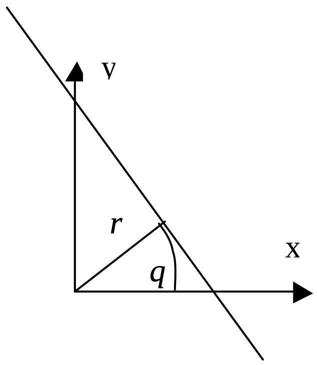
Boundary detection & region representation, texture analysis

Exercise 1 (Hough transform)

After performing edge detection on a 4x4 image suspected to contain a number of straight edges or lines, 7 pixels were found to lie on sufficiently strong edges, their coordinates are: (2,2),(2,1),(2,-1),(2,-2),(1,-1),(-2,2), and (-2,-2)
The equation used to describe a line is:

$$x \cos(q) + y \sin(q) = r$$

where x,y are the coordinates of a point on the line and the parameters q and r are defined as in the figure:



You are required to detect the two strongest lines in the image using the Hough Transform ($[r_1, q_1]$ and $[r_2, q_2]$).

Note: Use the following quantized values for r and q, respectively:
[-3 -2 -1 0 1 2 3] and [0 π/4 π/2 3π/4].

Exercise 2 (Moments)

a) Define the moments of order i,j of the function f(x,y)

b) Determine numerically the moment of order (1,0) for the 2 images below:

	0	1	2	3	4	5	6	7	8
0				3					
1		1	1	1	1	1			
2				2					
3			1	1	1				
4				2					
5									

	0	1	2	3	4	5	6	7	8
0							3		
1					1	1	1	1	1
2							2		
3						1	1	1	
4							2		
5									

c) If your solution in b) is correct, you can see that you get different values although the object in the two images is one and the same.

Suggest a modified version of the moments that in this case will give the same numerical value for the moment of order (2,0) and determine this value

Exercise 3 (Moments & CLCM)

Show that the central moments $\mu_{1,0}$ and $\mu_{0,1}$ always are invariant and equals to zero

Exercise 4 (Moments & GLCM)

The moments of order i,j of the function f(x,y) are defined as:

$$m_{i,j} = \sum_{x}\sum_{y} x^{i} y^{j} f(x,y)$$

a) Define the central moments

b)Why are they more useful ?

c) Determine numerically the central moment $\mu_{2,2}$ for object 1 specified below and for the same objects but shifted one pixel in the x and y directions respectively:

	0	1	2	3
0				
1			1	
2				2

d) Also compute moment $\mu_{2,2}$ for object 2 which is a rotated version of object 1. Notice that in object 2, in order to keep the distance between object points identical, the pixel with value 2 has x-coordinate $2+\sqrt{2}$:

	0	1	2	3	
0					
1			1		2
2					

e) Explain the results you get in c) and d)

Exercise 5 (Moments & GLCM)

a) Consider the image below and compute the sum of the central moments $\mu_{2,0} + \mu_{0,2}$:

	0	1	2	3
0			1	
1		2	3	2
2			1	
3				

b) Why may this sum be an interesting region feature?

c) Assume that each pixel value of the image is subject to additive noise which magnitude equals ε. Determine the largest value of ε that does not affect the numerical value of the feature $\mu_{2,0} + \mu_{0,2}$. The pixel values are assumed to be real but the value of the feature should be rounded to nearest integer.

d) Using Fourier descriptors based on the radius versus angular representation, how can you ensure that you have chosen the optimal midpoint?

e) Compute the contrast feature of the GLCM (Gray-Level Cooccurence Matrix) below:

5	2	1	0
2	4	3	1
1	3	4	1
0	1	1	5

f) Is this GLCM associated with high or low spatial frequencies? Motivate your answer.

Exercise 6 (GLCM)

a) Construct cooccurence matrices (second-order joint probabilities) for the image below. Take the directions 0, $\pi/4$, $\pi/2$, $3\pi/4$ and the distance $|d|=1$ into account:

0	0	1	1
0	0	1	1
0	2	2	2
2	2	3	3

b) Describe an application area for cooccurence matrices

c) Describe the characteristics of the cooccurence matrices, given an image with high spatial frequencies in the horizontal direction and low frequencies in the vertical direction

d) Define a cooccurence matrix feature capable of discriminating low-frequency images from high-frequencies images

e) Describe a Fourier-based feature for the same task as in d)

Exercise 7 (Region representation)

Consider the 3x3 image region below. There is reason to believe that the region may be modeled by a linear plance. Apply a least square solution to find the parameters of this plane:

17	23	40
15	19	30
10	16	26

Also, use the numerical value of the plane parameters to determine the slope and orientation of the plane.
Hint: the equation for a linear plane can be written:

$$g(x,y) = a_0 + a_1x + a_2y$$

Exercise 8 (GLCM)

An image (see below) is analyzed with respect to its spatial frequencies in the horizontal and vertical directions, respectively:

1	2	0	3
1	2	0	3
1	3	0	2
1	3	0	2

a) Determine the autocorrelation function r(a,b) for the lags (a,b) = (1,0), (2,0), (3,0), (0,1), (0,2), and (0,3)

b) Interpret the results and describe what in general can be understood from the autocorrelation function

c) Construct cooccurence matrices (2^{nd} order joint probabilities) for the same image as in a). Use the appropriate directions (for this problem) and the distance |d|=1

d) Interpret the results and describe what in general can be understood from the cooccurence matrices

e) Apply a quantitative texture feature extracted from the matrices in c) and describe numerically the difference as it comes to spatial frequencies in the horizontal and vertical directions

