Exercise 1 (Hough transform)
the histogram (hit/hough/accumulation) matrix is constructed:

	-3	-2	-1	0	1	2	3
0							
$\pi/4$							
$\pi/2$							
$3\pi/4$							

For each point on a strong edge we find the lines that it may belong to:
For the point P1(2,2) and q :

0: r =2*cos(0)+2*sin(0)=2 ==> line [2,0]
$\pi/4$: r =2*cos($\pi/4$)+2*sin($\pi/4$)=2.8284~3 ==> line [3,$\pi/4$]
$\pi/2$: r =2*cos($\pi/2$)+2*sin($\pi/2$)=2 ==> line [2,$\pi/2$]
$3\pi/4$: r =2*cos($3\pi/4$)+2*sin($3\pi/4$)=0 ==> line [0,$3\pi/4$]

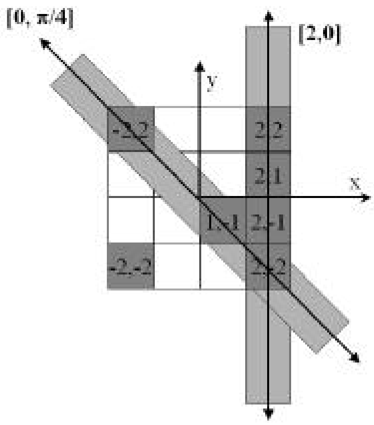
So the histogram matrix is updated as follows:

	-3	-2	-1	0	1	2	3
0						/	
$\pi/4$							/
$\pi/2$						/	
$3\pi/4$				/			

Finding the possible lines and updating the histogram matrix for the remaining 5 points:

	-3	-2	-1	0	1	2	3
0		//			/	////	
$\pi/4$	/			///	/	/	/
$\pi/2$		//	//		/	//	
$3\pi/4$	/	/	//	//			/

Identifying the two lines with the maximum number of hits:
Line 1: [r_1,q_1]=[2,0], Line 2: [r_2,q_2]=[0,$\pi/4$]
Examining the detected lines on the image with the strong edge points, we obtain agreeable results:



Exercise 2 (Moments)

$$m_{i,j} = \sum_x \sum_y x^i y^j f(x,y)$$

b) Image 1:

$$\begin{aligned} m_{1,0} &= 3\cdot 3 + \\ &\quad 1\cdot 1 + 2\cdot 1 + 3\cdot 1 + 4\cdot 1 + 5\cdot 1 + \\ &\quad 2\cdot 3 + \\ &\quad 1\cdot 2 + 1\cdot 3 + 1\cdot 4 + \\ &\quad 2\cdot 3 \\ &= 45 \end{aligned}$$

Image 2: $m_{1,0} = 90$

c) Central moments, invariant by translation:

$$\mu_{i,j} = \sum_x \sum_y (x-\bar{x})^i (y-\bar{y})^j f(x,y)$$

where $\bar{x} = \frac{m_{1,0}}{m_{0,0}}$ and $\bar{y} = \frac{m_{0,1}}{m_{0,0}}$

Image 1: $\mu_{2,0} = 12$

Image 2: $\mu_{2,0} = 12$

Exercise 3 (Moments & GLCM)

$$\begin{aligned}
\mu_{1,0} &= \sum_x \sum_y (x-\bar{x})^1 (y-\bar{y})^0 f(x,y) \\
&= \sum_x \sum_y (x-\bar{x}) f(x,y) \\
&= \underbrace{\sum_x \sum_y x\, f(x,y)}_{m_{1,0}} - \sum_x \sum_y \bar{x}\, f(x,y) \\
&= m_{1,0} - \underbrace{\bar{x}}_{m_{1,0}/m_{0,0}} \underbrace{\sum_x \sum_y f(x,y)}_{m_{0,0}} \\
&= m_{1,0} - \frac{m_{1,0}}{m_{0,0}} m_{0,0} = 0
\end{aligned}$$

