Exercises

SSY095
Image Analysis

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Problems

Gray Level Transformations

GLT-1

An three bit image with grayscale range [0,1] will be subject to histogram equalization. The eight discrete levels, the number of pixels of each level, and their relative occurrence is presented in the table below. Compute the new gray levels following equalization.

r_k	n_k	$p_r(r_k) = n_k/n$
$r_0 = 0$	790	0.19
$r_1 = 1/7$	1023	0.25
$r_2 = 2/7$	850	0.21
$r_3 = 3/7$	656	0.16
$r_4 = 4/7$	329	0.08
$r_5 = 5/7$	245	0.06
$r_6 = 6/7$	122	0.03
$r_7 = 1$	81	0.02

GLT-2

An eight bit image with grayscale range [0,255] and sized 512x512 should be histogram equalized. The number of input pixels, r_i, having grayscale values of 0, 1, 2 and 3 are given below

r_0: 962
r_1: 874
r_2: 603
r_3: 424

a) Determine the output pixel value s_2.

b) Given that the output pixel value s_4 equals 5, what is then the minimal and maximal number of input pixels, respectively, having a grayscale value of 4?

c) Why should any grayscale transformation $T(r) = s$ be a monotonically increasing function?

d) What happens to the output image contrast in the low grayscale range, say 0-50, if the transformation in this region is given by $T(r) = 2r$ for $0<r<50$.

e) What type of operation will be the result if $T(r) = 0$ for $0<r<50$ and $T(r) = 255$ otherwise?

f) Say that you iteratively and for a large number of iterations apply an averaging filter to an arbitrary image. How, in principle, will the histogram change as you successively apply more averaging operations?

GLT_3

An image sized 110x110 has 11 grayscales in the range [0,1]. The image should be histogram equalized. The number of pixels, r_i, with grayscale value 0, 1/10, 2/10, och 3/10 is given below:

r_i	n_i
$r_0=0$	962
$r_1=1/10$	874
$r_2=2/10$	603
$r_3=3/10$	424

a) Determine the output grayscale value s_2 (round to nearest tenth).

b) Say that the output grayscale value s_4 equals 5/10 (it may have been rounded to nearest tenth). What is the minimal and maximal number of input pixels having a value of 4/10?

c) Why should all transformations T(r) = s be a monotonically increasing function?

d) What happens to the output image contrast in the low grayscale range, say 0.0-0.5, if the transformation in this range is given by $T(r) = 2r$ for $0<r<0.5$.

e) What type of operation will be the result if $T(r) = 0$ for $0<r<0.5$ and $T(r) = 1$ for $0.5<r<1.0$?

1f) How, in principle, will the histogram change if the image is subject to a logarithmic type of transformation.

GLT-4

a) Consider the image grayscale, r, to be a continuous random variable with the range [0,1] and the probability density function $p(r) = ar^2$. The image is subject to a grayscale transformation so that the probability density function of the output image, p(s), becomes a constant. Determine the value of the output grayscale variable, s, if the value of the input variable, r, equals 0.5.

b) Why should all transformations T(r) = s be a monotonically increasing function?

c) What happens to the output image contrast in the low grayscale range, say 0.0-0.5, if the transformation in this range is given by T(r) = 2r för $0<r<0.5$.

d) What type of operation will be the result if T(r) = 0 for $0<r<0.5$ and T(r) = 1 for $0.5<r<1.0$?

e) How, in principle, will the histogram change if the image is subject to a logarithmic type of transformation.

GLT-5

a) Consider the image grayscale, r, to be a continuous random variable with the range [0,1] and the probability density function p(r). The function p(r) increases and decreases linearly in the intervals [0,0.5] and [0.5,1], respectively, and p(0)=p(1)=0. Hence, p(r) has a triangular shape. The image is subject to a grayscale transformation so that the probability density function of the output image, p(s), becomes a constant. Determine the value of the output grayscale variable, s, if the value of the input variable, r, equals 0.6.

b) Regardless of the image characteristics, why is the transformation s = T(r) = 0.5 x (cos(r x 2 pi) + 1) not appropriate?

c) Suggest a suitable transformation T(r) so that the output becomes 1 for input values equal to and above 0.5, otherwise zero.

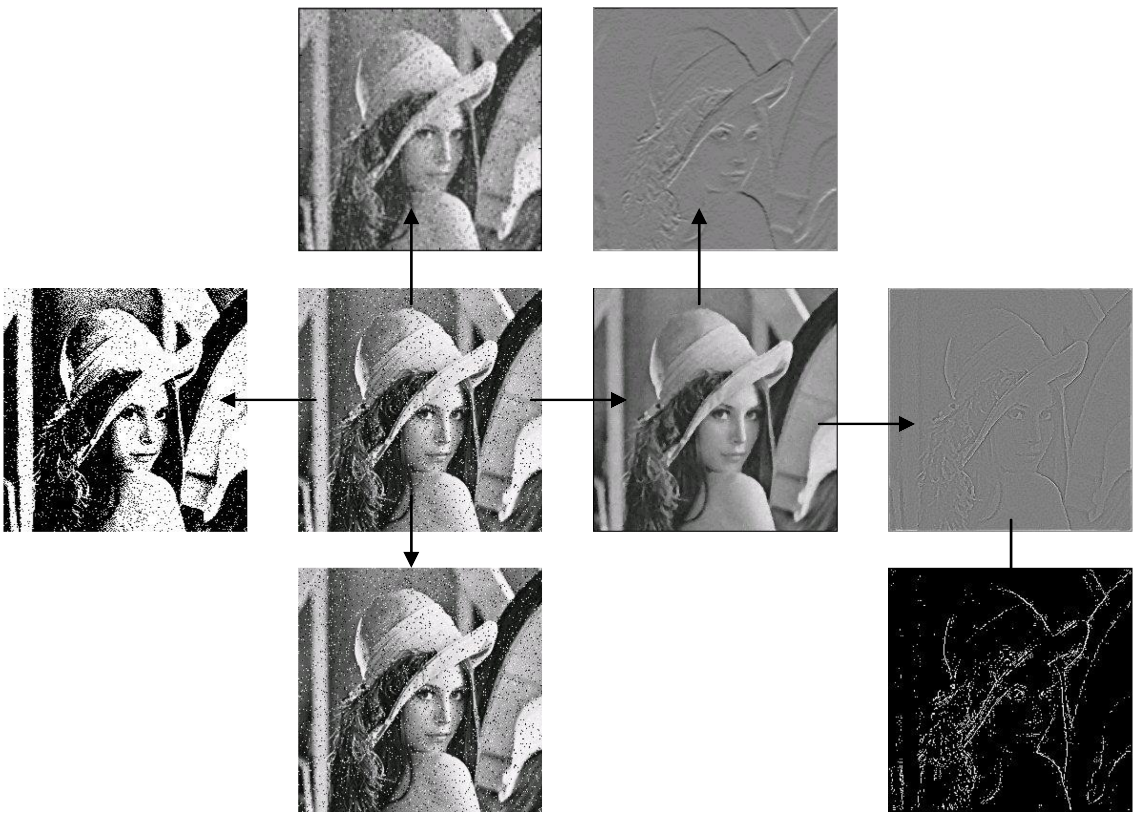
d) Consider an image being subject to repeated local averaging operations. Show in principle how the output histogram will change accordingly.

e) Why are, in general, grayscale transformations less powerful than neighborhood operations?

Image Filtering

IF-1

a) Several output images have been obtained from a noisy input image. Find out which filter corresponds to which image.



a) __

b) __ ← **original** → **d) __** → **f) __**

c) __

e) __

g) __

(original ↑ a), original ← b), original ↓ c), original → d), d) ↑ e), d) → f), d) ↓ g))

1) Binarization
2) Equalization
3) Laplace operator
4) Mean filter
5) Median filter
6) Sobel edge detector
7) Thresholding

IF-2

Consider the image pair below. Noise has been added to the original left image. What kind of noise is that?

* Gaussian
* Salt & Pepper
* Speckle

IF-3

a) Convolve the two filters f1 and f2 presented below so as to produce the new filter f3

f_1

$$\begin{matrix} 1 \\ \boxed{2} \\ 1 \end{matrix}$$

f_2

$$\begin{matrix} 1 & \boxed{2} & 1 \end{matrix}$$

b) Find the Fourier transform of f3 by carrying out the transform directly on the convolution kernel

c) Find the Fourier transform of f3 by first carrying out the transform of f1 and f2

d) What is the Fourier transform of the median filter? Motivate your answer.

IF-4

a) Derive and plot the Fourier transform of the mean filter defined by the operator:

	1/8	
1/8	**1/2**	**1/8**
	1/8	

b) Apply this filter to the input image below and compute the result (assume zeros outside the image, and round to nearest integer):

1	**1**	**1**
1	**8**	**1**
1	**1**	**1**

c) Apply a 3x3 median filter to the same image and compute the result

d) Describe (in words) in which situations the median filter may be superior to the mean filter

IF-5

a) The filters h1 and h2 are convolved so as to produce the filter h3.
Determine the coefficients a, b, and c:

h1

1	2	1
2	4	2
1	2	1

h2

c	b	c
b	a	b
c	b	c

h3

	1	2	1	
1	8	14	8	1
2	14	24	14	2
1	8	14	8	1
	1	2	1	

b) The Fourier transform of a spatial filter is (sampling distance=1):

$$a\cos(u) + b\cos(v) + c\cos(u+v) + d\cos(u-v) + e\cos(2u) + f\cos(2v) + g$$

where u and v are the spatial frequencies in the horizontal and vertical directions, respectively.
Determine the coefficients of the spatial filter and present it as an n x m operator in the way the operators are presented in a)

c) Apply a 3x3 median filter to the image below:

1	2	3
4	5	6
7	8	9

IF-6

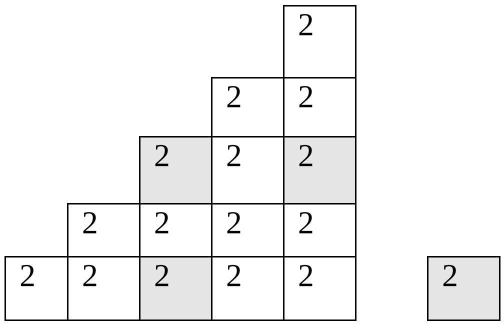
a) The two filters h1 and h2 should be convolved so as to produce the filter h3.
h1 is defined as:

1	2	1
2	4	2
1	2	1

h2 has the Fourier transform (sampling distance=1):
$4 + 2\cos(u) + 2\cos(v)$
where u and v are the spatial frequencies in the horizontal and vertical directions.
Determine the coefficients of filter h3 and present it as an operator similar to h1

b) Find the Fourier transform of h3

IF-7



a) Assume 0 outside the image, and sampling frequency be 1.

Convolve the image at the pixels filled with gray using the discrete approximation of the Laplacian:

$$\begin{matrix} & 1 & \\ 1 & -4 & 1 \\ & 1 & \end{matrix}$$

b) Comment and interpret your results

c) Determine the Fourier transform of the Laplacian above

d) Compute the magnitude of the transform you found at the frequencies:
$\pi, 0$ and π, π

e) Comment and interpret your results

IF-8

a) An image with additive gaussian noise will be thresholded after a gradient operation. Considering the noise, the gradient image will be created in 2 steps, using two of the following filters. Select two of the kernels for the operation, and motivate your choice:

$$h_1 = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad h_2 = \begin{bmatrix} & 1 & \\ 1 & -4 & 1 \\ & 1 & \end{bmatrix} \quad h_3 = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad h_4 = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad h_5 = \begin{bmatrix} & 1 & \\ 1 & 4 & 1 \\ & 1 & \end{bmatrix} \quad h_6 = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

b) Show how this 2-step filtering scheme can be replaced a single-step filtering scheme, by using these 2 kernels to create a new filter

c) Compute the coefficients of this new filter kernel.

d) What are the characteristics of this new filter in the frequency domain ?
(give the filter type or describe it with words)

e) Show that for a 512x512 pixels image, the number of mul Show that for a 512x512 pixels image, the number of multiplication operations is less if the filtering is computed in the image plane rather than in the frequency domain, given that the number of non-zero coefficients in the kernel is less than 40.

IF-9

a) Derive and plot the Fourier transform of the filter defined by the operator below:

$$\begin{array}{ccc} & 1/8 & \\ 1/8 & 1/2 & 1/8 \\ & 1/8 & \end{array}$$

b) Apply this filter to the input image below and compute the result (assume zeros outside the image, and round to nearest integer):

$$\begin{array}{ccc} 1 & 1 & 1 \\ 1 & 8 & 1 \\ 1 & 1 & 1 \end{array}$$

c) Apply a 3x3 median filter to the same image and compute the result

d) Comment upon the result of b) and c) and mention one advantage and one disadvantage of the median filter as compared to the mean filter.

IF-10

a) Show that applying the Laplace operator shown below to an image is equivalent (with except for a proportional factor) to locally subtracting a five point mean from the each original value of that image

	1	
1	–4	1
	1	

b) Show that local thresholding of an image can be replaced by global thresholding if the image first has been filtered by a Laplace operator.

c) In optical microscopy, let f_k be the 'true' image of a substance at depth k. Give a possible way to obtain f_k given that we have access to the digitized non-ideal unfocused images g_k, g_{k-1}, and g_{k+1}.

d) A 3x3 input image is given below:

1 1 1
2 <u>2</u> 2
2 2 2

Apply the edge detector filters:

1	1	1		-1	0	1
0	0	0		-1	0	1
-1	-1	-1		-1	0	1

at the underlined input pixel.

Compute the magnitude and the direction of the gradient?

e) What is the Fourier transform of the filter below?

1 0 1
0 1 0
1 0 1 (4p)

IF-11

a) Assume that we apply a linear filtering two times consecutively. Show that 2 linear filters with impulse response h_1 and h_2 applied consecutively can always be replaced by a filter h_3

b) Compute h_3 given that $h_1 = h_2$ and have the form given below:

$$h_1 = h_2 = \begin{bmatrix} & 1 & \\ 1 & 4 & 1 \\ & 1 & \end{bmatrix}$$

c) The output should in fact be in the form:

$$\begin{matrix} & 1/k & \\ 1/k & 4/k & 1/k \\ & 1/k & \end{matrix}$$

What is the meaning of k and what value should it have?

d) Assume that we would like to detect vertical edges in an image corrupted by noise. Give and motivate your choice for a 3x3 filter that would be applied before edge detection.

IF-12

a) Compute the Fourier transform H(u,v) of the filter h(x,y) below.

1 0 1
0 0 0
0 4 0
0 0 0
1 0 1

b) Apply the filter h to the underlined pixels in the two images below.

