Chalmers University of Technology

Active Contour Models & Active Shape Models (Snakes & Smart Snakes)

Image Analysis Group
Chalmers University of Technology

Contents

- Motivation, applications and problem description
- Active Contour Models (Snakes)
- Active Shape Models (Smart Snakes)

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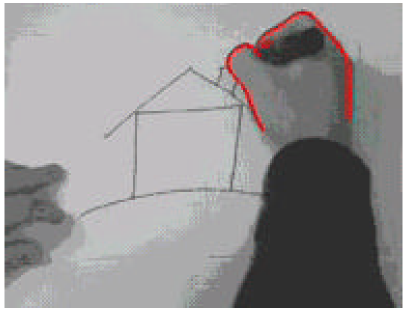
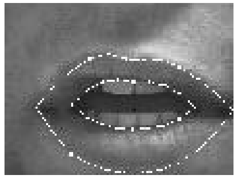
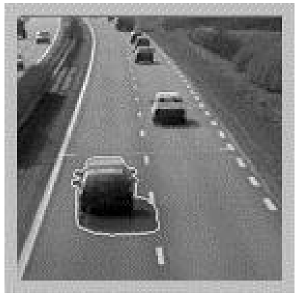
Applications



Human-computer interaction



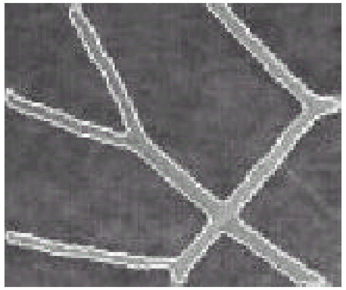
Lip-reading



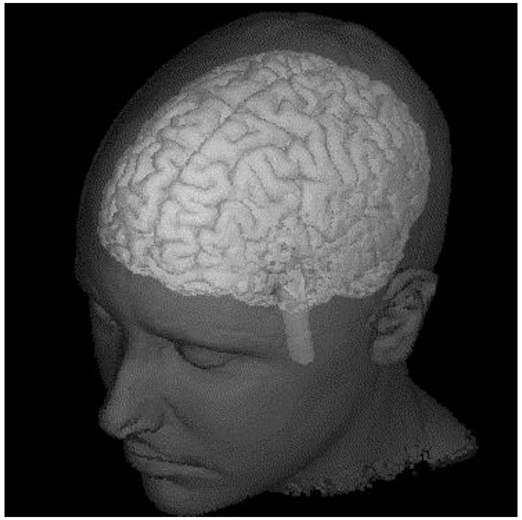
Surveillance/speed control

Face recognition

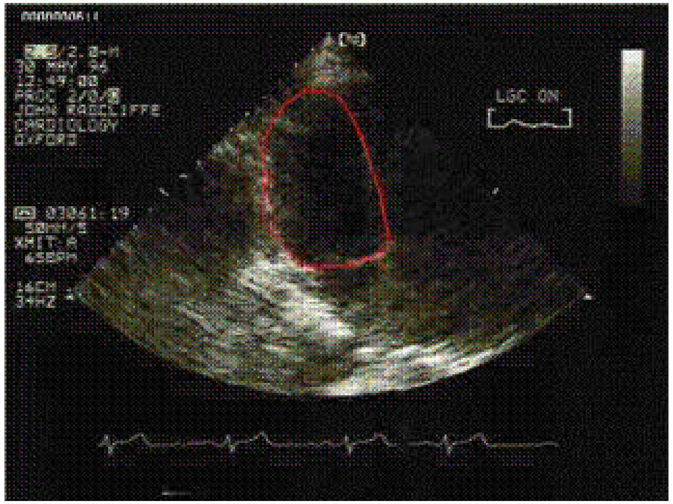
Applications - Medical



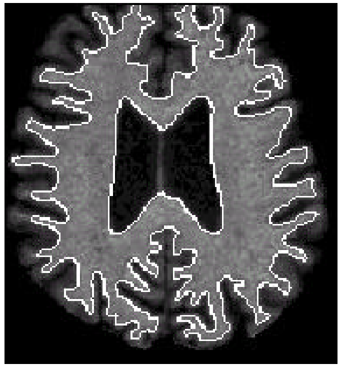
Blood vessels



3D brain

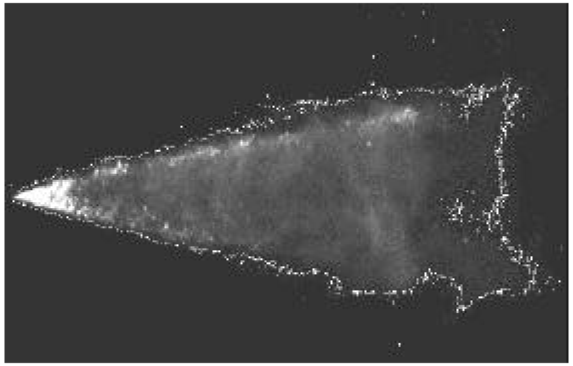


