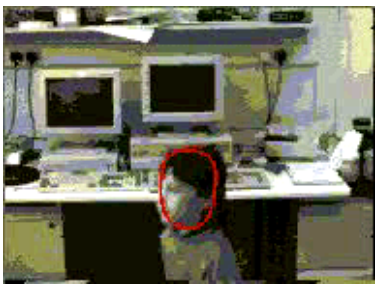
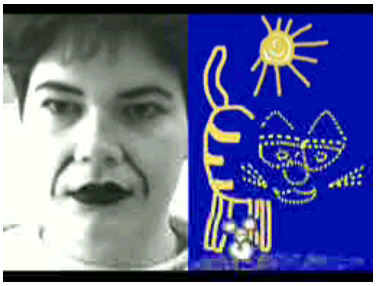
Motion Analysis

Example of applications

Security: tracking intruders

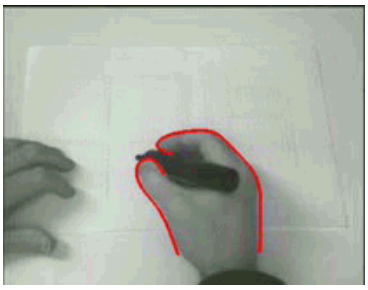


Actor-driven facial animation

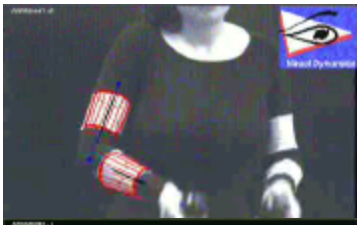


Automated video editing

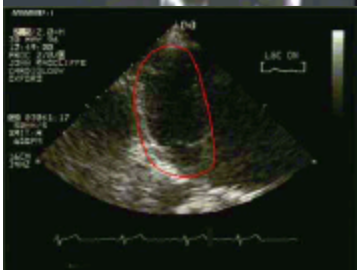
Computer user interaction



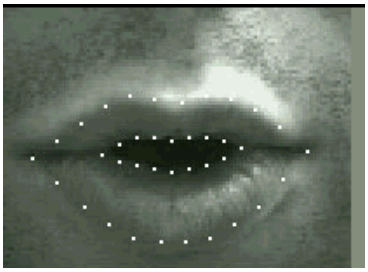
Body motion modeling



Medical applications: Echocardiogram



Audio-visual speech analysis



Optical Flow Computation

In what follows, the optical flow algorithm for measuring motion in images will be presented

Consider a time-varying image $f(x,y,t)$

Constraint 1: Brightness of any object point should be constant over time, i.e.

$$f(x+dx,y+dy,t+dt)=f(x,y,t)$$

Taylor series expansion
(neglecting higher order terms for small *dx*, *dy*, *dt*)

$$f(x,y,t)+f_x dx+f_y dy+f_t dt+o(\partial^2)=f(x,y,t)$$

$$u = \frac{dx}{dt}, v = \frac{dy}{dt}$$

$$f_x u + f_y v + f_t = 0$$

Because we have only one equation and have to resolve two unknowns (the so called aperture problem), the velocity variables u and v, we need an additional constraint.
The velocity smoothness
constraint means that two adjacent points in the image should have similar motion with respect to magnitude and direction. The second term of the equation below takes care of that.

The problem now is to minimize the square error

$$E^2(x,y) = \left(f_x u + f_y v + f_t\right)^2 + \lambda\left(u_x^2 + u_y^2 + v_x^2 + v_y^2\right)$$

Iterative solution

u(i,j)=0, v(i,j)=0, for all i, j

$$u = \bar{u} - f_x \frac{P}{D} \qquad v = \bar{v} - f_y \frac{P}{D}$$
$$P = f_x \bar{u} + f_y \bar{v} + f_t \qquad D = \lambda^2 + f_x^2 + f_y^2$$
$$E^2(x,y) > \varepsilon$$

Algorithm for dynamic image pairs
1 – Initialize velocity vectors c(i,j)=0 for all (i,j)
2 – Let k denote the number of iterations. Compute u^k, v^k, for all pixels (i,j):

$$u^k(i,j) = \bar{u}^{k-1}(i,j) - f_x(i,j)\frac{P(i,j)}{D(i,j)}$$

$$v^k(i,j) = \bar{v}^{k-1}(i,j) - f_y(i,j)\frac{P(i,j)}{D(i,j)}$$

The partial derivatives f_x, f_y, f_t can be estimated from the pair of consecutive images.
3 – Stop if:

Where ε is the maximum permitted error; return to step 2 otherwise

Algorithm for image sequence

1 - Evaluate starting values of the optical flow c(I,j) for all (I,j)

2 -Let m denote the sequence number of the currently processed image. For all pixels (i,j) of the next image, evaluate:

$$u^{m+1}(i,j) = \bar{u}^{m}(i,j) - f_x(i,j)\frac{P(i,j)}{D(i,j)}$$

$$v^{m+1}(i,j) = \bar{v}^{m}(i,j) - f_y i,j)\frac{P(i,j)}{D(i,j)}$$

3 – Repeat step 2 to process all images in the sequence