Exercise 1

a) Convolve the two filters f1 and f2 presented below so as to produce the new filter f3

f_1

1
2
1

f_2

1 2 1

b) Find the Fourier transform of f3 by carrying out the transform directly on the convolution kernel

c) Find the Fourier transform of f3 by first carrying out the transform of f1 and f2

d) What is the Fourier transform of the median filter? Motivate your answer.

Exercise 2

a) Derive and plot the Fourier transform of the mean filter defined by the operator:

	1/8	
1/8	**1/2**	**1/8**
	1/8	

b) Apply this filter to the input image below and compute the result (assume zeros outside the image, and round to nearest integer):

1	**1**	**1**
1	**8**	**1**
1	**1**	**1**

c) Apply a 3x3 median filter to the same image and compute the result

d) Describe (in words) in which situations the median filter may be superior to the mean filter

Exercise 3

a) The 2 filters h1 and h2 are convolved so as to produce the filter h3.
Determine the coefficients a, b, and c:

h1

1	2	1
2	4	2
1	2	1

h2

c	b	c
b	a	b
c	b	c

h3

	1	2	1	
1	8	14	8	1
2	14	24	14	2
1	8	14	8	1
	1	2	1	

b) The Fourier transform of a spatial filter is (sampling distance=1):

$$a\cos(u) + b\cos(v) + c\cos(u+v) + d\cos(u-v) + e\cos(2u) + f\cos(2v) + g$$

(where u and v are the spatial frequencies in the horizontal and vertical directions, respectively)
Determine the coefficients of the spatial filter and present it as an nxm operator in the way the operators are presented in a)

c) Apply a 3x3 median filter to the image below:

1	2	3
4	5	6
7	8	9

Exercise 4

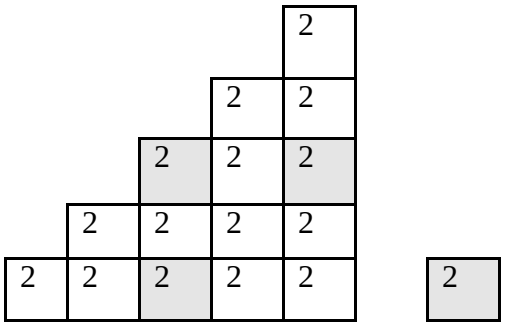
a) The two filters h1 and h2 should be convolved so as to produce the filter h3.
h1 is defined as:

1	2	1
2	4	2
1	2	1

h2 has the Fourier transform *(sampling distance=1):*
$4 + 2\cos(u) + 2\cos(v)$
(where u and v are the spatial frequencies in the horizontal and vertical directions)
Determine the coefficients of filter h3 and present it as an operator similar to h1

b) Find the Fourier transform of h3

Exercise 5



a) Assume 0 outside the image, and sampling freq. be 1.

Convolve the image at the pixels filled with gray using the discrete approximation of the Laplacian:

$$\begin{matrix} & 1 & \\ 1 & -4 & 1 \\ & 1 & \end{matrix}$$

b) Comment and interpret your results

c) Determine the Fourier transform of the Laplacian above

d) Compute the magnitude of the transform you found at the frequencies:
$(\pi, 0)$ and (π, π)

e) Comment and interpret your results

Exercise 6

a) An image with additive gaussian noise will be thresholded after a gradient operation. Considering the noise, the gradient image will be created in 2 steps, using two of the following filters. Select two of the kernels for the operation, and motivate your choice:

h_1 $\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ h_2 $\begin{bmatrix} & 1 & \\ 1 & -4 & 1 \\ & 1 & \end{bmatrix}$ h_3 $\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ h_4 $\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ h_5 $\begin{bmatrix} & 1 & \\ 1 & 4 & 1 \\ & 1 & \end{bmatrix}$ h_6 $\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

b) Show how this 2-step filtering scheme can be replaced a single-step filtering scheme, by using these 2 kernels to create a new filter

c) Compute the coefficients of this new filter kernel.

d) What are the characteristics of this new filter in the frequency domain ?
(give the filter type or describe it with words)

e) Show that for a 512x512 pixels image, the number of mul Show that for a 512x512 pixels image, the number of multiplication operations is less if the filtering is computed in the image plane rather than in the frequency domain, given that the number of non-zero coefficients in the kernel is less than 40.

Exercise 7

a) Derive and plot the Fourier transform of the filter defined by the operator below:

$$\begin{matrix} & 1/8 & \\ 1/8 & 1/2 & 1/8 \\ & 1/8 & \end{matrix}$$

b) Apply this filter to the input image below and compute the result (assume zeros outside the image, and round to nearest integer):

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 8 & 1 \\ 1 & 1 & 1 \end{matrix}$$

c) Apply a 3x3 median filter to the same image and compute the result

d) Comment upon the result of b) and c) and mention one advantage and one disadvantage of the median filter as compared to the mean filter.

Exercise 8

a) Show that applying the Laplace operator shown below to an image is equivalent (with except for a proportional factor) to locally subtracting a five point mean from the each original value of that image

$$\begin{matrix} & 1 & \\ 1 & -4 & 1 \\ & 1 & \end{matrix}$$

b) Show that local thresholding of an image can be replaced by global thresholding if the image has been filtered by a Laplace operator first.

c) In optical microscopy, let f_k be the 'true' image of a substance at depth k. Give a possible way to obtain f_k given that we have access to the digitized non-ideal unfocused images g_k, g_{k-1}, and g_{k+1}.

Exercise 9

a) Assume that we apply a linear filtering two times consecutively. Show that 2 linear filters with impulse response h_1 and h_2 applied consecutively can always be replaced by a filter h_3

b) Compute h_3 given that $h_1 = h_2$ and have the form given below:

$$h_1 = h_2 = \begin{bmatrix} & 1 & \\ 1 & 4 & 1 \\ & 1 & \end{bmatrix}$$

c) The output should in fact be in the form:

$$\begin{matrix} & 1/k & \\ 1/k & 4/k & 1/k \\ & 1/k & \end{matrix}$$

What is k meant for and what value should it have?

d) Assume that we would like to detect vertical edges in an image corrupted by noise. Give and motivate your choice for a 3x3 filter that would be applied before edge detection