Exercise 9 (GLCM)

A 2-bit 4x4 image region is given by:

4	5	4	6
7	6	5	5
6	5	4	5
7	5	6	4

Let u be a random variable representing a gray level in a given region of an image.

a) Define $p_u(x) \triangleq \mathrm{Prob}\left[u = x\right]$

b) Compute $p_u(5)$

c) Define the second order joint probability
$p_{u_1, u_2}(x_1, x_2) \triangleq \mathrm{Prob}\left[u_1 = x_1, u_2 = x_2\right]$

d) Compute $p_{u1,u2}(5,6)$ if $u_1=u(m,n)$ and $u_2=u(m+1,n+1)$ (no symmetry required)

e) Compute the texture feature contrast from the second order joint probabilities

f) Second order joint probabilities are useful in texture analysis. Mention two other common texture analysis methods (no mathematical definitions are needed)

Exercise 10 (GLCM)

Two images will be compared with respect to the amount of high spatial frequencies in the horizontal direction:

0	0	0	1
2	3	2	2
1	1	1	1
3	3	3	2

1	3	2	1
2	0	3	1
2	3	0	1
0	3	2	1

a) Apply a cooccurence matrices (second order joint probabilities, horizontal distance |d|=1) including an appropriate feature to carry out this comparison in a quantitative way

b) Describe in principle how the Fourier transform can be used for solving the same task as in a) and define a suitable Fourier-based feature which can be used for a similar quantitative comparison

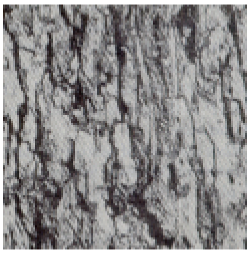
c) Two images have identical autocorrelation functions. Do these images have do be identical? Motivate your answer.

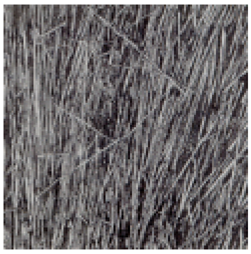
d) Describe how we can extract a first order grayscale difference statistic from the cooccurence matrices

Exercise 11 (GLMC & Fourier spectrum)

On the next page you can see nine images. Images labeled a, b, c depict peebles, bark and straw, respectively. Then you see pictures of the second order statistics captured by GLCM for a horizontal distance of one pixel. The brightnedd corresponds to the value of the GLCM elements. The next images illustrate the amplitude of the Fourier spectrum.


a)

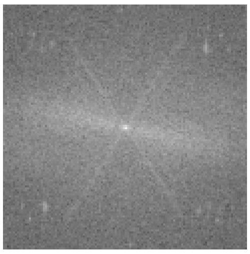

b)


c)

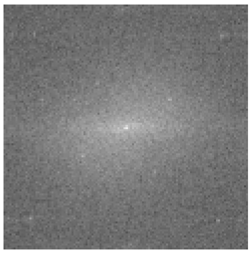

d)


e)


f)


g)


h)


i)

a) Find for each labeled image its corresponding GLCM and Fourier spectrum images. You get 1 point for each correct match, and 1 extra point per image if you provide an appropriate motivation. But you lose the points for that set of image if you provide a wrong motivation!

b) Use the two images below to show that the autocorrelation function is not necessarily a unique representation of an image:

Image 1				Image 2		
3	10	3		1	6	9

c) For the image below, compute the value of the GLCM texture feature useful for discriminating between low and high frequencies in the horizontal direction

0	0	0	1
2	3	2	2
1	1	1	1
3	3	3	2

Exercise 12 (Dynamic programming)

3	1	1	2	2
3	5	1	4	3
1	2	4	1	5
1	2	1	2	1

Given the 4x5 pixels image below,

Use dynamic programming to derive the optimum continuous line P with the following criteria:

-P is a raw vector composed of 5 pixels

-P minimizes the function below:

$$f(P) = -\sum_{x=1}^{5}(C_{grad}(p_x) + C_{int}(p_x)) + \sum_{x=2}^{5} C_{cont}(p_x, p_{x-1})$$

$$(p_1 \ldots, p_5 \in P)$$

where:

C_{grad} is computed using operator $\begin{bmatrix} 1 \\ \boxed{0} \\ -1 \end{bmatrix}$

C_{int} is computed using operator $\begin{bmatrix} 1 \\ [1] \end{bmatrix}$

i.e., the average of two adjecent pixels in the vertical direction

C_{cont} is computed using $(\Delta y)^2$, where Δy is the difference of y coordinates of two adjacent pixels on the line

Tip: for the computation of C_{grad} and C_{int}, at border pixels, add extra rows [3 1 1 2 2] and [1 2 1 2 1] on top and bottom of the image, respectively

Exercise 13 (Moments)

The central moments $\mu_{2,0}$ for the object below equals 2/3. What is the value of the pixel at (x=3, y=1) ?

	0	1	2	3
0				
1			1	?
2				