1 1 1 1 1
2 2 2 2 2
1 1 1 1 1
2 2 2 2 2
1 1 1 1 1
2 2 2 2 2

1 2 1 2 1 2
1 2 1 2 1 2
1 2 1 2 1 2
1 2 1 2 1 2
1 2 1 2 1 2

c) Compute H(pi,0) and H(0,pi).

d) Interpret the results in b) and c) by a discussion about the filter characteristics (e.g. is the filter low-pass, high-pass, or bandpass?). Your analysis should take both the spatial and frequency domains into account.

IF-13

a) The Fourier transform of a filter, h(x,y) is given by

F(u,v) = 1/2 + [cos(au + bv) + cos(cu + dv)] / 4

For u = pi and v = 0, F(u,v) = 1
For u = 0 and v = pi, F(u,v) = 0

The factors a,b,c, and d, are non-zero integers in the range [-3,3].

Determine h(x,y).

b) What type of filter is this (low-pass, band-pass, high-pass, etc)? Motivate your answer clearly.

c) Compute the 3D median given the image volume f(x,y,z) below.

3	1	4
5	2	1
2	3	4

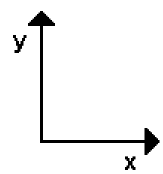
z=-1

2	5	1
4	3	2
3	2	5

z=0

4	3	1
2	5	4
2	3	1

z=1

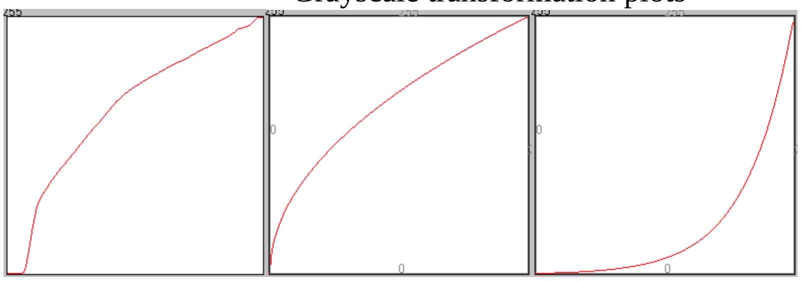


d) Mention a drawback with the median filter as compared to the mean.

IF 14

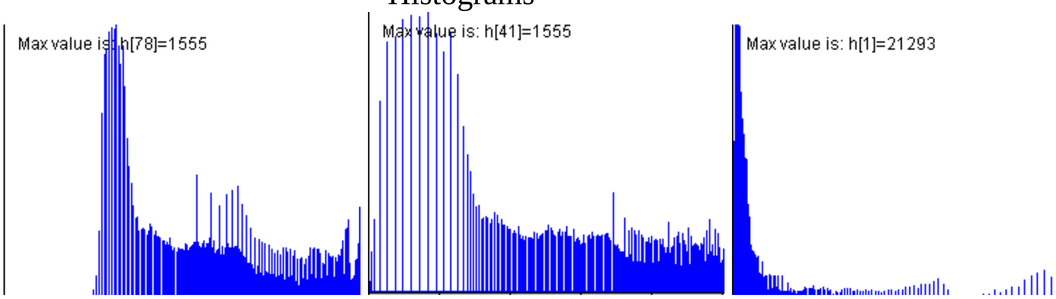
a) Combine the grayscale transformation plot, histogram, and transformation type (e.g. 1 – I – a). Each correct combination gives 1 point. If you can motivate your choice, you get an extra point for each correct motivation. If the motivation is clearly wrong, you get a minus point.

Grayscale transformation plots



1 2 3

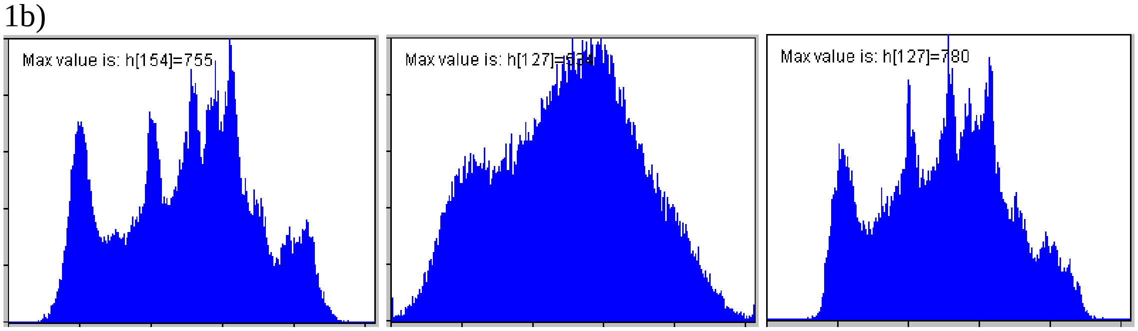
Histograms



I II III

Transformation types: a) Logarithm - b) Exponential- c) Equalize

1b)

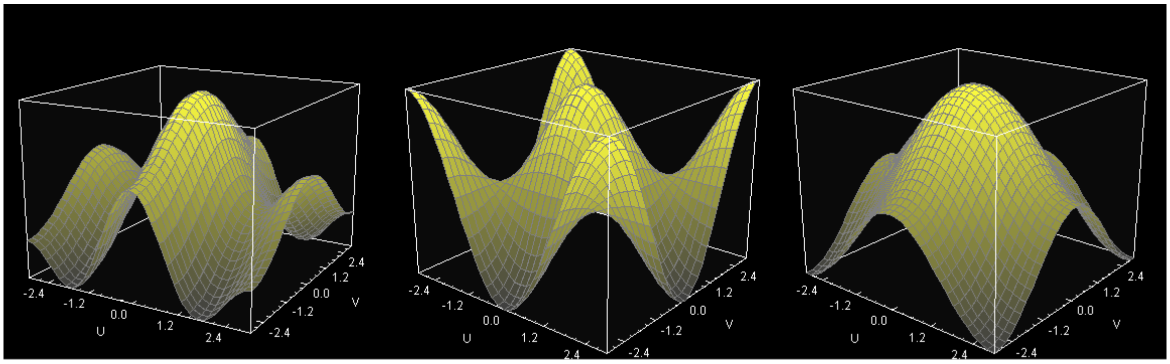


X Y Z

The histogram of the original image is denoted X. Suggest and motive (1 + 1 point, see problem 1a) the operations (applied to the image) that lead to the modified histograms Y and Z respectively (e.g. Y-1 and Z-2). Chose among the following operations:
1) Gaussian noise with zero mean and a reasonably small variance. 2) Equalization. 3) Low-pass filtering. 4) 10 % salt and pepper noise. 5 Bandpass filtering. 6) High-pass filtering 7) Logarithm grayscale transformation. 8) Exponential grayscale transformation.

IF-15

a) Combine the Fourier plot, Fourier transform, and filter kernel (e.g. 1 – I – a). Each correct combination gives 1 point. If you can motivate the choice of plot, you get an extra point for each correct plot motivation. If the motivation is clearly wrong, you get a minus point. (6p)



1 2 3

I	II	III
0 1 0	1 0 1	0 0 1 0 0
1 $\underline{1}$ 1	0 $\underline{1}$ 0	1 1 $\underline{1}$ 1 1
0 1 0	1 0 1	0 0 1 0 0

a) 1 + 2cos(u) + 2cos(v)
b) 1 + 2cos(u) + 2cos(v) + 2cos(2u)
c) 1 + 2cos(u+v) + 2cos(u-v)

b) What is the Fourier transform of the filter h_1 below

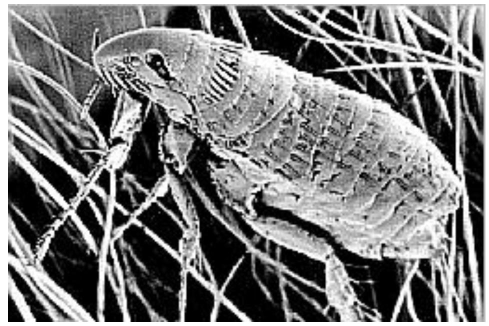
0 0 1
0 $\underline{0}$ 0
0 0 0

c) What is the effect of this filter (what happens to the image)?

d) Suggest a filter h_2 so that the Fourier transform of $h_3 = h_1 * h_2$ equals 1

IF-16


Original


a


b


c


d


e

Combine each image a-e with one of the filters 1-10. Each correct combination gives 1 point. If you can motivate your choice, you get an extra point for each correct motivation. If the motivation is clearly wrong, you get a minus point.

1)	2)	3)	4)	5)
-1 -1 -1	-1 -1 -1	-1 -1 -1	0 -1 0	A 5 x 5 filter in which all
-1 10 -1	-1 8 -1	-1 16 -1	-1 4 -1	coefficients equals one
-1 -1 -1	-1 -1 -1	-1 -1 -1	0 -1 0	

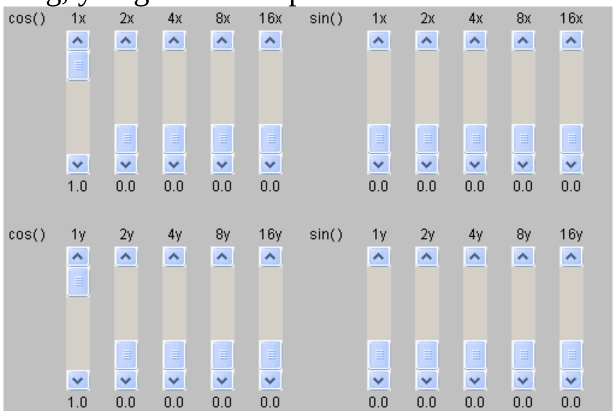
6) A 5 x 1 horizontal filter in which all coefficients equals one

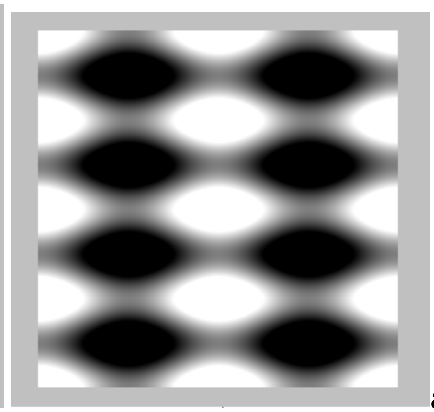
7) Similar as 6) but vertical

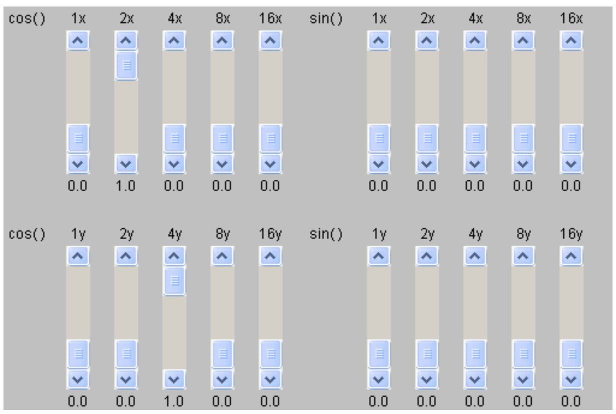
8)	9)	10)
-1 -1 -1	-1 0 1	0 1 0
0 0 0	-1 0 1	1 -4 1
1 1 1	-1 0 1	0 1 0

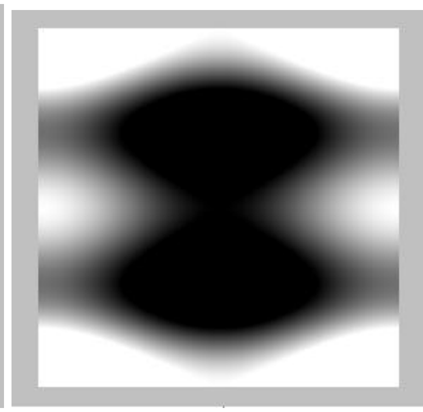
IF-17

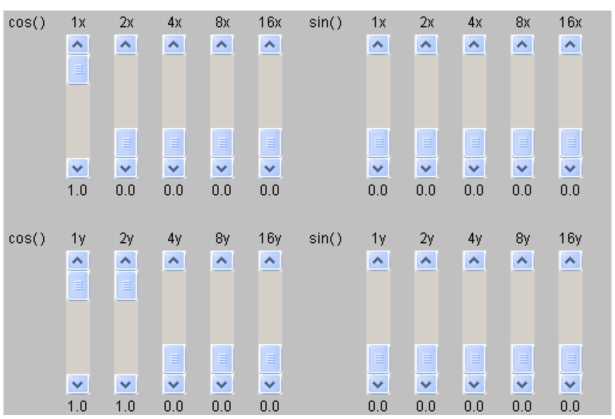
a) Combine the illustrations (1-3) with the images (a-c). Each correct combination gives 1 point. If you can motivate your choice, you get an extra point for each correct motivation. If the motivation is clearly wrong, you get a minus point.

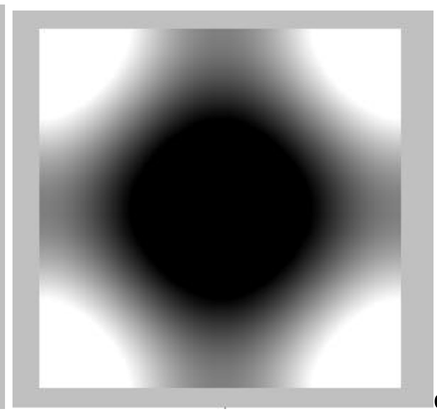
1 

 a

2 

 b

3 

 c

b) Determine the coefficient a of the normalized filter kernel:

1 a 1
a 4 a / 16
1 a 1

c) Determine the coefficient b of the vertical edge detector:

b b b
0 0 0
3 3 3

IF-18

Below you find one input image and five output images. There is also a list of ten image analysis procedures. The task is to link each output image to a specific procedure (1-a, 2-b, etc.). Each correct link will give one point. For each link, an additional point will be given if you can clearly motivate your choice. Any incorrect motivation will lead to subtraction by one point.