Exercise 4 (Moments &GLCM)

a)

$$\mu_{i,j} = \sum_x \sum_y (x-\bar{x})^i (y-\bar{y})^j f(x,y)$$

where $\bar{x} = \frac{m_{1,0}}{m_{0,0}}$ and $\bar{y} = \frac{m_{0,1}}{m_{0,0}}$

b) They are invariant to translation

c)

$$\left.\begin{array}{l} m_{0,0} = 1+2=3 \\ m_{1,0} = 2\cdot 1 + 3\cdot 2 = 8 \\ m_{0,1} = 1\cdot 1 + 2\cdot 2 = 5 \end{array}\right\} \Rightarrow \begin{array}{l} \bar{x} = 8/3 \\ \bar{y} = 5/3 \end{array}$$

$$\mu_{2,2} = (2-8/3)^2(1-5/3)^2\cdot 1 + (3-8/3)^2(2-5/3)^2\cdot 2 = 18/81$$

$$\left.\begin{array}{l} m_{0,0} = 1+2=3 \\ m_{1,0} = 3\cdot 1 + 4\cdot 2 = 11 \\ m_{0,1} = 2\cdot 1 + 3\cdot 2 = 8 \end{array}\right\} \Rightarrow \begin{array}{l} \bar{x} = 11/3 \\ \bar{y} = 8/3 \end{array}$$

$$\mu_{2,2} = (3-11/3)^2(2-8/3)^2\cdot 1 + (4-11/3)^2(3-8/3)^2\cdot 2 = 18/81$$

d)

$$\left.\begin{array}{l} m_{0,0} = 1+2=3 \\ m_{1,0} = 2\cdot 1 + (2+\sqrt{2})\cdot 2 = 6+2\sqrt{2} \\ m_{0,1} = 1\cdot 1 + 1\cdot 2 = 3 \end{array}\right\} \Rightarrow \begin{array}{l} \bar{x} = \frac{6+2\sqrt{2}}{3} \\ \bar{y} = 1 \end{array}$$

$$\mu_{2,2} = (2-\frac{6+2\sqrt{2}}{3})^2(1-1)^2\cdot 1 + (2+\sqrt{2}-\frac{6+2\sqrt{2}}{3})^2(1-1)^2\cdot 2 = 0$$

e)

The central moments $\mu_{p,q}$ are always invariant under translation whereas they are not invariant under rotation

Exercise 5 (Moments & GLMC)

a)

$$\mu_{i,j} = \sum_x \sum_y (x - \bar{x})^i (y - \bar{y})^j f(x,y)$$

where $\bar{x} = \frac{m_{1,0}}{m_{0,0}}$ and $\bar{y} = \frac{m_{0,1}}{m_{0,0}}$

$\bar{x} = 2$ and $\bar{y} = 1$

$$\left.\begin{array}{l} \mu_{2,0} = 4 \\ \mu_{0,2} = 2 \end{array}\right\} \quad \mu_{2,0} + \mu_{0,2} = 6$$

b) It is invariant under rotation

c) $\bar{x}$ and $\bar{y}$ will not be affected by the noise term

$$\left.\begin{array}{l} \mu_{2,0} = 4 + 2\varepsilon \\ \mu_{0,2} = 2 + 2\varepsilon \end{array}\right\} \quad \mu_{2,0} + \mu_{0,2} = 6 + 4\varepsilon$$

$$4\varepsilon < 0.5 \Rightarrow \varepsilon < 0.5/4 = 0.125$$

d) The magnitude of the first harmonic should be zero

e) $2 \cdot (2+3+1) + 4 \cdot 2^2 \cdot 1 = 28$

f) It is associated with low frequencies because most of the values are along the main diagonal

