4/3}$
- (b) Fourier phase, shifted
- (c) Fourier phase, not shifted
- (d) Gaussian filter with σ=49 pixels and kernel size 49 x 49
- (e) $|x - 0.5|$
- (f) reconstruction from Fourier phase
- (g) $log_2(2^{x})$
- (h) $2 \cdot |x - 0.5|$
- (i) $\tanh(x)/2$
- (j) median filter with kernel size 49 x 49

$$tanh(x) = (exp(x) - exp(-x))/(exp(x) + exp(-x))$$

Problem 4 (totally 10 points)

Problem 4.1 Assume that we are looking for thin long structures in images that can be on bright or dark background. We know that the background is changing from dark to bright exactly in the middle of the image as shown in the figure below left. The figure below right shows a small part of the figure on the left.



We are looking for thin horizontal structures/lines.

0	0	4	4
0	1	3	4
2	2	5	2
0	0	4	4

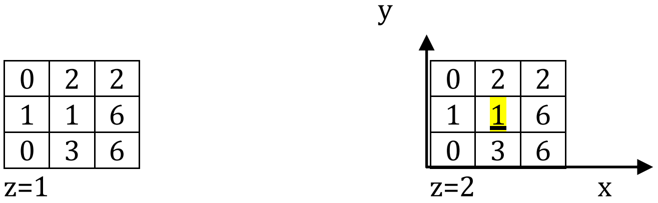
Figure 4.

The image pixel intensity values (dark background: columns 1 & 2, bright background: columns 3 & 4) are shown in Figure 4 (right).

Find the position of the long horizontal structure using a **dynamic programming** approach. Assume that the smoothness cost is $C_2(p_{i-1}, p_i) = |\Delta y|$, where Δy is a change in the y-coordinate. The cost at pixel p_i is $C(p_i) = C_1(p_i) + C_2(p_{i-1}, p_i)$, where p_i= pixel from layer i. (You should choose a suitable $C(p_i)$ cost yourself).
The solution should consist of the **accumulation matrix**, the **minimum cost path pointers**, and an example showing how you calculate the accumulation matrix (= cumulative cost matrix).

(5p)

Problem 4.2 The small gray-scale three dimensional (in spatial domain) 1-band image (volume) below, of size 3x3x3 pixels, has to be segmented using the mean-shift algorithm. Calculate the convergence point in the range domain of the underlined/yellow marked pixel located in the center of the volume (coordinates x=2, y=2, z=2). Use two iterations with **Epanechnikov** kernel with bandwidth h_r = 3. (Note: the complete segmentation is **not required**).



0	2	2
1	1	6
0	3	6

z=1

0	2	2
1	1	6
0	3	6

z=2

0	2	2
1	1	6
0	3	6

z=3

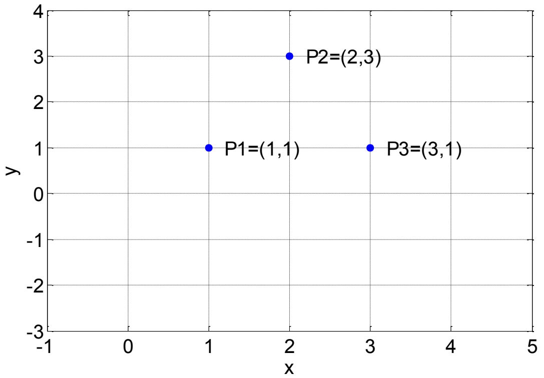
Use the following formula (Sonka, page 260) for calculating the convergence point and successive locations of the kernel:

$$\boldsymbol{y}_{j+1} = \sum_{i=1}^{n} \boldsymbol{x}_i g\left(\left\|\frac{\boldsymbol{y}_j - \boldsymbol{x}_i}{h_r}\right\|^2\right) \Big/ \sum_{i=1}^{n} g\left(\left\|\frac{\boldsymbol{y}_j - \boldsymbol{x}_i}{h_r}\right\|^2\right)$$

(5p)

Problem 5 (totally 10 points)

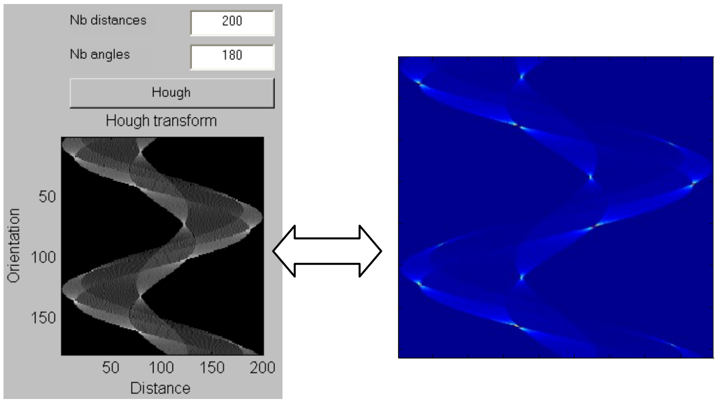
Problem 5.1 We are looking for straight lines in an image and we know that the lines may intersect the y-axis at $y = \{-1, 1, 2, 3\}$ only. Find these lines using the Hough Transform and the $y = k \cdot x + q$ representation, i.e. use Hough Transform in (k, q) parameter space. The edgepoint coordinates are: $P1 = (1, 1)$, $P2 = (2, 3)$, $P3 = (3, 1)$ (5p)



Note: Numerical calculations are needed, not just sketching. You should demonstrate the voting procedure, the voting matrix (Hough Transform) and the equation(s) of the detected line(s) i.e. (k, q) values.