Input image


1


2


3


4


5

a) Canny edge detection b) Sobel edge detection c) Histogram equalization
d) Ideal low-pass filtering e) Butterworth low-pass-filtering f) Fourier phase
g) Fourier magnitude h) Reconstruction using Fourier magnitude
i) Reconstruction using Fourier phase j) Ideal high-pass filtering

IF-19

An adaptive filter computes the output as:

$$g(x,y) = f(x,y) - \frac{\sigma_n^2}{\sigma^2(x,y)}\left[f(x,y) - \bar{f}(x,y)\right]$$

where σ_n^2 is an estimate of the overall image noise variance, $\sigma^2(x,y)$ is the gray-level variance computed in a neighborhood centered on (x,y,) and $\bar{f}(x,y)$ is the mean gray-level in that neighborhood. An image depicts a rectangular object against a background. The true object and background have homogenous grayscale values but the image is corrupted by additive noise. In an area truly outside the object and sized 5X5, the pixel intensities are:

32,45,37,38,41
40,42,39,36,45
38,42,43,39,40
41,38,45,39,42
38,40,38,42,40

The three first columns in the image below belong to the background, the rest to the object. Compute the output at the underlined pixel position using the adaptive filter described above. Use a 3 X 3 neighborhood for computing $\bar{f}(x,y)$ and $\sigma^2(x,y)$ for the three pixel positions.

32,42,43,61,66,63
39,40,42,68,68,61
40,<u>45</u>,<u>41</u>,<u>68</u>,60,62
38,37,45,65,67,68
41,38,37,65,60,62

4b) Use a conventional filter below to compute the output values of the underlined pixels.

	1	
1	4	1
	1	

4c) Interpret your results in a-b)

IF-20

a) An input image depicts a cell against a background. The average intensities of the cell, I_c, and background, I_b , are 80 and 50, respectively. The signal-to-noise ratio, defined as of the ratio between the squared cell and background intensity difference, i.e. $(I_c - I_b)^2$ and the overall image noise variance, σ_n^2, has been estimated to zero (expressed in decibel). An adaptive filter computes the output as:

$$g(x,y) = f(x,y) - \frac{\sigma_n^2}{\sigma^2(x,y)}\left[f(x,y) - \bar{f}(x,y)\right]$$

where $\sigma^2(x,y)$ is the local image variance computed in a 5 x 5 neighborhood centered on (x,y,) and $\bar{f}(x,y)$ is a local unweighted image mean of that neighborhood. Determine the fraction of information in g(x,y) (in percentage) that comes from f(x,y) given that the local image variance, $\sigma^2(x,y)$, equals 90^2. Determine the same fraction if $\sigma^2(x,y)$ equals 30^2.

b) Interpret the results.

c) What is the drawback of applying non-linear square shaped median filters to images depicting square shaped objects?

d) What is the minimal size of a square shaped object that can survive an N x N symmetric median filtering?

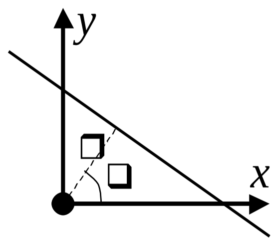
e) What is the effect of the minimum filter as being applied to an image consisting of long thin black structures on a bright background?

f) What is the effect of the maximum filter as being applied to an image with the same characteristics?

Boundary Detection

BD-1

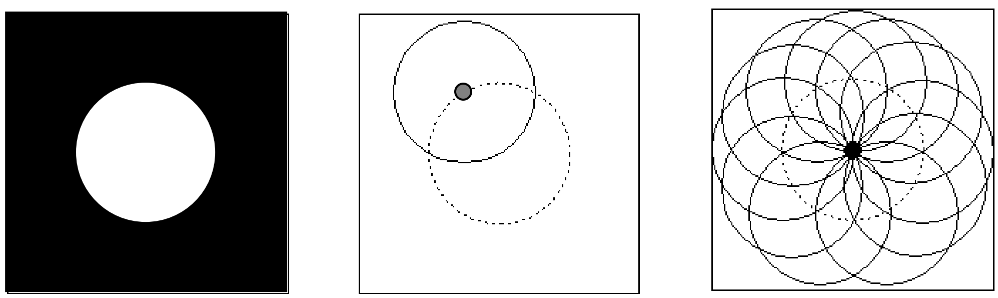
After performing edge detection on a 5x5 image suspected to contain a number of straight edges or lines, 7 pixels were found to lie on sufficiently strong edges, their coordinates are: (-2,2),(-1,2),(1,2),(2,2),(1,1),(-2,-2), and (2,-2)
The equation used to describe a line is: $x\cos(\theta) + y\sin(\theta) = \rho$
where x, y are the coordinates of a point on the line and the parameters ρ and θ are defined as in the figure:



You are required to detect the two strongest lines in the image using the Hough Transform ([$\rho 1, \theta 1$] and [$\rho 2, \theta 2$]). *Note*: Use the following quantized values for ρ and θ, respectively: [-3 -2 -1 0 1 2 3] and [0 π/4 π/2 3π/4]

BD-2

The following figures illustrates the principles of using the Hough Transform for circle detection. Assuming the circle's radius **r** is known, explain what you see in the plots (a, b, c). (3p)



a) b) c)

BD-3

Consider a gradient image f(x,y) where:

f(0,0) = 1, f(1,0) = 6, f(2,0) = 2, f(3,0) = 7
f(0,1) = 2, f(1,1) = 3, f(2,1) = 1, f(3,1) = 2
f(0,2) = 9, f(1,2) = 6, f(2,2) = 9, f(3,2) = 9
f(0,3) = 2, f(1,3) = 1, f(2,3) = 3, f(3,3) = 4

Apply the Canny edge detector with T_{low} = 5 and T_{high} = 8. Neighboring pixels are the eight pixels surrounding the center pixels. Which of pixel positions (x,y) will be part of the resulting edge map?

BD-4

3	1	1	2	2
3	5	1	4	3
1	2	4	1	5
1	2	1	2	1

Given the 4x5 pixels image below, apply dynamic programming to derive the optimum continuous line P with the following criteria:

-P is a raw vector composed of 5 pixels

-P minimizes the function below:

$$f(P) = -\sum_{x=1}^{5}(C_{grad}(p_x) + C_{int}(p_x)) + \sum_{x=2}^{5} C_{cont}(p_x, p_{x-1})$$

$$(p_1 ..., p_5 \in P)$$

where:

C_{grad} is computed using operator $\begin{bmatrix} 1 \\ \boxed{0} \\ -1 \end{bmatrix}$

C_{int} is computed using operator $\begin{bmatrix} 1 \\ \boxed{1} \\ 0 \end{bmatrix}$

C_{cont} is computed using Δy^2, where Δy is the difference of y coordinates of two adjacent pixels on the line

Tip: For the computation of C_{grad} and C_{int}, at border pixels, add extra rows [3 1 1 2 2] and [1 2 1 2 1] on top and bottom of the image, respectively.

BD-5

A 4 x 7 image is shown below. As can be seen, the image consists of "concentric" layers. The coordinates of the inner layer is (1,4), (1,5), (1,6), and (1,7). For each position x,y in the image there is an associated cost c, e.g. c(2,1) = 7. The problem is to find the path from the inner layer to the outer layer (three travel steps is required for each path) so that the accumulative cost is minimized. Each layer may only be visited once. The allowed travel directions are north, east, and north-east. There is a penalty term p = 1 associated with the travel directions north and east. For the travel direction north-east p = 0. The result should present the optimal path, e.g by specifying the four coordinate pairs of this path. It should also present the values of the accumulated travel costs in the form of a 4 x 7 accumulation matrix with clearly indicated back tracing pointers. (10p)

	1	2	3	4
1	4	7	8	8
2	6	5	5	6
3	3	6	6	4
4	6	6	6	7
5	5	4	4	6
6	7	4	5	7
7	6	5	3	8

BD-6

Apply Dynamic Programming for computing the optimal path from the left to the right border of the image, f, below. The brightness cost $C_1(p_i) = \max(f) - f(p_i)$ and the smoothness cost $C_2(p_{i-1}, p_i) = \Delta y$. The total cost $C(p_i) = w_1 * C_1(p_i) + w_2 * C_2(p_{i-1}, p_i)$, where the weights $w_1 = 2$ and $w_2 = 3$. The answer should include the cost accumulation matrix with back tracing pointers and the coordinates of the optimal path.

f(x, y):

2	0	2	2
2	1	4	5
3	4	3	1
2	3	1	1

BD-7

Consider the 3x3 image region below. There is reason to believe that the region may be modeled by a linear plane. Apply a least square solution to find the parameters of this plane:

17	23	40
15	19	30
10	16	26

Also, use the numerical value of the plane parameters to determine the slope and orientation of the plane.
Hint: the equation for a linear plane can be written:

$$g(x,y) = a_0 + a_1x + a_2y$$

Region Description

RD-1

a) Determine numerically the moment of order (1,0) for the 2 images below:

	0	1	2	3	4	5	6	7	8
0				3					
1		1	1	1	1	1			
2				2					
3			1	1	1				
4				2					
5									

	0	1	2	3	4	5	6	7	8
0							3		
1					1	1	1	1	1
2							2		
3						1	1	1	
4							2		
5									

b) If your solution in b) is correct, you can see that you get different values although the object in the two images is one and the same.

Suggest a modified version of the moments that in this case will give the same numerical value for the moment of order (2,0) and determine this value.

RD-2

Show that the central moments $\mu_{1,0}$ and $\mu_{0,1}$ are always invariant and equal to zero

RD-3

a) Determine numerically the central moment $\mu_{2,2}$ for object 1 specified below and for the same objects but shifted one pixel in the x and y directions respectively:

	0	1	2	3
0				
1			1	
2				2

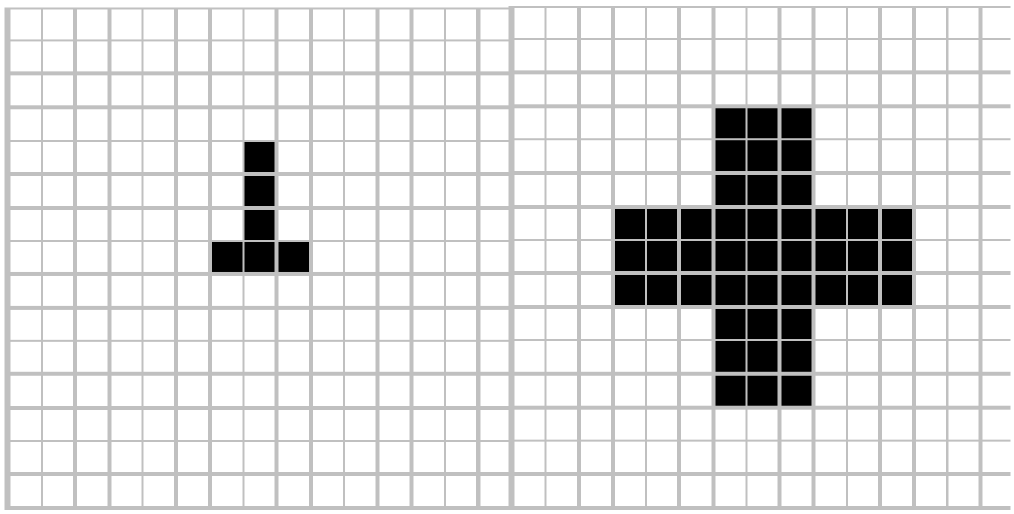
b) Also compute moment $\mu_{2,2}$ for object 2 which is a rotated version of object 1. Notice that in object 2, in order to keep the distance between object points identical, the pixel with value 2 has x-coordinate $2+\sqrt{2}$:

	0	1	2	3
0				
1			1	2
2				

c) Explain the results you get in c) and d)

RD-4

a) Compute numerically a second order moment that is invariant to translation and scale and which is the most effective for discriminating between the shape of the two objects below.



b) What can moments in general be used for?

c) Why is the moment $u_{2,0} + u_{0,2}$ particularly useful?

d) What is the main difference between moments and Fourier descriptors?

e) Using the real valued Fourier descriptors based on a radius versus angular representation, how can you ensure that you have chosen the optimal midpoint of the contour to be analyzed?

Answer *correct, not correct, or inconclusive*.

f) Moments can not be used if difference in shape is the only thing that matters.

g) Fourier Shape Descriptors can not be used for object characterization and classification if not only shape but also difference in structure inside the object matters.

h) For any rectangular shape, the second harmonic will contribute a lot to shape reconstruction from Fourier decomposition.

i) For any ellipsoid shape, the second harmonic will contribute a lot to shape reconstruction from Fourier decomposition.

RD-5

The central moments $\mu_{2,0}$ for the object below equals 2/3. What is the value of the pixel, a, at x=3, y=1?

	0	1	2	3
0				
1			1	a
2				

RD-6

a) Consider the image below and compute the sum of the central moments $\mu_{2,0} + \mu_{0,2}$:

	0	1	2	3
0			1	
1		2	3	2
2			1	
3				

b) Why may this sum be an interesting region feature?

c) Assume that each pixel value of the image is subject to additive noise which magnitude equals ε. Determine the largest value of ε that does not affect the numerical value of the feature $\mu_{2,0} + \mu_{0,2}$. The pixel values are assumed to be real but the value of the feature should be rounded to nearest integer.

RD-7

a) A small object is specified by the table below

x	y	$f \;\; x, y$	$\bar{f} \;\; 1,1$
1	1	a	3
2	1	b	
1	2	c	
2	2	d	

The sum of the pixel values equals 10 and the x and y coordinates of the center of gravity equals 1.4 and 1.3, respectively. The mean value $\bar{f} \;\; 1,1$ has been computed using the operator below (origo is underlined).

$$\frac{1}{6}\begin{bmatrix} \underline{3} & 1 \\ 1 & 1 \end{bmatrix}$$

Determine a, b, c, and d.

Hint: The x-coordinate of the center of gravity is $m_{1,0}/m_{0,0}$

If you manage to formulate and motivate the correct equations needed for the solution, you will get 4 points. Therefore, if you run out of time, try to focus on these equations more than on the actual solution.

b) How can moments be used in image analysis?

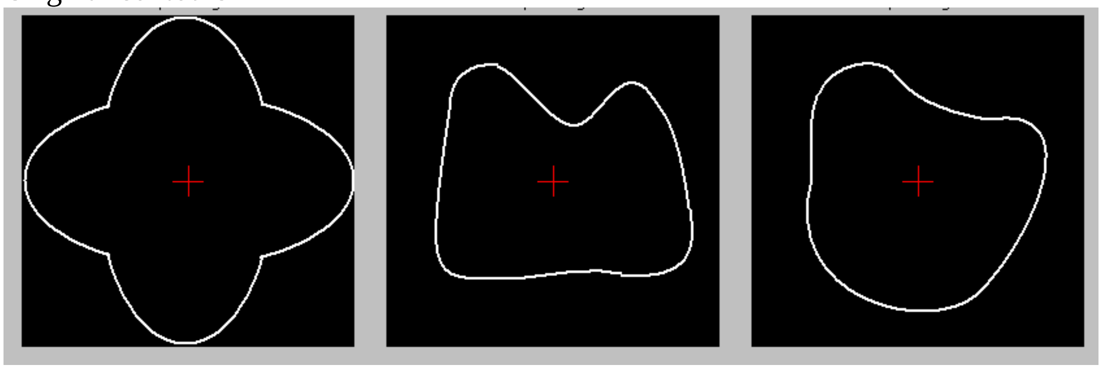
c) How will the second order moments $m_{2,2}$ change (quantitatively) if the image size increases (in both x and y) by a factor of 2 (the answer should be exact).

d) What is the main difference between moments and Fourier descriptors?