Exercise 6 (GLCM)

a)

$$0: \begin{matrix} 4 & 2 & 1 & 0 \\ 2 & 4 & 0 & 0 \\ 1 & 0 & 6 & 1 \\ 0 & 0 & 1 & 2 \end{matrix} \qquad \pi/4: \begin{matrix} 4 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 \\ 0 & 2 & 4 & 1 \\ 0 & 0 & 1 & 0 \end{matrix}$$

$$\pi 2: \begin{matrix} 6 & 0 & 2 & 0 \\ 0 & 4 & 2 & 0 \\ 2 & 2 & 2 & 2 \\ 0 & 0 & 2 & 0 \end{matrix} \qquad 3\pi/4: \begin{matrix} 2 & 1 & 3 & 0 \\ 1 & 2 & 1 & 0 \\ 3 & 1 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{matrix}$$

b) Texture analysis

c) horizontal direction: values are spread out
vertical direction: values are concentrated on the diagonal

d) Contrast $\sum_{i,j}(i-j)^2 p(i,j)$

e)

$$\int_0^{2\pi}\int_0^{r'} F(r,\varphi)\,dr\,d\varphi$$

This feature measures the energy in the low-frequency range of the Fourier spectrum

Exercise 7 (Region representation)

Let ε denote the error between data f(x,y) and model g(x,y). By the least square solution, we would like to minimize the squared error:

$$\varepsilon^2 = \sum\left[f(x,y) - g(x,y)\right]^2 = \sum\left[f(x,y) - a_0 - a_1x - a_2y\right]^2$$

We determine the parameters a_0, a_1 and a_2 by solving the following equations:

$$\frac{\partial\varepsilon^2}{\partial a_0} = 0 \quad \frac{\partial\varepsilon^2}{\partial a_1} = 0 \quad \frac{\partial\varepsilon^2}{\partial a_2} = 0$$

$$\begin{aligned}\frac{\partial\varepsilon^2}{\partial a_0} &= 2\cdot\sum\left[f(x,y) - a_0 - a_1x - a_2y\right] \\ &= 2\cdot\sum f(x,y) - 2\underbrace{\sum a_0}_{Na_0} - 2a_1\underbrace{\sum x}_{0} - 2a_2\underbrace{\sum y}_{0}\end{aligned}$$

$$a0 = \frac{1}{N}\sum f(x,y) \text{ (mean intensity)}$$

(N is the number of pixels)

$$\begin{aligned}\frac{\partial\varepsilon^2}{\partial a_1} &= 2\cdot\sum\left[xf(x,y) - a_0x - a_1x^2 - a_2xy\right] \\ &= 2\cdot\sum xf(x,y) - 2a_1\sum x^2 - 2a_0\underbrace{\sum x}_{0} - 2a_2\underbrace{\sum xy}_{0}\end{aligned}$$

$$a_1 = \frac{\sum x\ f(x,y)}{\sum x^2} \text{ and similarly } a_2 = \frac{\sum y\ f(x,y)}{\sum y^2}$$

Notice that a_1 and a_2 can be looked upon as f_x' and f_y', respectively, in the image region:

$$f_x^{'}: \begin{matrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{matrix} \ /6 \qquad\qquad f_y^{'}: \begin{matrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{matrix} \ /6$$

The slope of the plane then equals the gradient magnitude:

$$\left|G_f\right| = \sqrt{f_x^{'2} + f_y^{'2}}$$

The orientation equals the gradient orientation:

$$\theta = \arctan\left(f_y^{'} / f_x^{'}\right) = \angle G_f$$

Numerically, we obtain f_x’ and f_y’ by convolving the image region with the operators above:

$$f_x^{'} = (-17 - 15 - 10 + 40 + 30 + 26)/26 = 54/6$$

$$f_y^{'} = (-10 - 16 - 26 + 17 + 23 + 40)/6 = 28/6$$

$$\left|G_f\right| \approx 9.3 \text{ (grayscale units per length unit)}$$

$$\theta \approx 30^o$$

Exercise 8

a)

$$
\begin{aligned}
r(a,b) &= \frac{L_x L_y}{(L_x - |a|)(L_y - |b|)} \frac{\sum_x \sum_y f(x,y) f(x-a, y-b)}{\sum_x \sum_y f^2(x,y)} \\
r(1,0) &= (1\cdot 2 + 1\cdot 2 + 1\cdot 3 + 1\cdot 3)\frac{16}{12\cdot 56} \approx 0.24 \\
r(2,0) &= (3\cdot 2 + 3\cdot 2 + 2\cdot 3 + 2\cdot 3)\frac{16}{8\cdot 56} \approx 0.86 \\
r(3,0) &= (3+3+2+2)\frac{16}{4\cdot 56} \approx 0.71 \\
r(0,1) &= (1+9+4+1+6+6+1+4+9)\frac{16}{12\cdot 56} \approx 0.98 \\
r(0,2) &= (1+6+6+1+6+6)\frac{16}{8\cdot 56} \approx 0.93 \\
r(0,3) &= (1+6+6)\frac{16}{4\cdot 56} \approx 0.93
\end{aligned}
$$