Heart in ultrasound



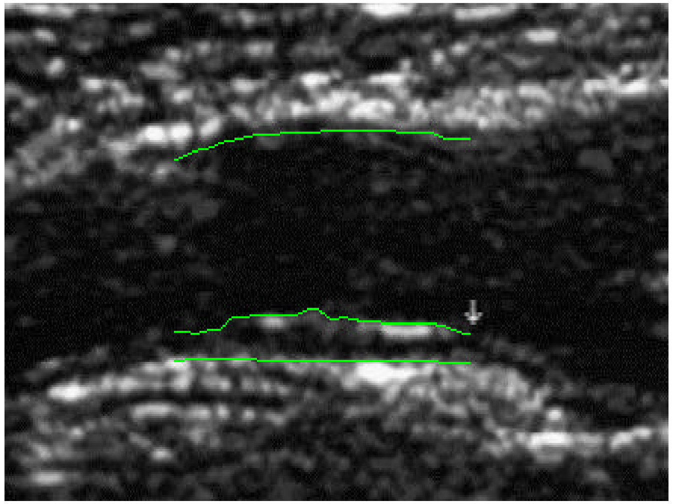
brain in MR

Applications - Our group

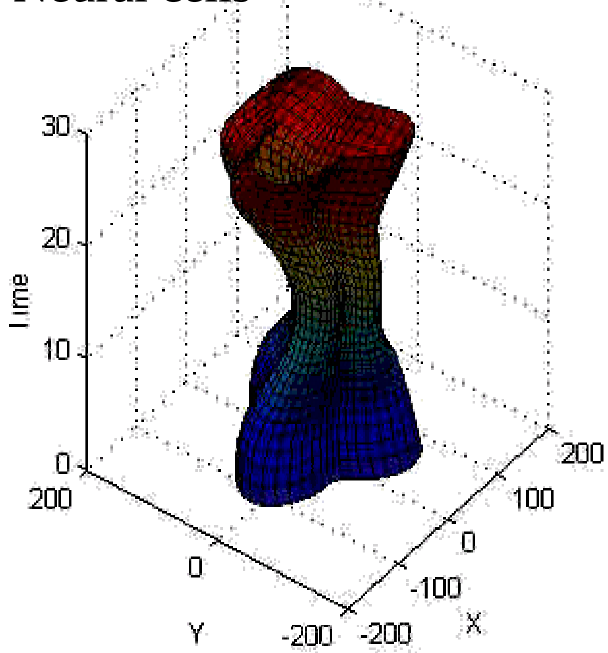


Spray segmentation

Neural cells



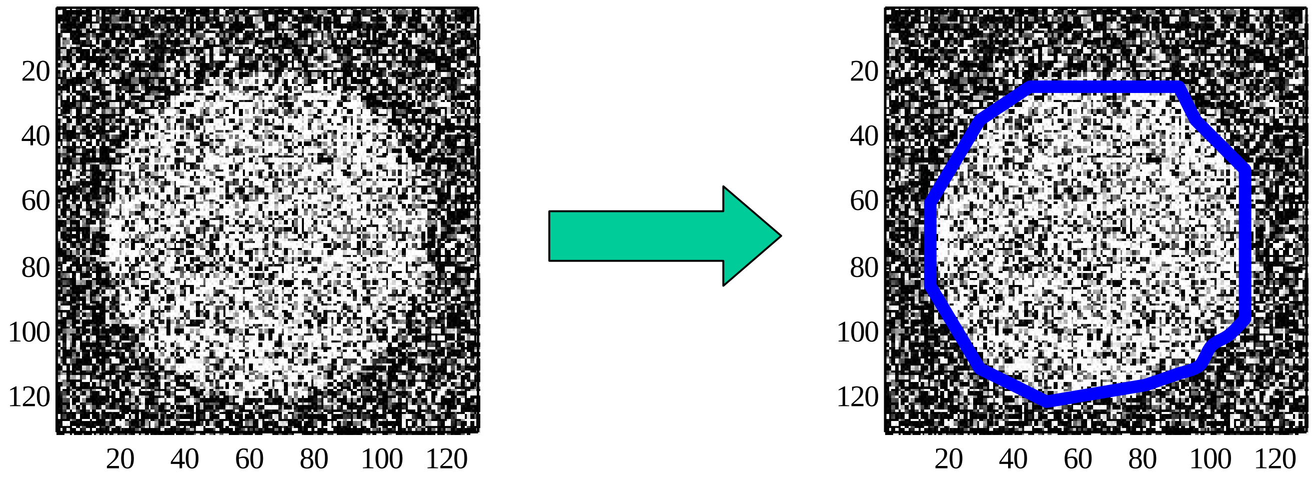
Boundary detection



Spatio-Temporal Segmentation

Problem

The basic problem is to locate an object in an image (segmentation, detection)



Part I

Active Contour Models (Snakes)

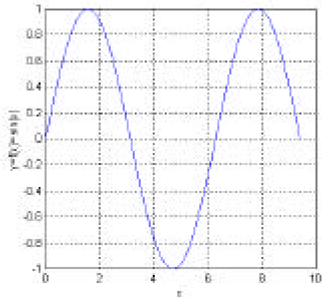
Contents for ACM

- Active Contour Models (Snakes)
  - Contour representation
  - Contour Energy
  - Formulation
  - Discretization
  - Segmentation
  - Notes

Contour Representation

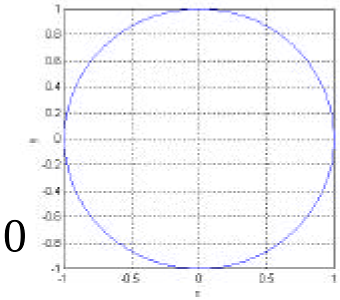
Although some curves can be represented explicitly by equations of the form:

$$y = f(x)$$
$$y = \cos(x)$$



or...

$$f(x, y) = 0$$
