Solution

a) $$\mu_{i,j} = \sum_x \sum_y (x - \bar{x})^i (y - \bar{y})^j f(x,y)$$

where $\bar{x} = \dfrac{m_{1,0}}{m_{0,0}}$ and $\bar{y} = \dfrac{m_{0,1}}{m_{0,0}}$

Solution

b) They are invariant to translation

Solution

c)

$$\left.\begin{aligned} m_{0,0} &= 1+2=3 \\ m_{1,0} &= 2\cdot 1+3\cdot 2=8 \\ m_{0,1} &= 1\cdot 1+2\cdot 2=5 \end{aligned}\right\} \Rightarrow \begin{aligned} \bar{x} &= 8/3 \\ \bar{y} &= 5/3 \end{aligned}$$

$$\mu_{2,2} = (2-8/3)^2(1-5/3)^2\cdot 1+(3-8/3)^2(2-5/3)^2\cdot 2=18/81$$

$$\left.\begin{aligned} m_{0,0} &= 1+2=3 \\ m_{1,0} &= 3\cdot 1+4\cdot 2=11 \\ m_{0,1} &= 2\cdot 1+3\cdot 2=8 \end{aligned}\right\} \Rightarrow \begin{aligned} \bar{x} &= 11/3 \\ \bar{y} &= 8/3 \end{aligned}$$

$$\mu_{2,2} = (3-11/3)^2(2-8/3)^2\cdot 1+(4-11/3)^2(3-8/3)^2\cdot 2=18/81$$

Solution

d)

$$\left.\begin{aligned} m_{0,0} &= 1 + 2 = 3 \\ m_{1,0} &= 2 \cdot 1 + (2 + \sqrt{2}) \cdot 2 = 6 + 2\sqrt{2} \\ m_{0,1} &= 1 \cdot 1 + 1 \cdot 2 = 3 \end{aligned}\right\} \Rightarrow \begin{aligned} \bar{x} &= \frac{6 + 2\sqrt{2}}{3} \\ \bar{y} &= 1 \end{aligned}$$

$$\mu_{2,2} = (2 - \frac{6 + 2\sqrt{2}}{3})^2 (1 - 1)^2 \cdot 1 + (2 + \sqrt{2} - \frac{6 + 2\sqrt{2}}{3})^2 (1 - 1)^2 \cdot 2 = 0$$

Solution

e) The central moments $\mu_{p,q}$ are always invariant under translation whereas they are not invariant under rotation