RD-8

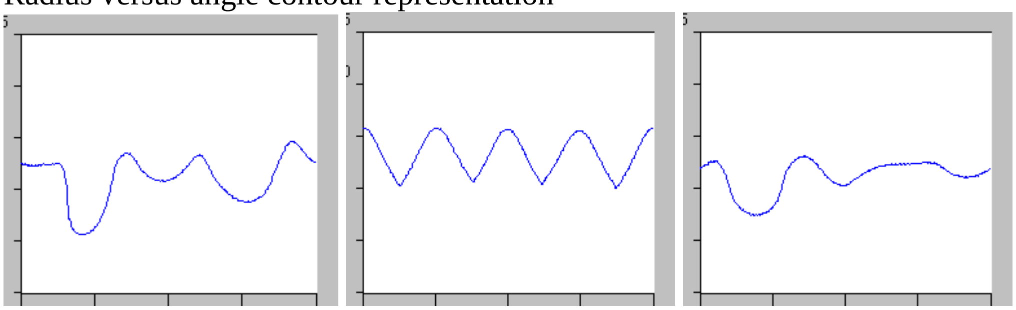
Combine the original contour, radius versus angle contour representation and Fourier transform of the radius versus angle contour representation (e.g. 1 – I – a).

Original contours



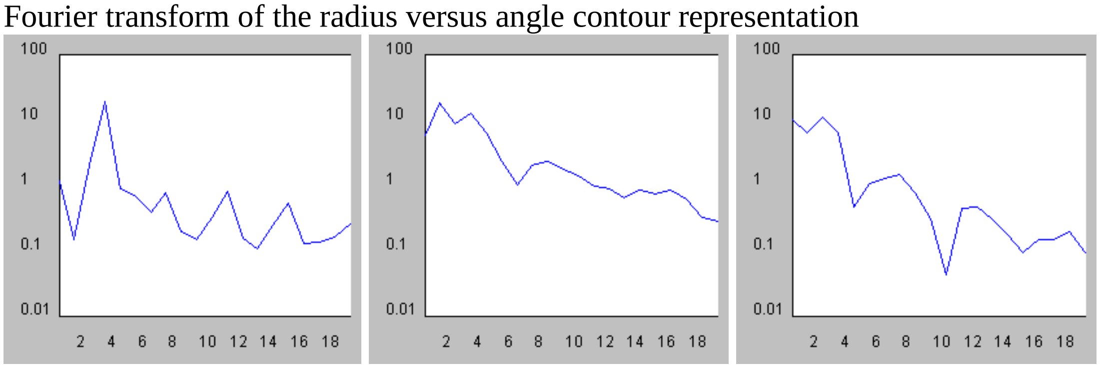
1 2 3

Radius versus angle contour representation



I II III

Fourier transform of the radius versus angle contour representation



a b c

TA-1

a) Construct cooccurence matrices (second-order joint probabilities) for the image below. Take the directions 0, $\pi/4$, $\pi/2$, $3\pi/4$ and the distance $|d|=1$ into account:

0	0	1	1
0	0	1	1
0	2	2	2
2	2	3	3

b) Describe an application area for cooccurence matrices

c) Describe the characteristics of the cooccurence matrices, given an image with high spatial frequencies in the horizontal direction and low frequencies in the vertical direction

d) Define a co-occurence matrix feature capable of discriminating low-frequency images from high-frequencies images

e) Describe a Fourier-based feature for the same task as in d)

TA-2

Two images will be compared with respect to the amount of high spatial frequencies in the horizontal direction:

0	0	0	1
2	3	2	2
1	1	1	1
3	3	3	2

1	3	2	1
2	0	3	1
2	3	0	1
0	3	2	1

a) Apply a cooccurence matrices (second order joint probabilities, horizontal distance |d|=1) including an appropriate feature to carry out this comparison in a quantitative way

b) Describe in principle how the Fourier transform can be used for solving the same task as in a) and define a suitable Fourier-based feature which can be used for a similar quantitative comparison

c) Two images have identical autocorrelation functions. Do these images have do be identical? Motivate your answer.

d) Describe how we can extract a first order grayscale difference statistic from the cooccurence matrices.

TA-3

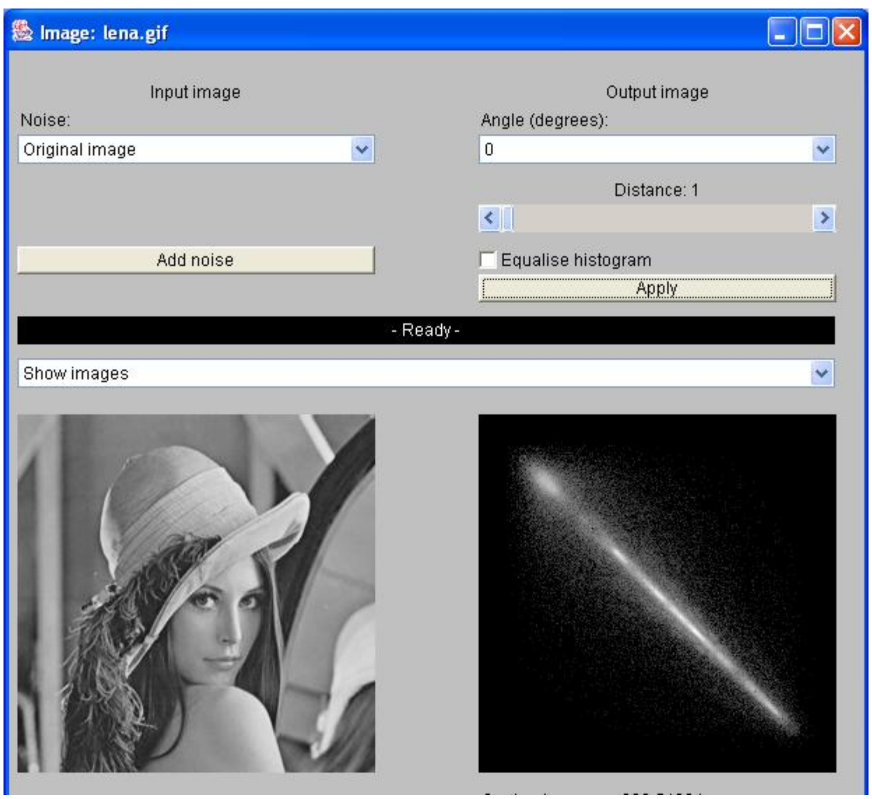
a) Given the 5 X 3 image below, compute the Gray Level Co-occurence Matrices for the horizontal and vertical directions, respectively, and distance = 1.

b) Compute the texture feature Contrast for both matrices.

Image
3 5 4 3 4
3 5 4 3 4
3 5 4 3 4

c) Interpret this result by describing the image with respect to its frequency characteristics (low, medium, or high frequency) in the horizontal and vertical directions, respectively.

The applet below presents the GLCM matrix for the horizontal direction and distance = 1. Illustrate in principle (by a simple sketch) how the GLCM will change if you
d) increase the distance to 8.
e) add noise with mean = 0 and variance = 20
f add noise with mean = 20 and variance = 0



TA-4

An image (see below) is analyzed with respect to its spatial frequencies in the horizontal and vertical directions, respectively:

1	2	0	3
1	2	0	3
1	3	0	2
1	3	0	2

a) Determine the autocorrelation function r(a,b) for the lags (a,b) = (1,0), (2,0), (3,0), (0,1), (0,2), and (0,3)

b) Interpret the results and describe what in general can be understood from the autocorrelation function

c) Construct cooccurence matrices (2nd order joint probabilities) for the same image as in a). Use the appropriate directions (for this problem) and the distance |d|=1

d) Interpret the results and describe what in general can be understood from the cooccurence matrices

e) Apply a quantitative texture feature extracted from the matrices in c) and describe numerically the difference as it comes to spatial frequencies in the horizontal and vertical directions

TA-5

A 2-bit 4x4 image region is given by:

4	5	4	6
7	6	5	5
6	5	4	5
7	5	6	4

Origo is at the upper-left corner.
Let u be a random variable representing a gray level in a given region of an image.

a) Define $p_u(x) \triangleq \text{Prob}\ u = x$

b) Compute $p_u(5)$

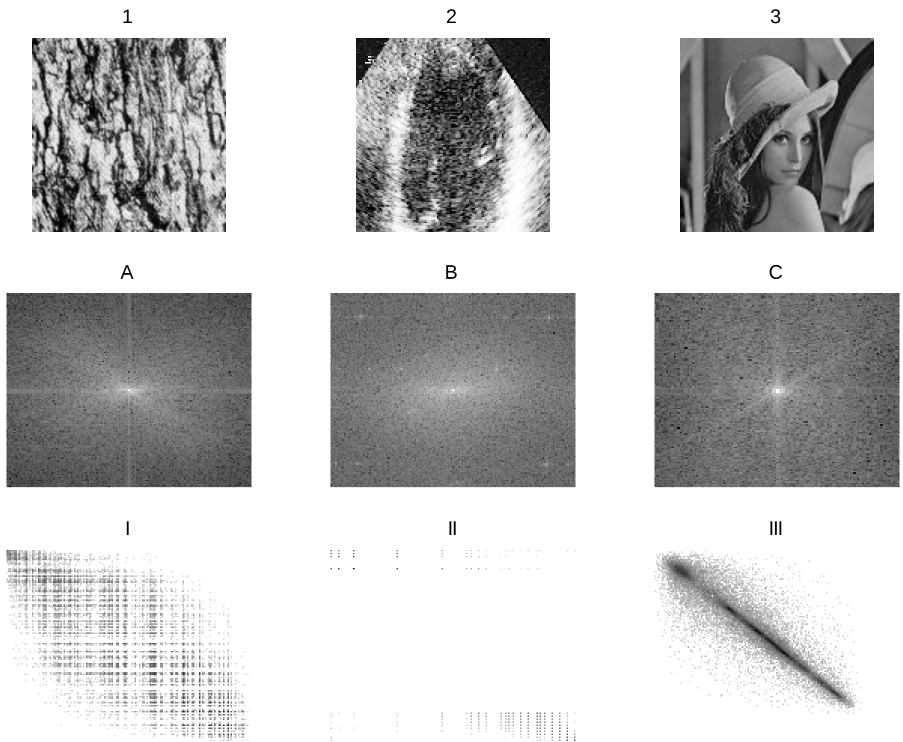
c) Define the second order joint probability
$p_{u_1,u_2}(x_1, x_2) \triangleq \text{Prob}\ u_1 = x_1, u_2 = x_2$

d) Compute $p_{u1,u2}(5,6)$ if $u_1=u(m,n)$ and $u_2=u(m+1,n+1)$ (no symmetry required)

e) Compute the texture feature contrast from the second order joint probabilities

f) Second order joint probabilities are useful in texture analysis. Mention two other common texture analysis methods (no mathematical definitions are needed).

TA-6



a) Combine the images above with their associated spectra and GLCM's. A correct combination gives you one point and you may get an additional point if you provide the correct motivation. An obviously erroneous motivation, however, gives you a minus point. Maximum number of points for this problem is:

b) An image sized 512 x 512 has 8 bits per pixel. What are the dimensions of its GLCM?

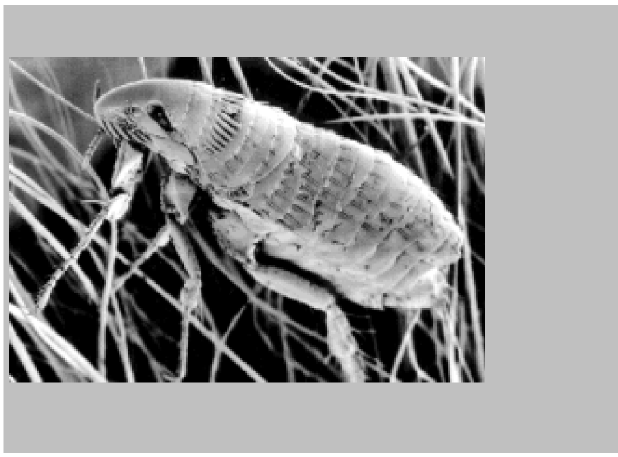
c) An image presents a light object against a dark background. The object extends to the right border of the image. Why may this cause a problem if the image is subject to a Fourier transform.

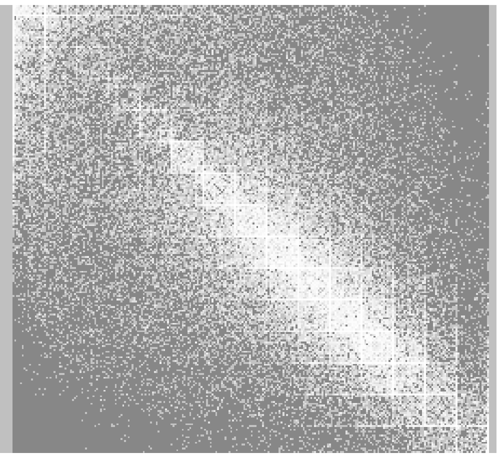
d) An unknown image sized 4 X 4 has 2 bits per pixel with range [0,3]. Below you will find its GLCM for the direction specified by dx = 1, dy = 0. Determine the image, i.e. specify the grayscale value for each image position.

0	0	12	0
0	0	0	0
12	0	0	0
0	0	0	0

TA-7

a) Below you see an image and a representation of its Gray Level Cooccurence Matrix (GLCM). The GLCM is computed at a pixel spacing of one. Which of the angles 0, 45, 90, and 135 degrees will give the highest value for the texture feature Contrast? A correct answer gives one point. A correct motivation gives an additional point but if this motivation is clearly wrong, the first point is taken away





b) Given the 3 X 5 image below, compute the Gray Level Co-occurence Matrices for the horizontal and vertical directions, respectively, and distance = 1.

c) Compute the texture feature Contrast for both matrices.

Image
4 4 4
3 3 3
4 4 4
5 5 5
3 3 3

d) Describe how you would use the autocorrelation r(a,b) in order to find out the answer to question a)

TA-8

a) Show by computing the GLCM feature Contrast that the two images, I_1 and I_2, have the same texture as considered for the highest frequency in the horizontal direction.