$$f(x, y) = x^2 + y^2 - 1 = 0$$



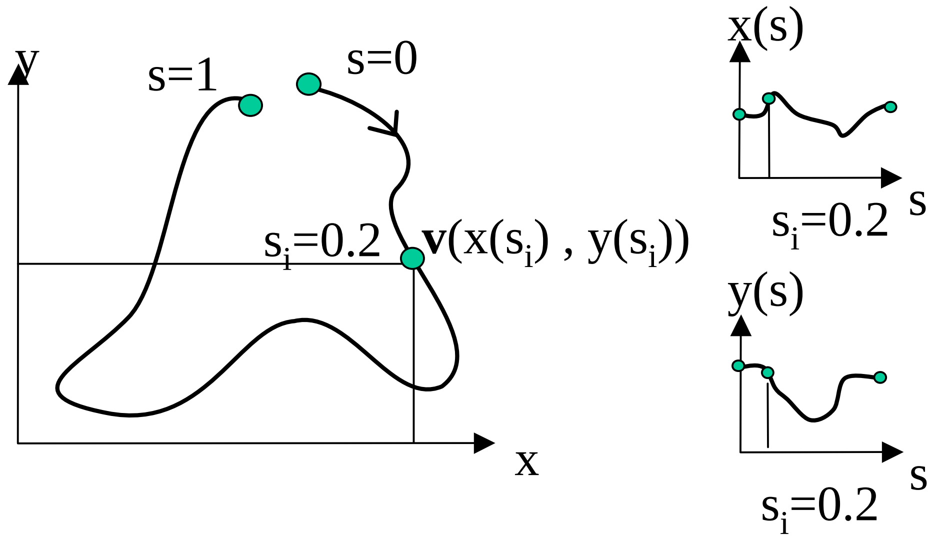
When working with complex contours usually implicit and parametric representations are used

$x = x(s); y = y(s)$ for a parameter *s*.

E.g. an ellipse for example is represented by

$x = a\cos(s); y = b\sin(s), s \in [-p, p]$

$$\mathbf{v}(s) = [x(s) \quad y(s)]^T, s \in [0,1]$$

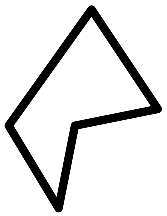


Contour Energy - Internal

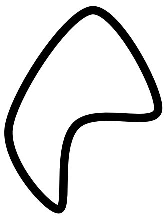
Assign low energy values to ‘good’ contours and high energy to ‘bad’ ones

Internal energy

In natural objects we usually require smooth contours



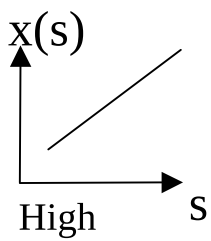
High energy

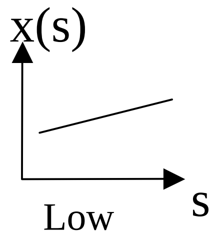


Low energy

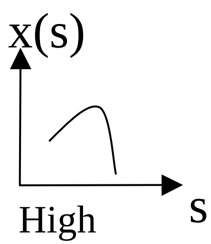
Internal Energy depends on the shape of the contour itself.

