OPTIMAL THRESHOLDING

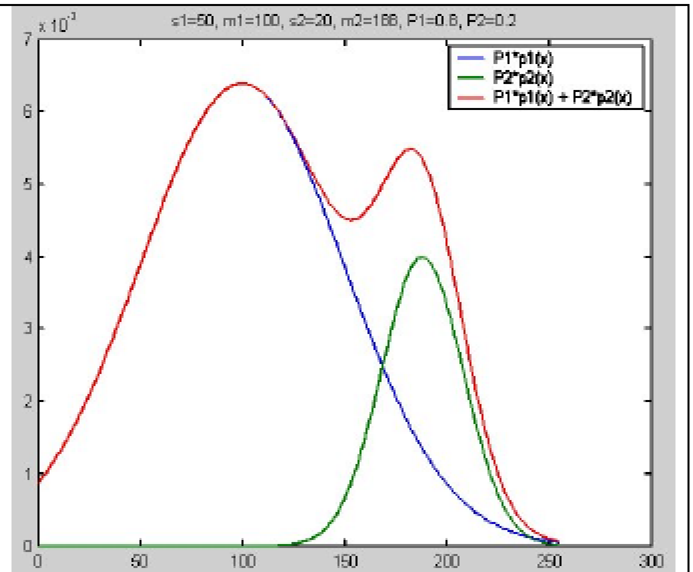
Assume that the original image contains 2 gray-scale components that represent the background and the object, respectively. Further, assume that the sum of these components can be modeled as an additive mixture of Gaussian distributions, i.e.

$$p(x) = P_1 p_1(x) + P_2 p_2(x)$$

$$p_1(x) = \mathrm{Gauss}(\sigma_1, \mu_1)$$
$$p_2(x) = \mathrm{Gauss}(\sigma_2, \mu_2)$$

P_1 and P_2 represents the a priori probabilities for the background and the object, respectively

($P_1 + P_2 = 1$)



The probability of misclassifying an object pixel as a background pixel is:

$$E_1(T) = \int_{-\infty}^{T} p_2(x)\,dx$$

The probability of misclassifying a background pixel as an object pixel is:

$$E_2(T) = \int_{T}^{\infty} p_1(x)\,dx$$

The total probability of misclassification is thus:

$$E(T) = P_2 \cdot E_1(T) + P_1 \cdot E_2(T)$$

Differentiating E(T) with respect to *T*, and setting to 0, yields:

$$P_1 \cdot p_1(T) = P_2 \cdot p_2(T)$$

Taking the log of the expression, and simplifying, we obtain a 2nd degree equation:

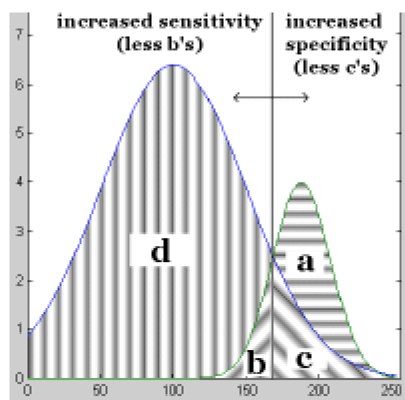
$$AT^2 + BT + C = 0 \quad \begin{cases} A = \sigma_1^2 - \sigma_2^2 \\ B = 2\left(\mu_1\sigma_2^2 - \mu_2\sigma_1^2\right) \\ C = \sigma_1^2\mu_2^2 - \sigma_2^2\mu_1^2 + \sigma_1^2\sigma_2^2 \ln(\sigma_1 P_1/\sigma_2 P_2) \end{cases}$$

If the variances are equal $\sigma_1^2 = \sigma_2^2 = \sigma^2$), then we get a single solution for *T* (otherwise, we get 2 solutions):

$$T = \frac{\mu_1 + \mu_2}{2} + \frac{\sigma^2}{\mu_1 - \mu_2}\ln(P_2/P_1)$$

If P_1=P_2, then T is the mean of the mean values: $T = \dfrac{\mu_1 + \mu_2}{2}$

a=true positive (true object)
b=false negative (false background)
c=false positive (false object)
d=true negative (true background)
Sensitivity=a/(a+b)
Specificity=d/(c+d)



Increased sensitivity: You will miss less true object pixels but the price you pay is more false object pixels. This is good if you do not want to miss object pixels

Increased specificity: You will get less false object pixels but may miss true object pixels. This is good if you do not want to detect false object pixels