I_1:

0 1 0 1
2 3 2 3
0 2 0 2
1 3 1 3

I_2:

2 0 2 0 2 0
0 1 0 1 0 1
1 3 1 3 1 3
2 3 2 3 2 3
3 1 3 1 3 1
1 0 1 0 1 0

b) Consider the GCLM to represent the second order joint probabilities p(i,j). Compute p(2,3) for I_1 and I_2.

c) Consider the first order difference statistics p(i). Compute p(1) for I_1 and I_2.

d) Comment and interpret the results in a), b), and c).

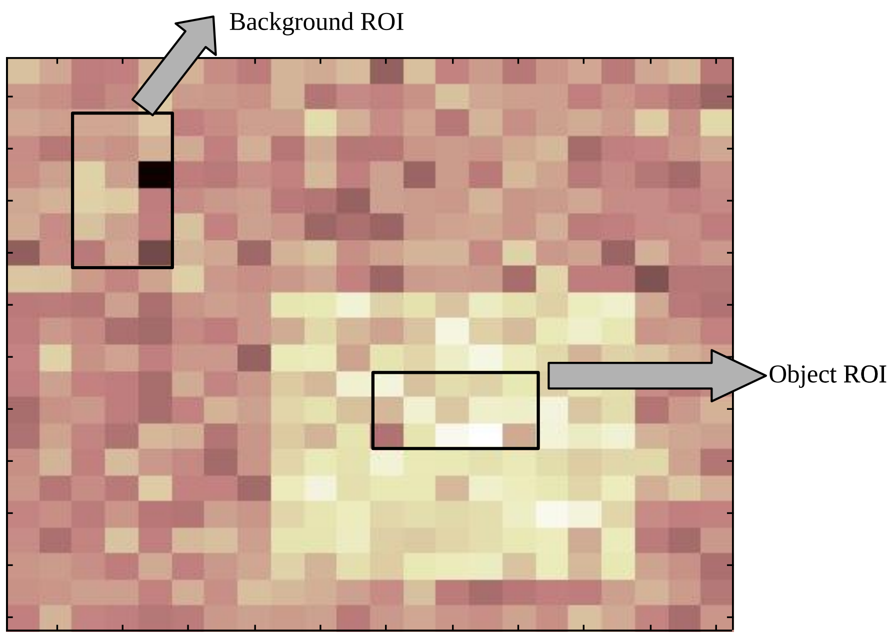
Optimal Thresholding

OT-1

An image depicts objects against a background. Because the objects are known as to their approximate shape and scale we know that on the average, 40% of the pixels belong to the objects. The distribution of grayscale values for objects as well as background is assumed to be Gaussian but the values of the distribution parameters are unknown. However, the variances are assumed to be the same. The distribution of grayscale values in the observed image is considered to be an additive mixture of the individual Gaussians. The Table below contains representative grayscale value ensembles captured from regions-of-interests (ROI) from positions clearly inside an object and clearly outside an object. Your task is to segment the image into objects and background by applying a single threshold chosen so that the number of misclassified pixels is being minimized.

a) Derive the optimal threshold value expressed as a function of the Gaussian parameters.
b) Solve numerically.
c) Express the optimal threshold value as a function of the Gaussian parameters given that on the average there are as many object pixels as background pixels. Solve numerically.

ROI for Object Pixels	98	62	36	66	94	82	78	69	105	74	92	112	84	91	57
ROI for Background Pixels	52	74	73	66	40	49	53	70	54	56	60	2	42	43	16



OT-2

A microscope image of size 512 x 512 depicts a cell with a known area of 1180 cm^2. The scale is 0,1 cm per pixel (in both directions). You are modeling the image as being an additive mixture of two Gaussian distributions describing the background and cell pixels, respectively. The variances are assumed to be equal. The signal-to-noise ratio (S/N), defined as 20 times the logarithm of the squared difference of the two means divided by the variance, is assumed to be zero. The optimal threshold value, i.e. the one that minimizes the total number of misclassified pictures, is 142. By cropping the image so that the cell covers half of the total image area, the optimal threshold value becomes 130.

Determine the numerical values of the Gaussian parameters μ_1, μ_2, σ_1, and σ_2.

OT-3

A microscope image that presents cells against background should be segmented by thresholding. The thresholding should be carried out so that the total number of misclassified pixels is minimized. Preliminary studies show that the histogram of background pixels has got a triangular and symmetric shape. The histogram of the cell pixels has got the same shape but a different scale. The intensity of the darkest background pixel is zero. The number of cell pixels is estimated to be one forth of the number of background pixels. Samples of background and object pixels are given below.

Background
58, 63, 54, 66, 57, 60, 61, 58, 62, 61

Object
122, 117, 126, 114, 123, 120, 119, 122, 118, 119

a) Compute the threshold value

b) Compute the threshold so that the sensitivity becomes 17/18 (there is no longer a requirement that the threshold should be determined so that the number of misclassified pixels is minimized)

Watershed Algorithms

WA-1

Fig. X and Y show an image at two different times: t_1 and t_2. The local minima are underlined.

a) Segment the objects in both images using the watershed algorithm.

b) Use the auction algorithm to associate the three objects in Fig. X with the three objects in Fig. Y. To calculate the assignment weights use: $a(i,j) = a_{dist}(i,j) + a_{area}(i,j)$

	$d(i,j)<0.5$	$0.5\leq d(i,j)<2$	$2\leq d(i,j)<4$	$4\leq d(i,j)<6$	$6\leq d(i,j)<10$	$10\leq d(i,j)<17$
$a_{dist}(i,j)$	10	6	4	3	2	1

where *d(i,j)* is the Euclidean distance between object *i*'s center point in image t_1 and object *j*'s center point in image t_2.

n(i,j)	0	1	2	3	4
$a_{area}(i,j)$	9	5	3	2	1

where *n(i,j)* is the difference in area (=number of pixels) between object *i* in image t_1 and object *j* in image t_2. Initial prices are zero, and $\varepsilon=0.1$.

9	9	9	8	8	7	7	6	6	6	6	6
9	15	15	15	16	8	6	6	6	6	6	6
9	14	13	12	12	8	6	6	6	6	6	6
8	14	13	10	11	8	7	12	12	12	12	5
8	14	12	11	11	8	7	11	10	10	11	5
8	14	14	13	13	9	7	11	10	9	11	5
8	7	7	7	7	7	7	11	10	12	12	5
8	13	13	14	13	8	8	8	7	7	6	5
8	12	10	10	14	8	8	8	5	5	4	4
8	12	9	11	14	9	9	6	5	4	4	4
8	10	10	14	14	9	9	6	5	4	3	2

Figure X. Image at t_1.

9	9	9	9	9	9	9	10	10	10	10	10
9	9	9	10	11	12	12	12	12	12	10	10
7	8	11	10	8	8	10	9	9	11	8	8
7	8	12	6	12	12	11	7	7	13	4	5
7	14	13	13	13	13	11	6	5	13	4	5
6	14	10	11	15	7	11	12	12	13	4	4
6	14	10	9	15	5	5	4	4	4	3	3
6	14	11	13	15	5	5	4	4	4	3	2
6	13	13	13	5	5	5	4	4	4	3	3
6	6	6	5	5	4	4	4	4	4	3	3
6	6	6	5	5	4	4	4	4	4	3	3

Figure Y. Image at t_2.

WA-2

a) The image shown in Fig. X contains two rectangular objects. Use the **watershed algorithm** to separate the objects from each other. Assume that the background is already segmented as one object.

0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	0	0	0	0	0	0	0
0	1	1	1	1	1	1	0	0	0	0	0	0	0
0	1	1	1	1	1	1	0	0	0	0	0	0	0
0	1	1	1	1	1	1	0	0	0	0	0	0	0
0	1	1	1	1	1	1	0	0	0	0	0	0	0
0	1	1	1	1	1	1	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	1	1	1	1	1	0
0	0	0	0	0	1	1	1	1	1	1	1	1	0
0	0	0	0	0	1	1	1	1	1	1	1	1	0
0	0	0	0	0	1	1	1	1	1	1	1	1	0
0	0	0	0	0	1	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure X.

b) Segment the image shown in Fig. Y using the **watershed algorithm**. The two seed pixels are underlined.

1	1	1	1	1	1	1	1
1	11	13	10	10	12	13	10
1	11	13	**9**	10	13	14	10
1	11	13	12	12	13	15	12
1	11	13	14	14	15	15	11
1	12	12	12	12	13	3	11
0	4	3	3	3	2	2	2
1	2	2	2	2	2	2	2

Figure Y.

c) Segment the image shown in Fig. Y using **region growing**. The two seed pixels are underlined. Use $\Delta=3$.

d) Comment on the result of b) and c).

Motion Analysis

MA-1

5) Given two input images, I_1 and I_2, compute the optical flow value, i.e. the velocity components U and V, at point (3,3). The upper left corner of the images has the coordinate (1,1). Run the algorithm for two iterations and set the weighting factor λ=0.5.

$$I_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad I_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Applied to I_1, calculate I_x and I_y using the filters $\begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$ / 2 and $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ / 2, respectively.

For calculating the average velocities, use the filter $L = \begin{bmatrix} \frac{1}{12} & \frac{1}{6} & \frac{1}{12} \\ \frac{1}{6} & 0 & \frac{1}{6} \\ \frac{1}{12} & \frac{1}{6} & \frac{1}{12} \end{bmatrix}$.

MA-2

Motion Correction

Two images are recorded of the same object (for example stem cells in a microscopy). Between the two sampling occasions the cells have moved only a little (time delay 1 min), and the images can be regarded as identical. However, due to vibrations the camera has moved relative to the object causing a horizontal shift. The algorithm for estimating this shift makes use of correlation. This is defined (for the one-dimensional case) as the convolution of

$c_{f_1 f_2}(x) = f_1(x) * f_2(-x) = \int f_1(x-u) f_2(-u) du$

(Notice the minus sign in $f_2(-x)$)

a) What is the Fourier transform, $C_{f_1 f_2}(\omega)$, of $c_{f_1 f_2}(x)$,?

b) If the functions are identical except for a translation ($f_1(x) = f(x),\ f_2(x) = f(x-x_0)$), what is then $C_{f_1 f_2}(\omega)$? (1p)

c) From the result in a2, where is the information about the translation: in $\left|C_{f_1 f_2}(\omega)\right|$, $\arg\ C_{f_1 f_2}(\omega)$, or in both?

d) Write a formula for x_0 using the previous results and

$x_0 = \frac{\sum x\delta(x-x_0)}{\sum \delta(x-x_0)} \quad \delta(x-x_0) = ...?$

Estimation of velocity field

Again, two cell images are recorded at different occasions but this time with very long time in-between (1 hour), During this time, cells have moved slightly but the camera has <u>not</u> moved. The so called Optical Flow algorithm makes use of two constraints, C_1 and C_2.

e) What are the names of these constraints, what are the mathematical definitions, and what is the meaning of each of them (describe by words)?

f) The second constraint, C_2, makes use of a Lagrangian multiplier, λ. Explain the importance of λ?

g) Which are the limitations of the Optical Flow algorithm? What may cause the algorithm to fail?

Binary Operations

BO-1

The boundary of a set of object pixels, denoted $\beta(A)$, can be obtained by first eroding A by the structure element B, and then performing the set difference between A and its erosion that is,

$\beta(A)=A\setminus (A\exists B)$

A (object)

X	X	X		X	X	X	X	X	
X	X	X		X	X	X	X	X	
X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X	X

B (structure element, origin is shaded)

X	X	X
X	X	X
X	X	X

a) Apply the boundary extraction algorithm described above. This means you should first erode A by the structure element B and present the result $(A\exists B)$. After this you should find out what is the set difference between the original object A and its eroded version $(A\exists B)$. This set difference should be presented as the final result $\beta(A)$.

b) Apply the “open” operation to the object below. Use the same structure element as in a)

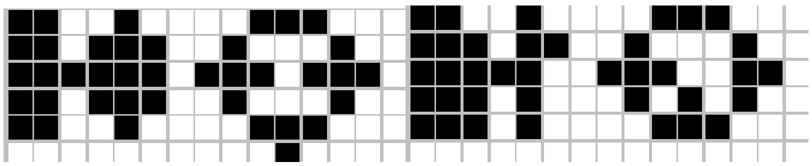
X	X	X	X	X	X	X	X	X
X	X	X		X	X	X	X	X
X	X	X		X	X	X	X	X
							X	

c) Apply the “close” operation to the same object as in b) using the same structure element as in a).

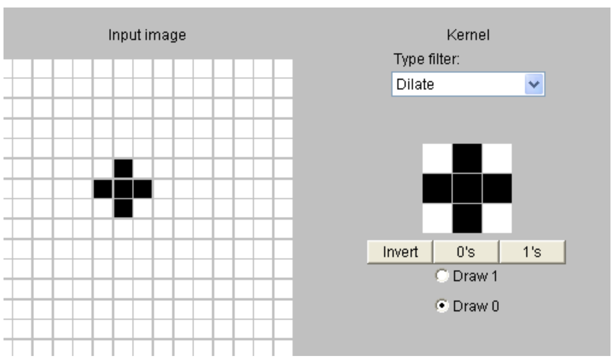
Apply the attack-from-different directions in order to shrink the object in b) into a skeleton.

BO-2

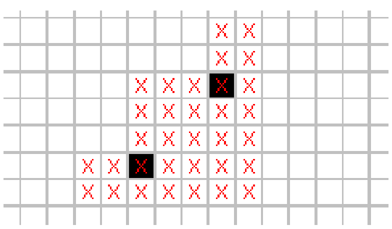
a) The four letters below have been disturbed by an imperfect thresholding operation. Try to restore these letters by applying the attack-from-different-directions skeleton algorithm. Hint: There is a Christmas message hidden in these four letters.



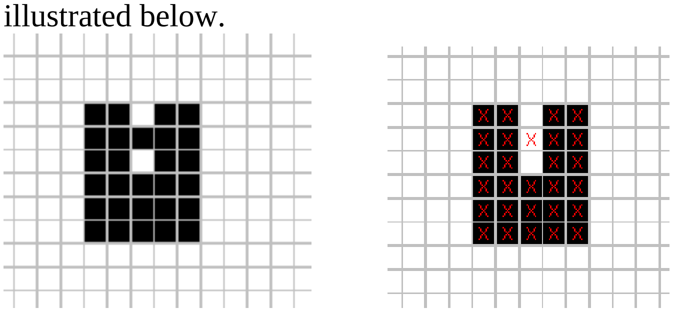
b) What is the result after applying the operation Dilate twice using the structure element presented below?



c) Design a hit-miss structure element that detects inner corners as illustrated below.

