Solution

a) $p_u(x) \triangleq \mathrm{Prob}[u = x] \approx \dfrac{\text{number of pixels with graylevel x}}{\text{total number of pixels in the region}}$

Solution

b) $p_u(5)=3/8$

Solution

c) $\quad p_{u_1, u_2}(x_1, x_2) @ \text{Prob}\left[u_1 = x_1, u_2 = x_2\right] \approx$

$$\frac{\text{number of pair of pixels } u_1 = x_1, u_2 = x_2}{\text{total number of such pairs of pixels in the region}}$$

Solution

d) $p_{u1,u2}(5,6)=1/9$

(there are 9 such pixel pairs out of which only one with $u_1=5$ and $u_2=6$)

Solution

e)   Second order joint probabilities:

$$\frac{1}{9} \cdot \begin{vmatrix} 1 & 1 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix}$$

$$\text{Contrast} \equiv \sum_{X_1} \sum_{X_2} (X_1 - X_2)^2 p_{u_1, u_2}(X_1, X_2)$$

$$= 3 \cdot 1^2 \cdot 1 + 3 \cdot 2^2 \cdot 1 = 15$$

or $15 \big/ 9$ if matrix is normalized

Solution

f)   Autocorrelation and features from the Fourier domain