Problem 5.2 An image analysis student implemented the Hough Transform for line detection using the (s, θ) representation and obtained the Hough Transform shown in the Figure below, with twelve (12) high intensity maximum points ($nT = 180, nS = 200$, $\theta \in (-180°, 360°]$). How many lines have been detected? Motivate your answer. (nT is the orientation resolution, nS is the distance resolution).

(2p)

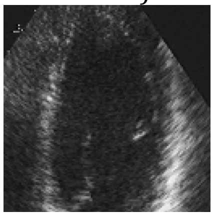


Nb distances 200

Nb angles 180

Problem 5.3 In extracting textural features (Lab3) we extracted features in different directions (like 0, 45, 90, 135 degrees) and then we calculated a new feature as the average of all directional features. What is the advantage of the "average" feature compared to the directional features? (1p)

Problem 5.4 Which of the images below (a, b, c) has the **lowest** and **highest** textural entropy (based on the Gray Level Cooccurrence Matrix, $d = 1, \varphi = 0°$) ? Motivate your answer (calculations not needed). (2p)



(a)

(b)

(c)

Problem 6 (totally 10 points)

Problem 6.1 An image that presents objects against background should be segmented by thresholding. The thresholding should be carried out so that the total number of misclassified pixels is minimized. We approximate the histograms of the objects and background with quadratic and triangular functions as in the Figure below. The background histogram is proportional to $P_2 \cdot p(x|C_2)$, while the objects' histograms are proportional to $P_1 \cdot p(x|C_1)$, P_1, P_2 - *a priori* probabilities of objects and background, respectively, $p(x|C_i), i = 1, 2$ are class-conditional probability density functions, and x is a grayscale value.

$P_2 p(x|C_2) \sim \quad 4 - 20\,|x - 0.4| \qquad for\ x\ in\ [0.2, 0.6]$
Otherwise 0

$P_1 p(x|C_1) \sim \quad -50 \cdot x \cdot (x - 0.3) \qquad for\ x\ in\ [0, 0.3]$
Otherwise 0

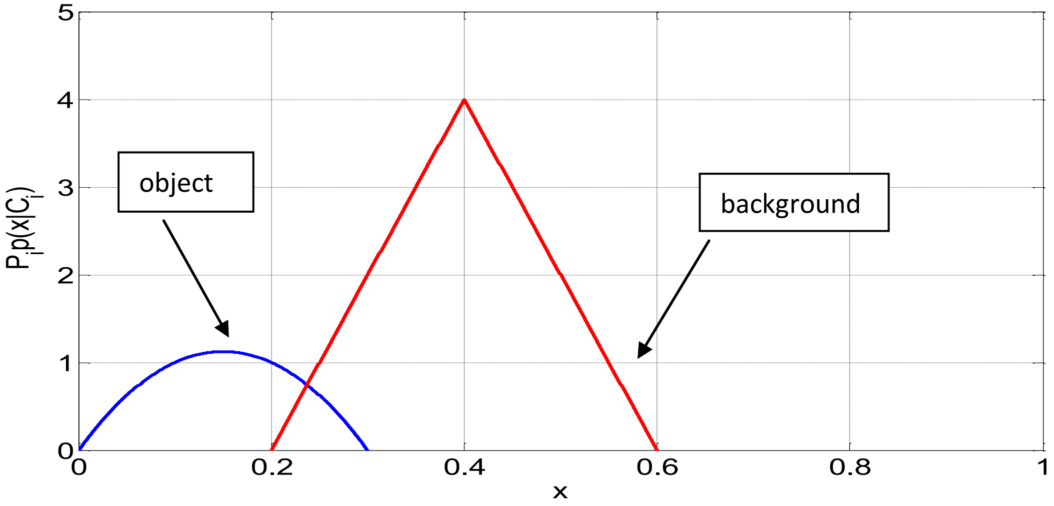


Fig. 6: Class-conditional histograms of the object and background approximated with quadratic and triangular distributions, respectively.

The task is to discriminate the dark objects from the light background (two-class problem):

(a) Calculate the optimal threshold value T, and the corresponding **value of minimum error**. (2p)
(b) Calculate the **sensitivity** and **specificity** at the point of minimum error. (1p)
(c) Calculate the sensitivity and specificity as a function of threshold T. (2p)
(d) Plot the **ROC-curve** (ROC = Receiver Operating Characteristic) using values from (c) (1p)
(e) For which values of T, 50% of the object pixels will be correctly classified? (1p)
(f) What is the value of specificity when sensitivity is equal 1? (1p)

Problem 6.2 Mention at least six (two per guest lecture) different image analysis techniques or applications presented on the guest lectures this year.

(2p)