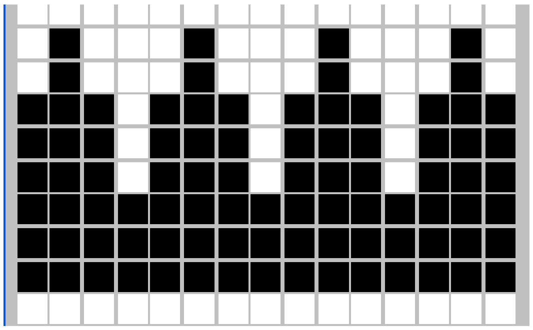


d) Design a structure element for an Open operation that will remove the midpixel as illustrated below.

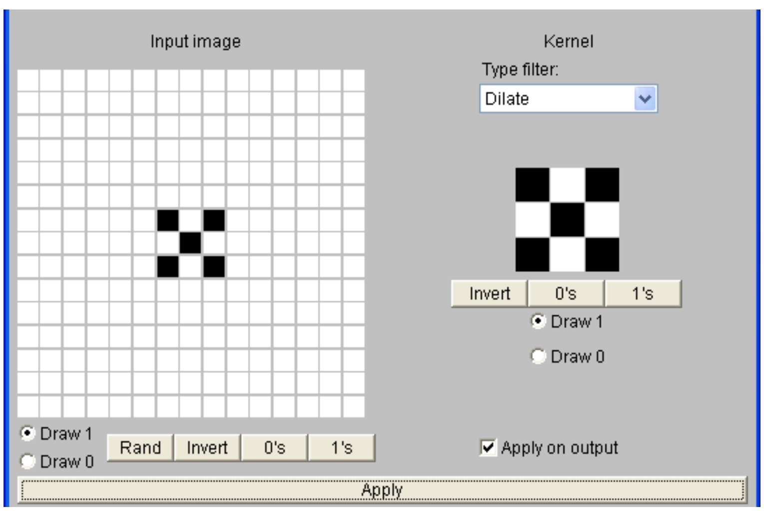


BO-3

Apply the attack-from-different-directions algorithm for construct a skeleton of the Christmas candlestick below. You should present the intermediate (after each phase) as well as the final result.

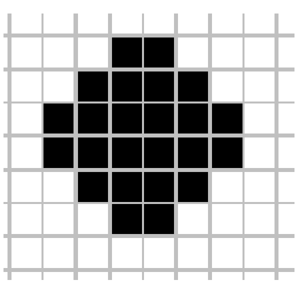


b) What is the result after applying the operation Dilate twice using the structure element presented below?



c) What is the result after applying the hit-miss operation one using the same structure element?

d) Compute the chessboard distance to all pixels inside the object below



ACM-1

a) Consider a snake attempting to segment a circular object as shown below. Associate the correct description (A,B,C,D) of the energy terms of the snake with each of the figures (1,2,3,4).

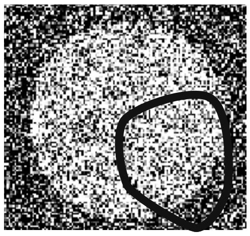
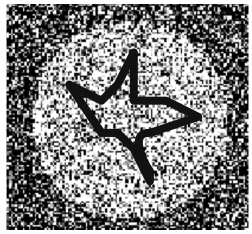
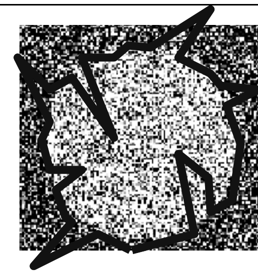
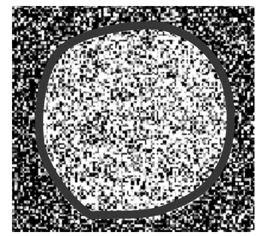
			

Figure 1.	Figure 2.	Figure 3.	Figure 4.

A. high-internal energy & low-external energy
B. high-internal energy & high-external energy
C. low-internal energy & low-external energy
D. low-internal energy & high-external energy

b)
Let $\mathbf{v}_i(t) = (x_i(t), y_i(t))$ be the snake nodes where $i = 1,2,\ldots,N$, and $I_s(x,y)$ is a smoothed version of the image. Associate the correct force effect (a,b,c) and force equation (I,II,III) with each of the following forces (1,2,3). (3p)

$\mathbf{F}_i^{flexural}(t)$: tensile force	$\mathbf{F}_i^{tensile}(t)$: flexural force	$\mathbf{F}_i^{external}(t)$: external force
Force 1.	Force 2.	Force 3.

a. Pulls the snake towards edges in the image
b. Resists stretching in the snake
c. Resists bending in the snake

I. $2\mathbf{v}_i(t) - \mathbf{v}_{i-1}(t) - \mathbf{v}_{i+1}(t)$
II. $\nabla(-\|\nabla I_s(x,y)\|)$
III. $6\mathbf{v}_i(t) - 4\mathbf{v}_{i-1}(t) - 4\mathbf{v}_{i+1}(t) + \mathbf{v}_{i-2}(t) + \mathbf{v}_{i+2}(t)$

ACM-2

Given 5 shapes, each of which is being represented by two parameters, x and y, an alignment procedure was used to produce the following new set of aligned shapes:

$$\mathbf{X}_1 = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix},\ \mathbf{X}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \mathbf{X}_3 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \mathbf{X}_4 = \begin{bmatrix} 1 \\ 5 \end{bmatrix}, \mathbf{X}_5 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}.$$

a) Find the PDM (Point Distribution Model) for the above shapes

b) What is the percentage of shape variation explained by each of the variation modes?

c) First reduce the PDM by using only one variation mode. Then, find the weight of the variation mode that will give a shape satisfying the reduced model and is closest to the new shape:

$$\mathbf{X}_{new} = \begin{bmatrix} x_{new} \\ y_{new} \end{bmatrix}$$

Exam Problems 051213

Problem 1

Consider the image grayscale, r, to be a continuous random variable with the range [0,1]. The image is subject to a piece-wise linear grayscale transformation, T(r), uniquely defined by

T(0) = 0.0
T(0.2) = 0.2
T(0.4) = 0.6
T(0.8) = 0.8
T(1) = 1.0

Answer *correct, not correct, or inconclusive* (it is not possible to answer the question given the problem formulation). Each correct answer gives you one point. You may motivate your answer but no additional points are given.

a) The contrast increases in the range [0.0,0.1]
b) The contrast increases in the range [0.25,0.35]
c) Edges in the image will be enhanced.
d) If one point in the input image is brighter than another point (located elsewhere) in the image), the first point will still be brighter in the output image.
e) The mean image intensity will be preserved by the transformation
f) A small object in the image will be better visualized against it local background (local contrast in the neighborhood of the small object will increase).

The input image, following grayscale transformation as defined above, is subject to a filtering step applying the operator

10 20 10
20 40 20
10 20 10

Consider the filter to be normalized before it is being applied. Compare the two images before and after filtering and answer correct, not correct, inconclusive. Each answer may give you one point.

g) The histogram of the output image will be more smooth
h) The mean image intensity will be preserved by the filtering
i) If one point in the input image is brighter than another point (located elsewhere) in the image), the first point will still be brighter in the output image.
j) The contrast increases in the range [0.25,0.35]

Problem 2

Consider two images I_1 and I_2. Extracted from the Fourier magnitude spectrum, F, nine texture features have been computed for both images. Consider a polar representation of the magnitude spectrum, F(r, fi). The frequency range is normalized to [0,1]. The values of the computed features are as follows:

$f_{1,1}\ (0.0<r<0.1,\ 0<fi<360) = 100$	$f_{2,1}\ (0.0<r<0.1,\ 0<fi<360) = 10$
$f_{1,2}\ (0.4<r<0.5,\ -10<fi<10) = 10$	$f_{2,2}\ (0.4<r<0.5,\ -10<fi<10) = 10$
$f_{1,3}\ (0.4<r<0.5,\ 35<fi<55) = 100$	$f_{2,3}\ (0.4<r<0.5,\ 35<fi<55) = 10$
$f_{1,4}\ (0.4<r<0.5,\ 80<fi<100) = 10$	$f_{2,4}\ (0.4<r<0.5,\ 80<fi<100) = 10$
$f_{1,5}\ (0.4<r<0.5,\ 135<fi<155) = 100$	$f_{2,5}\ (0.4<r<0.5,\ 135<fi<155) = 10$
$f_{1,6}\ (0.9<r<1.0,\ -10<fi<10) = 10$	$f_{2,6}\ (0.9<r<1.0,\ -10<fi<10) = 10$
$f_{1,7}\ (0.9<r<1.0,\ 35<fi<55) = 10$	$f_{2,7}\ (0.9<r<1.0,\ 35<fi<55) = 10$
$f_{1,8}\ (0.9<r<1.0,\ 80<fi<100) = 10$	$f_{2,8}\ (0.9<r<1.0,\ 80<fi<100) = 100$
$f_{1,9}\ (0.9<r<1.0,\ 135<fi<155) = 10$	$f_{2,9}\ (0.9<r<1.0,\ 135<fi<155) = 100$

Answer *correct*, *not correct*, *or inconclusive* (it is not possible to answer the question given the problem formulation). Each correct answer gives you one point. You may motivate your answer but no additional points are given.

a) I_1 is overall smoother (has got more slowly varying pixel intensities) than I_2.
b) I_1 is more high-frequency at 45 degrees.
c) I_1 is more high-frequency at -135 degrees.
d) I_1 is more high-frequency at 145 degrees.
e) Counting from left to right, I_1 has got more transitions from darker to brighter pixels in the horizontal direction
f) Counting from top to bottom, I_2 has got more extremely rapid transitions from darker to brighter pixels in the vertical direction

Consider two 2-bit images I_3 and I_4 with range [0,3] The associated non-normalized GLCM (dx=+-1, dy=0) are presented below (origo, is at the underlined position):

GLCM I_3	GLCM I_4
01 19 10 20	40 40 40 80
19 01 19 10	40 40 40 40
10 19 01 10	40 40 40 40
20 10 10 01	80 40 40 40

g) By computing the GLCM texture Contrast, it can be seen that I_3 is more low-frequency in the horizontal direction.
h) The relative occurrence of pixel transitions from graylevel 2 to 3 is the same for the two images.
i) By computing the GLCM texture Contrast, it can be seen that I_3 is more high-frequency in the vertical direction.
j) I_3 is the only image that could have produced GLCM I_3

Problem 3

Consider the filter kernel presented below.

1 1 1
0 0 0
1 1 1
0 0 0
1 1 1

Answer *correct*, *not correct*, *or inconclusive* (it is not possible to answer the question given the problem formulation). Each correct answer gives you one point. You may motivate your answer but no additional points are given.

a) The filter performs smoothing in all directions and at all frequencies.
b) The filter performs smoothing in all directions.
c) The filter performs high-pass filtering in the vertical direction at the highest possible frequency.
d) The filter performs low-pass filtering in the vertical direction at the lowest possible frequency.
e) If the underlined mid-coefficient is changed from zero to six, it will not change any important characteristic of the filter.
f) The filter performs less well as compared to an unweighted averaging filter.

g) What is the Fourier transform of this filter? (2p)
h) Apply this filter (normalized) to the small pattern (only at underlined pixels) presented below (2p)

3 3 3 2 2 3 3
2 2 2 2 2 3 3
3 3 3 2 2 3 3
2 2 2 2 2 3 3
3 3 3 2 2 3 3
2 2 2 2 2 3 3

Problem 4

Consider the two patterns below:

```
00000000000000        0000000000000
00000000111000        0011111111000
00111111100000        0011111111000
00000000000000        0011111111000
00000000000000        0000000000000
```

a) Apply a 3x3 max filter of rank type to the left patter. Explain what happens (1+1 p)
b) Apply a 3x3 min filter of rank type to the right patter. Explain what happens (1+1 p)
For both a) and b), consider points outside the patterns as zeroes.

Consider the pattern below:

```
00000000111111111
00000000111111111
00000000111111111
00000000111111111
```

c) Apply a 3x3 range filter. Explain what happens (1+1 p)

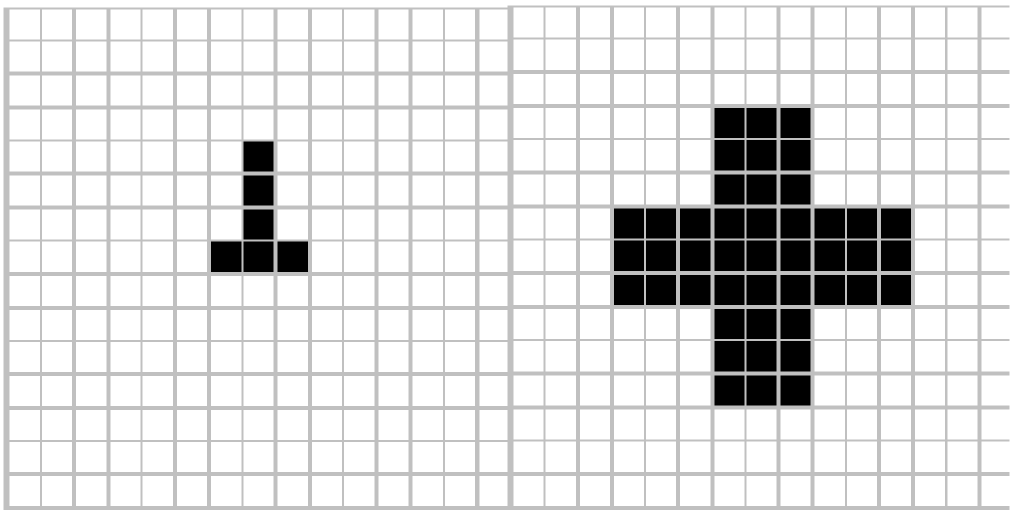
Answer *correct*, *not correct*, *or inconclusive* (it is not possible to answer the question given the problem formulation). Each correct answer gives you one point. You may motivate your answer but no additional points are given.

d) The median filter performs better than the mean in the presence of salt-and-pepper noise.
e) The median filter performs better than the mean in the presence of Gaussian noise.
f) The Fourier transform of the median is very close to that of the mean
g) The median may remove single pixels at object corners.

Problem 5

a) Compute numerically a scale invariant version of the second order moment $u_{2,0}$.