b) The autocorrelation function drops off more quickly in the horizontal direction which means we have higher frequencies

c) Horizontal

	0	1	2	3
0	0	0	4	4
1	0	0	2	2
2	4	2	0	0
3	4	2	0	0

Vertical

	0	1	2	3
0	6	0	0	0
1	0	6	0	0
2	0	0	4	2
3	0	0	2	4

d) The CLCM computed for the horizontal direction has a lot of values outside the diagonal: high frequencies in the horizontal direction. Vice-versa for GLCM computed for the vertical direction.

e) Contrast:

Horizontal direction:

$$
\begin{aligned}
\sum_i \sum_j (i-j)^2 p(i,j) &= (2^2\cdot 4 + 3^2\cdot 4 + 1^2\cdot 2 + 2^2\cdot 2)\cdot 2 \\
&= (4\cdot 4 + 9\cdot 4 + 1\cdot 2 + 4\cdot 2)\cdot 2 \\
&= (16+36+2+8)\cdot 2 \\
&= 124
\end{aligned}
$$

Vertical direction:

$$\sum_i \sum_j (i-j)^2 p(i,j) = 1^2\cdot 2 + 1^2\cdot 2 = 4$$

Exercise 9 (GLCM)

a)

$$p_u(x) \triangleq \text{Prob}[u = x] \approx \frac{\text{number of pixels with graylevel x}}{\text{total number of pixels in the region}}$$

b)

$p_u(5)=3/8$

c)

$$p_{u_1,u_2}(x_1, x_2) \triangleq \text{Prob}[u_1 = x_1, u_2 = x_2] \approx$$

$$\frac{\text{number of pair of pixels } u_1=x_1,\ u_2=x_2}{\text{total number of such pairs of pixels in the region}}$$

d)

$p_{u1,u2}(5,6)=1/9$

(there are 9 such pixel pairs out of which only one with $u_1=5$ and $u_2=6$)

e)

Second order joint probabilities: $\frac{1}{9}\cdot\begin{vmatrix}1 & 1 & 1 & 0\\ 0 & 2 & 1 & 0\\ 1 & 1 & 0 & 0\\ 0 & 1 & 0 & 0\end{vmatrix}$

$$\text{Contrast} \equiv \sum_{X_1}\sum_{X_2}(X_1 - X_2)^2 p_{u_1,u_2}(X_1, X_2)$$

$$= 3\cdot 1^2\cdot 1 + 3\cdot 2^2\cdot 1 = 15$$

or $15/9$ if the matrix is normalized

f) Autocorrelation and features from the Fourier domain

Exercise 10 (GLCM)

a) GLCM for dist=1:

$$\begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 4 & 1 & 0 & 0 \\ 1 & 1 & 6 & 0 & 0 \\ 2 & 0 & 0 & 2 & 3 \\ 3 & 0 & 0 & 3 & 2 \end{array} \qquad \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & 1 & 1 & 3 \\ 1 & 1 & 0 & 2 & 2 \\ 2 & 1 & 2 & 0 & 3 \\ 3 & 3 & 2 & 3 & 0 \end{array}$$

The appropriate feature should be contrast $\sum_{i} (i-j)^2 p(i,j)$

Image 1:

$2 \cdot (1^2 \cdot 1) + 2 \cdot (1^2 \cdot 3) = 8$

Image 2:

