a)
$$r(a,b) = \frac{L_x L_y}{(L_x - |a|)(L_y - |b|)} \frac{\sum_x \sum_y f(x,y) f(x - a, y - b)}{\sum_x \sum_y f^2(x,y)}$$

$$r(1,0) = (1 \cdot 2 + 1 \cdot 2 + 1 \cdot 3 + 1 \cdot 3) \frac{16}{12 \cdot 56} \approx 0.24$$

$$r(2,0) = (3 \cdot 2 + 3 \cdot 2 + 2 \cdot 3 + 2 \cdot 3) \frac{16}{8 \cdot 56} \approx 0.86$$

$$r(3,0) = (3 + 3 + 2 + 2) \frac{16}{4 \cdot 56} \approx 0.71$$

$$r(0,1) = (1 + 9 + 4 + 1 + 6 + 6 + 1 + 4 + 9) \frac{16}{12 \cdot 56} \approx 0.98$$

$$r(0,2) = (1 + 6 + 6 + 1 + 6 + 6) \frac{16}{8 \cdot 56} \approx 0.93$$

$$r(0,3) = (1 + 6 + 6) \frac{16}{4 \cdot 56} \approx 0.93$$

b) The autocorrelation function dropps off more quickly in the horizontal direction: higher frequencies

c) Horizontal

 0
 1
 2
 3

 0
 0
 4
 4

 0
 0
 2
 2

 4
 2
 0
 0

2

Vertical

d) The GLCM computed for the horizontal direction has a lot of values outside the diagonal: high frequencies in the horizontal direction. Vice-versa for GLCM computed for the vertical direction

e) Contrast:

Horizontal direction:

$$\sum_{i} \sum_{j} (i - j)^{2} p(i, j) = (2^{2} \cdot 4 + 3^{2} \cdot 4 + 1^{2} \cdot 2 + 2^{2} \cdot 2) \cdot 2$$

$$= (4 \cdot 4 + 9 \cdot 4 + 1 \cdot 2 + 4 \cdot 2) \cdot 2$$

$$= (16 + 36 + 2 + 8) \cdot 2$$

$$= 124$$

Vertical direction:

$$\sum_{i} \sum_{j} (i-j)^{2} p(i,j) = 1^{2} \cdot 2 + 1^{2} \cdot 2 = 4$$