(5p)



b) Why are the scale-invariant values of $u_{2,0}$ different for the two objects (1p)

Answer *correct, not correct, or inconclusive* (it is not possible to answer the question given the problem formulation). Each correct answer gives you one point. You may motivate your answer but no additional points are given.

c) Moments can not be used if difference in shape is the only thing that matters.

d) Fourier Shape Descriptors can not be used for object characterization and classification if not only shape but also difference in structure inside the object matters.

d) For any rectangular shape, the second harmonic will contribute a lot to shape reconstruction from Fourier decomposition.

e) For any ellipsoid shape, the second harmonic will contribute a lot to shape reconstruction from Fourier decomposition.

Problem 6

Answer *correct, not correct, or inconclusive* (it is not possible to answer the question given the problem formulation). Each correct answer gives you one point. You may motivate your answer but no additional points are given.

a) Active Shape Modeling (ASM) performs better for object segmentation than Active Contour Modeling (ACM).
b) Dynamic Programming always gives one unique solution.
c) Dynamic Programming is computationally less expensive than ACM.
d) Using weighted variation modes of the Point Distribution Model, it is impossible to synthesize shapes which have not been seen in the training set.
e) The histogram of an image depicting a dark object against a bright background is modeled as an additive mixture of Gaussian distributions. Setting the threshold to a value lower than that associated with minimizing the number of misclassified pixels leads to an increase in specificity.
f) Local thresholding is equivalent to point-wise subtraction between the input image and a small local average.
g) Local thresholding is superior to global thresholding.
h) The Canny Edge Detection algorithm makes use of two thresholds in order to obtain edges which are one pixel thick.
i) A sequence of erode and dilate followed by another sequence of dilate and erode will always take you back to the original image.
j) The course on Image Analysis is the best course currently available in the curriculum.

Exam Problems 061218

Problem 1. Consider the image grayscale, r, to be a continuous random variable with the range [0,1]. The image is subject to a grayscale transformation, s=T(r), where T(r) is single valued and s has got the range [0,1]. The transformation is such that the probability density function (referred to as the histogram in the discrete case) of s becomes a constant.

Answer *correct, not correct, or inconclusive* (it is not possible to provide an unmistakable answer to the question given the problem formulation). Each correct answer gives you one point. You may motivate or clarify your answer but no additional points will be given.

k) Given the probability density function of an image, it is always possible to reconstruct the image in a unique way.
l) A rotation of the image will not change its probability density function.
m) A mean filter applied to the image will not change its probability density function.
n) A median filter applied to the image will not change its probability density function.
o) A high-frequency painting is being close-up pictured by a digital camera resulting in a 6 bit 64 x 64 pixel image. The histogram of this result image is very close to the probability density function of a faithfully (in the sense of the Nyqvist sampling theorem) pictured version of the same scene.
p) The histogram turns into the probability density function (becomes the same) if the discrete image has got unlimited number of pixels per size unit and unlimited number of graylevels per pixel.

An arbitrary 512 x 512 input image with 8 bits per pixel is subject to a filtering operation applying the operator

00 01 00
01 96 01
00 01 00

Consider the filter to be normalized before it is being applied. Answer correct, not correct, or inconclusive to the questions below. Each correct answer gives you one point.

q) The histogram of the output image will be more smooth.
r) The mean image intensity will be preserved by the filtering
s) Following one filtering step, for each position in the output image the fraction of information that comes from the corresponding position in the input image is 96 percentage.
t) Following two filtering steps applying the same type of filter in a sequence, for each position in the output image the fraction of information that comes from the corresponding position in the original input image increases as compared to the case presented in i).

Problem 2

Consider two images I_1 and I_2. Extracted from the Fourier magnitude spectrum, F, nine texture features have been computed for both images. Consider a polar representation of the magnitude spectrum, F(r, fi). The frequency range is normalized to [0,1]. The values of the computed features are as follows:

$f_{1,1}$ $(0.0<r<0.1, 0<fi<360) = 100$	$f_{2,1}$ $(0.0<r<0.1, 0<fi<360) = 100$
$f_{1,2}$ $(0.4<r<0.5, -10<fi<10) = 10$	$f_{2,2}$ $(0.4<r<0.5, -10<fi<10) = 10$
$f_{1,3}$ $(0.4<r<0.5, 215<fi<235) = 100$	$f_{2,3}$ $(0.4<r<0.5, 215<fi<235) = 10$
$f_{1,4}$ $(0.4<r<0.5, -80<fi<-100) = 10$	$f_{2,4}$ $(0.4<r<0.5, -80<fi<-100) = 10$
$f_{1,5}$ $(0.4<r<0.5, 135<fi<155) = 100$	$f_{2,5}$ $(0.4<r<0.5, 135<fi<155) = 10$
$f_{1,6}$ $(0.9<r<1.0, -10<fi<10) = 10$	$f_{2,6}$ $(0.9<r<1.0, -10<fi<10) = 10$
$f_{1,7}$ $(0.9<r<1.0, 215<fi<235) = 10$	$f_{2,7}$ $(0.9<r<1.0, 215<fi<235) = 10$
$f_{1,8}$ $(0.9<r<1.0, -80<fi<100) = 10$	$f_{2,8}$ $(0.9<r<1.0, 80<fi<-100) = 100$
$f_{1,9}$ $(0.9<r<1.0, 135<fi<155) = 10$	$f_{2,9}$ $(0.9<r<1.0, 135<fi<155) = 100$

Answer *correct*, *not correct*, *or inconclusive* (it is not possible to answer the question given the problem formulation). Each correct answer gives you one point. You may motivate your answer but no additional points are given.

k) I_1 is overall smoother (has got more slowly varying pixel intensities) than I_2.
l) I_1 is more high-frequency at 45 degrees.
m) I_1 is more high-frequency at -135 degrees.
n) Both I_1 and I_2 present dominant edges oriented at 135 degrees, pretty much like the bug image discussed during the lecture.
o) Counting from left to right, I_1 has got more intensity variations in the horizontal direction
p) I_1 is a rotated (+45 degrees) version of I_2.

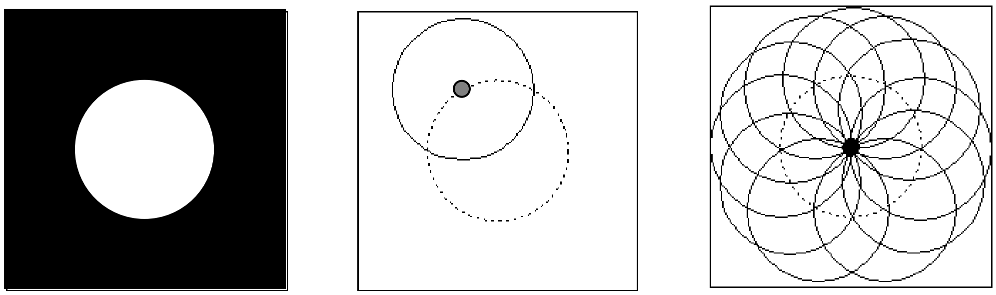
Consider two 2-bit images I_3 and I_4 with range [0,3] The associated non-normalized GLCM, direction is given by (dx=-1, dy=-1) and (dx=1, dy=1) for both images, are presented below (origo, is at the underlined position):

GLCM I_3	GLCM I_4
02 00 00 05	23 00 00 00
00 02 00 00	00 23 00 00
10 00 02 00	00 00 23 00
05 00 00 02	00 00 00 23

q) The size of I_4 is 8 x 8.
r) I_3 is more high-frequency in the horizontal direction than I_4.
s) By computing the GLCM texture Contrast, it can be seen that I_3 is more high-frequency in the horisontal direction than I_4.
t) I_3 has got more gray levels than I_4.

Problem 3

The figure below illustrates the principles of using the Hough Transform for circle detection. Assuming the circle's radius **r** is known, explain what you see in the plots (a, b, c). (3 x 2p)

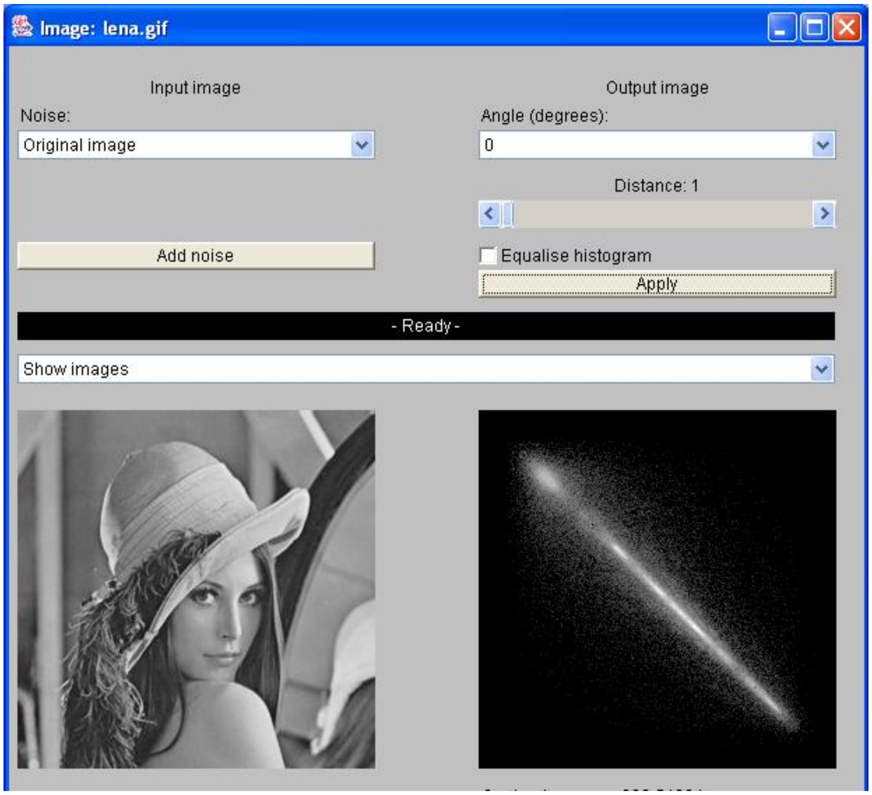


a) b) c)

In what situations do you think the following thresholding schemes would perform well:

a) Fixed thresholding. (1p)
b) The Triangle algorithm, where a line is constructed between the maximum of the histogram at brightness b_{max} and the lowest value b_{min} in the image. The distance **d** between the line and the histogram h[b] is computed for all values of b from $b = b_{min}$ to $b = b_{max}$. The brightness value b_o where the distance between $h[b_o]$ and the line is maximal is chosen the threshold value. (2p)
c) Local thresholding. (1p)

Problem 4



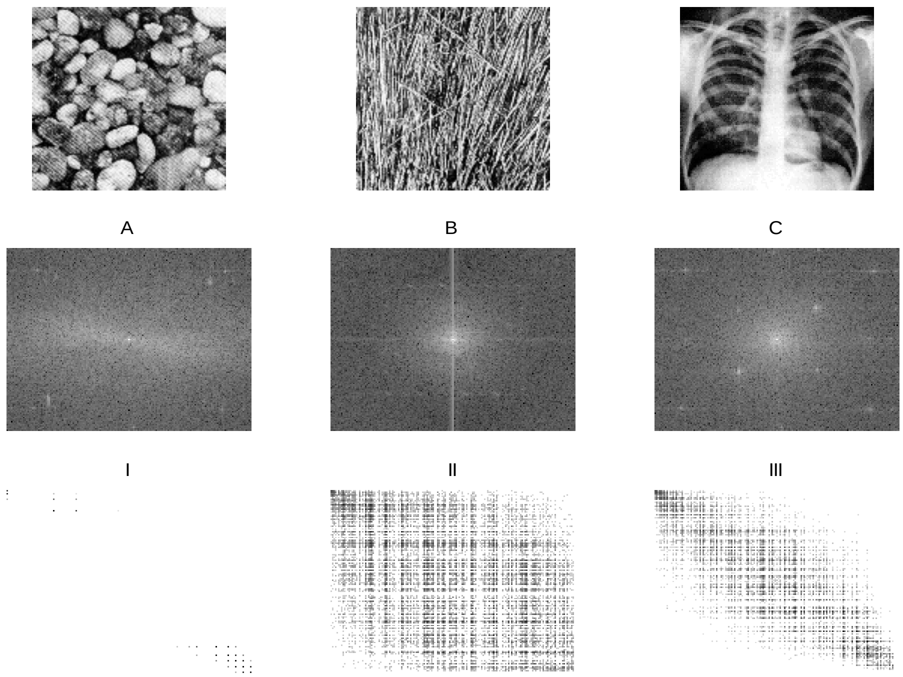
Consider the image above. Answer *correct*, *not correct*, *or inconclusive* (it is not possible to answer the question given the problem formulation). Each correct answer gives you one point. You may motivate your answer but no additional points are given.

a) If we increase Distance to 4, the pattern in the GLCM plot will shift downwards along the diagonal.
b) If we change Angle to 90 degress, the GLCM plot will change slightly.
c) If we add Gaussian noise with mean = 0 and variance = 20, the GLCM plot will loose much of its structure (type of shape).
d) If we add 10% of Salt-and-Pepper noise, the GLCM plot will look pretty much the same but add intensities in the upper left and lower right corners.
e) If we add Gaussian noise with mean = 30 and variance = 0, the GLCM plot will keep its structure (type of shape) but become wider.
f) If we rotate the image 90 degrees, the GLCM plot will change slightly.

g) Describe by words how texture analysis by GLCM and Fourier-based texture analysis both can be carried out for non-oriented low-frequency region and 8 (4 x 2) oriented medium- and high-frequency regions. (2p)

h) Discuss pros and cons with the GLCM and Fourier based texture analysis algorithms. (2p)

Problem 5



1 2 3

Consider the three images A, B, C, the three Fourier Spectra I, II, and III, and the three GLCM plots 1, 2, and 3. Answer *correct, not correct, or inconclusive* (it is not possible to answer the question given the problem formulation). Each correct answer gives you one point. You may motivate your answer but no additional points are given.

a) Image A should be associated with Spectrum III because in this spectrum there is no dominant orientation. Furthermore, the four dots equally distributed around the origo of the spectrum plot can be explained by an oriented raster phenomena in image A.
b) Image A should be associated with Spectrum II because in this spectrum there is no dominant orientation. Furthermore, the vertical and horisontal line structures visible in the spectrum plot can be explained by an oriented raster phenomena in image A.
c) Image B should be associated with GLCM 3 because the orientation of the GLCM is such that it represents a 90 degree rotation of the almost vertically oriented grass straws.
d) Image B should be associated with the elongated Spectrum I because this image presents elongated structures.
e) Image C should be associated with GLCM 1 because of its gray level distribution.
f) The low-frequency content of Image A and B is different.

g) Describe Image A in terms of frequency content (magnitude, orientation) (2p)
h) Describe Image B in terms of GLCM appearance (given a certain pixel-pair distance and orientation. (2p)

Problem 6

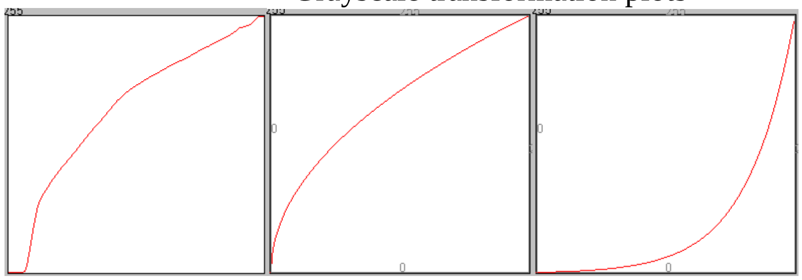
Answer *correct, not correct, or inconclusive* (it is not possible to answer the question given the problem formulation). Each correct answer gives you one point. You may motivate your answer but no additional points are given.

k) Active Shape Modeling (ASM) generally performs better for object segmentation than Active Contour Modeling (ACM) if shape statistics are available.
l) As a cost minimizing computational tool, Dynamic Programming and Exhaustive Search would typically not lead to the same result.
m) Dynamic Programming can not be used for circle detection or detection of any closed contour.
n) Using un-weighted variation modes of the Point Distribution Model, it is always possible to synthesize shapes which have been seen in the training set.
o) The ACM algorithm converges faster than does the ASM algorithm.
p) Dynamic Programming converges faster than does the ACM.
q) In the Optical Flow algorithm, spatial and temporal derivatives can be estimated from an image pair and used for estimating a vector image in which each image point contains a vector where magnitude represents velocity for that point and orientation represents motion direction for the same point.
r) Dynamic Programming for boundary detection can not be generalized for detection of 2D surfaces.
s) Subregion local image correlation will give the same results as the Optical Flow algorithm.
t) The lecturer presenting the course on Image Analysis is nuts.