$2 \cdot (1^2 \cdot 1) + 2 \cdot (2^2 \cdot 1) + 2 \cdot (3^2 \cdot 3) + 2 \cdot (1^2 \cdot 2) + 2 \cdot (2^2 \cdot 2) + 2 \cdot (1^2 \cdot 3) = 90$

b) A suitable feature is: $\int_{r1}^{r2} \int_{\varphi 1}^{\varphi 2} F(r,\varphi) dr d\varphi$ i.e. we compute the energy in a segment of the spectrum

c) No, the autocorrelation function does not cover spectral magnitude and phase simultaneously

d) Sum up the values along diagonals parallel to the main diagonal

$$\begin{array}{|cccc} \hline 0 & 0 & 0 & 1 \\ 2 & 3 & 2 & 2 \\ 1 & 1 & 1 & 1 \\ 3 & 3 & 3 & 2 \end{array}$$

e) Contrast:

Horizontal direction:

$$\begin{aligned}\sum_i \sum_j (i-j)^2 p(i,j) &= (2^2 \cdot 4 + 3^2 \cdot 4 + 1^2 \cdot 2 + 2^2 \cdot 2) \cdot 2 \\ &= (4 \cdot 4 + 9 \cdot 4 + 1 \cdot 2 + 4 \cdot 2) \cdot 2 \\ &= (16 + 36 + 2 + 8) \cdot 2 \\ &= 124\end{aligned}$$

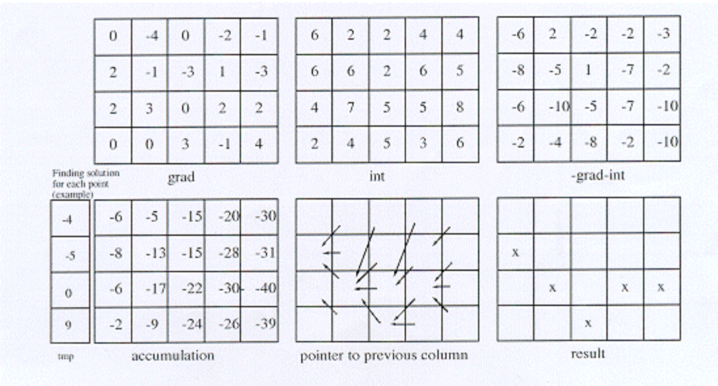
Vertical direction:

$$\sum_i \sum_j (i-j)^2 p(i,j) = 1^2 \cdot 2 + 1^2 \cdot 2 = 4$$

Exercise 11

saknas

Exercise 12


0 -4 0 -2 -1
2 -1 -3 1 -3
2 3 0 2 2
0 0 3 -1 4
grad
6 2 2 4 4
6 6 2 6 5
4 7 5 5 8
2 4 5 3 6
int
-6 2 -2 -2 -3
-8 -5 1 -7 -2
-6 -10 -5 -7 -10
-2 -4 -8 -2 -10
-grad-int
Finding solution for each point (example)
-4
-5
0
9
tmp
-6 -5 -15 -20 -30
-8 -13 -15 -28 -31
-6 -17 -22 -30 -40
-2 -9 -24 -26 -39
accumulation
pointer to previous column
x
x x x
x
result

Exercise 13 (Moments)

$$\mu_{i,j} = \sum_x \sum_y (x - \bar{x})^i (y - \bar{y})^j f(x, y)$$

where $\bar{x} = \dfrac{m_{1,0}}{m_{0,0}}$ and $\bar{y} = \dfrac{m_{0,1}}{m_{0,0}}$

$$\mu_{2,0} = \sum_x \sum_y (x - \bar{x})^2 f(x,y)$$

$$\bar{x} = \frac{2+3a}{1+a}$$

$$\mu_{2,0} = \left(2 - \frac{2+3a}{1+a}\right)^2 + \left(3 - \frac{2+3a}{1+a}\right)^2 \cdot a = \frac{2}{3}$$

$$3\frac{(2+2a-2-3a)^2}{(1+a)^2} + 3\frac{a\cdot(3+3a-2-3a)^2}{(1+a)^2} = 2$$

$$(a \neq -1) \Rightarrow 3a^2 + 3a = 2(1+a)^2 \Rightarrow a^2 - a + 2 = 0 \Rightarrow a = 2$$