Exam 031213

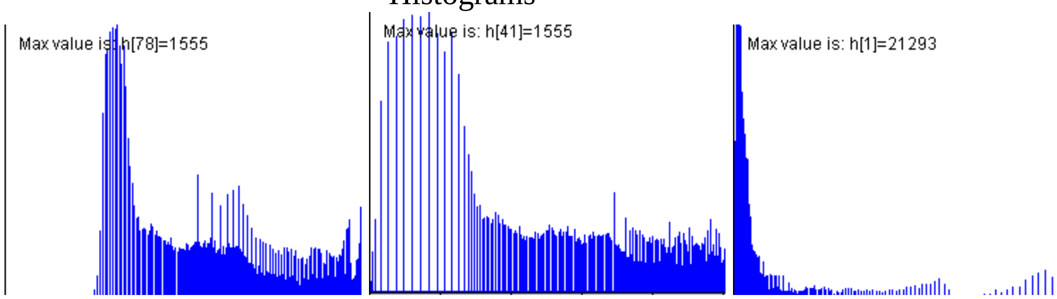
1a) Combine the grayscale transformation plot, histogram, and transformation type (e.g. 1 – I – a). Each correct combination gives 1 point. If you can motivate your choice, you get an extra point for each correct motivation. If the motivation is clearly wrong, you get a minus point. (6p)

Grayscale transformation plots



1 2 3

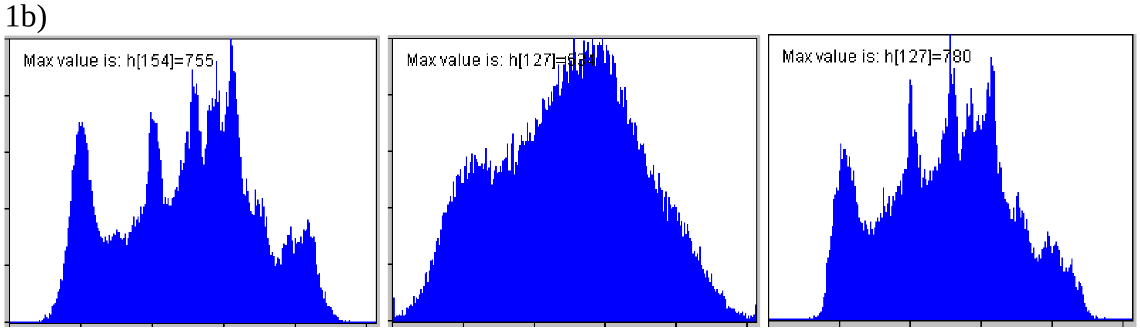
Histograms



I II III

Transformation types: a) Logarithm - b) Exponential- c) Equalize

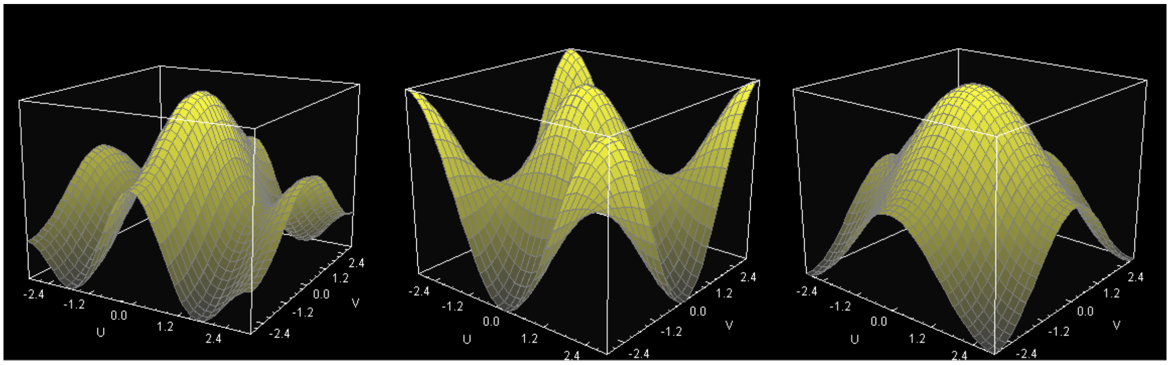
1b)



X Y Z

The histogram of the original image is denoted X. Suggest and motive (1 + 1 point, see problem 1a) the operations (applied to the image) that lead to the modified histograms Y and Z respectively (e.g. Y-1 and Z-2). Chose among the following operations:
1) Gaussian noise with zero mean and a reasonably small variance. 2) Equalization. 3) Low-pass filtering. 4) 10 % salt and pepper noise. 5 Bandpass filtering. 6) High-pass filtering 7) Logarithm grayscale transformation. 8) Exponential grayscale transformation. (4p)

2a) Combine the Fourier plot, Fourier transform, and filter kernel (e.g. 1 – I – a). Each correct combination gives 1 point. If you can motivate the choice of plot, you get an extra point for each correct plot motivation. If the motivation is clearly wrong, you get a minus point. (6p)



1 2 3

0 1 0	1 0 1	0 0 1 0 0
1 $\underline{1}$ 1	0 $\underline{1}$ 0	1 1 $\underline{1}$ 1 1
0 1 0	1 0 1	0 0 1 0 0
I	II	III

a) $1 + 2\cos(u) + 2\cos(v)$
b) $1 + 2\cos(u) + 2\cos(v) + 2\cos(2u)$
c) $1 + 2\cos(u+v) + 2\cos(u-v)$

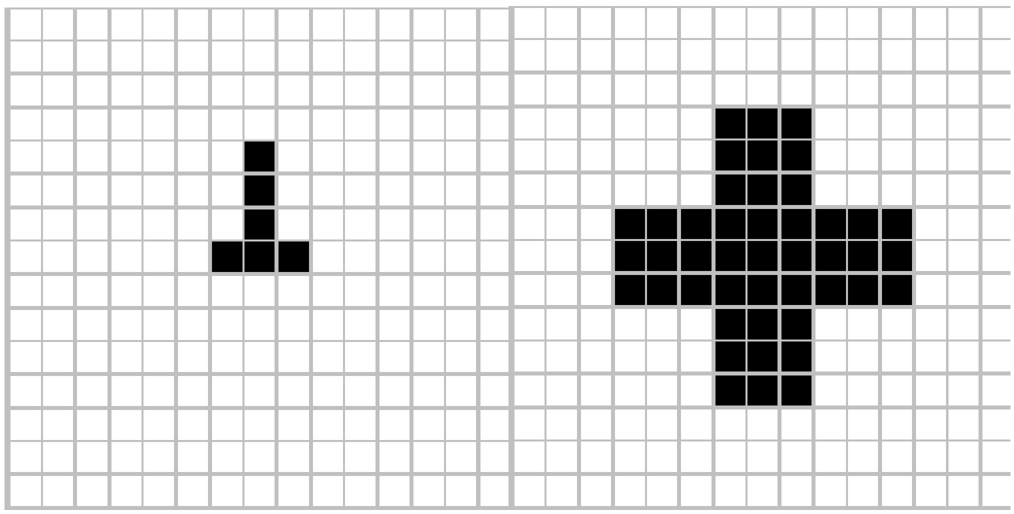
b) What is the Fourier transform of the filter h_1 below (1)

0 0 1
0 $\underline{0}$ 0
0 0 0

c) What is the effect of this filter (what happens to the image)? (1)

d) Suggest a filter h_2 so that the Fourier transform of $h_3 = h_1 * h_2$ equals 1 (2)

3a) Compute numerically a second order moment that is invariant to translation and scale and which is the most effective for discriminating between the shape of the two objects below. (6p)



b) What can moments in general be used for? (1p)

c) Why is the moment $u_{2,0} + u_{0,2}$ particularly useful? (1p)

d) What is the main difference between moments and Fourier descriptors? (1p)

e) Using the real valued Fourier descriptors based on a radius versus angular representation, how can you ensure that you have chosen the optimal midpoint of the contour to be analyzed? (1p)

4a) Given the 5 X 3 image below, compute the Gray Level Co-occurence Matrices for the horizontal and vertical directions, respectively, and distance = 1. (4p)

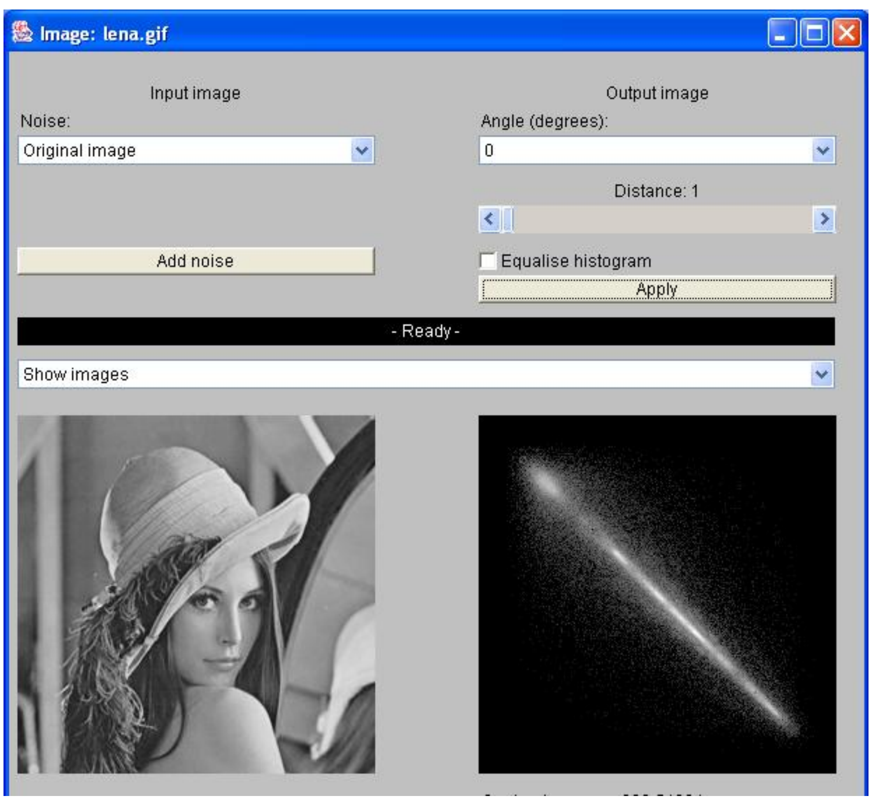
b) Compute the texture feature Contrast for both matrices. (2p)

Image
3 5 4 3 4
3 5 4 3 4
3 5 4 3 4

c) Interpret this result by describing the image with respect to its frequency characteristics (low, medium, or high frequency) in the horizontal and vertical directions, respectively. (1p)

The applet below presents the GLCM matrix for the horizontal direction and distance = 1. Illustrate in principle (by a simple sketch) how the GLCM will change if you

d) increase the distance to 8. (1p)
e) add noise with mean = 0 and variance = 20 (1p)
f add noise with mean = 20 and variance = 0 (1p)



5) A 4 x 7 image is shown below. As can be seen, the image consists of “concentric” layers. The coordinates of the inner layer is (1,4), (1,5), (1,6), and (1,7). For each position x,y in the image there is an associated cost c, e.g. c(2,1) = 7. The problem is to find the path from the inner layer to the outer layer (three travel steps is required for each path) so that the accumulative cost is minimized. Each layer may only be visited once. The allowed travel directions are north, east, and north-east. There is a penalty term p = 1 associated with the travel directions north and east. For the travel direction north-east p = 0. The result should present the optimal path, e.g by specifying the four coordinate pairs of this path. It should also present the values of the accumulated travel costs in the form of a 4 x 7 accumulation matrix with clearly indicated back tracing pointers. (10p)

	1	2	3	4
1	4	7	8	8
2	6	5	5	6
3	3	6	6	4
4	6	6	6	7
5	5	4	4	6
6	7	4	5	7
7	6	5	3	8

Problem 6

a) Describe one important difference between the snake and dynamic programming algorithm (1p)

b) Describe the difference between the moment and Fourier descriptor methods for object recognition (1p)

c) For optimal thresholding, assuming an additive mixture of two Gaussian distributions, describe the meaning of the parameters P_1 and P_2. (1p)

d) Describe the difference between Active Contour Models (snakes) and Active Shape Models (smart snakes). (1p)

e) Describe a disadvantage with the median filter (1p)

f) Describe the brightness constraint of the optical flow algorithm. (1p)

g) Describe the smoothness constraint of the optical flow algorithm. (1p)

h) What is the dimension of the GLCM matrix given that the image size is 1024 x 1024 and there are 12 bits per pixel? (1p)

i) How come there are two threshold parameters in the Canny edge detector? (1p)

j) Why is it sometimes preferable to apply filtering in the Fourier domain rather than the image domain? (1p)