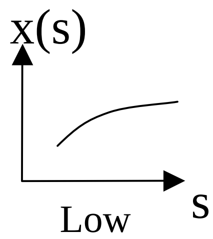
small $\frac{\partial \mathbf{v}}{\partial s}$ → tension in the contour, low internal energy





small $\frac{\partial^2 \mathbf{v}}{\partial s^2}$ → No bending in the contour, low internal energy



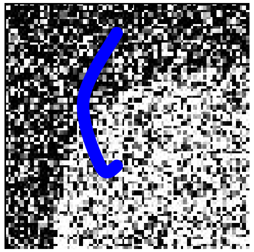


Contour Energy - External

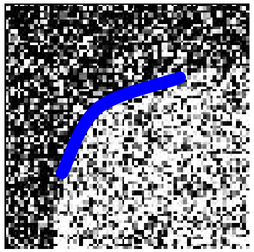
Assign low energy values to 'good' contours and high energy to 'bad' ones

External energy

When locating an object in an image...



High energy



Low energy

External Energy is derived from the image data

Look for high intensity gradient
(watch out for noise - gradient of smoothed image)

high $\nabla[G_s * I(x,y)]$ ⇒ Low external energy

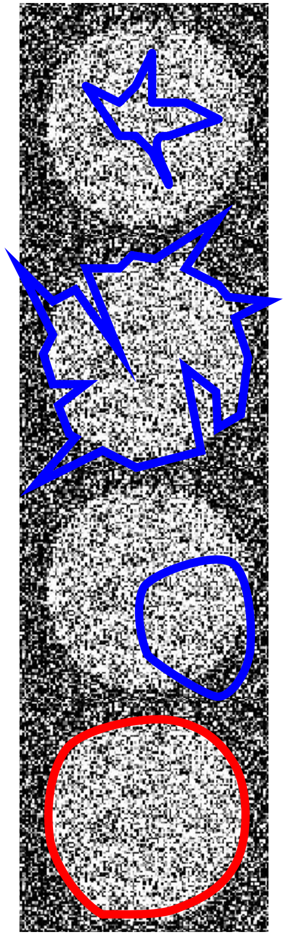
$G_s * I$ image convolved with a smoothing (ex. Gaussian) filter.

$I(x,y)$ image intensity

s parameter controlling the extent of the smoothing (ex. variance of Gaussian).

could have other than gradient e.g. intensity, termination, corners...

Contour Energy - Examples



☒ high-internal energy ☒
high-external energy ☒
(not smooth and away from target)

☒ high-internal energy ☒
low-external energy ☑
(not smooth and close to target)

☒ low-internal energy ☑
high-external energy ☒
(smooth and away from target)

☑ low-internal energy ☑
low-external energy ☑
(smooth and very close to target)

Formulation

Our problem is to find the contour that minimizes the total energy (internal+external) along the contour.

$$x(\mathbf{v}) = a(\mathbf{v}) + b(\mathbf{v})$$

$$a(\mathbf{v}) = \int_0^1 w_1(s)\left|\frac{\partial \mathbf{v}}{\partial s}\right|^2 + w_2(s)\left|\frac{\partial^2 \mathbf{v}}{\partial s^2}\right|^2 ds$$

$$b(\mathbf{v}) = \int_0^1 P(\mathbf{v}(s))ds$$

$$\mathbf{v}(s) = [\,x(s) \quad y(s)\,]^T, s \in [0,1]$$

$$P(x,y) = -c|\nabla[G_s * I(x,y)]|$$

w_1 and w_2 control the tension and rigidity

In accordance with the *calculus of variations*, the contour must satisfy vector valued partial differential (Euler-Lagrange) equation:

$$-\frac{\partial}{\partial s}\left(w_1 \frac{\partial \mathbf{v}}{\partial s}\right) + \frac{\partial^2}{\partial s^2}\left(w_2 \frac{\partial^2 \mathbf{v}}{\partial s^2}\right) + \nabla P(\mathbf{v}(s)) = \mathbf{0}$$

first two terms:
internal stretching and bending forces
the third term:
external forces

minimization balance of forces

Discretization

Require a discrete setting:

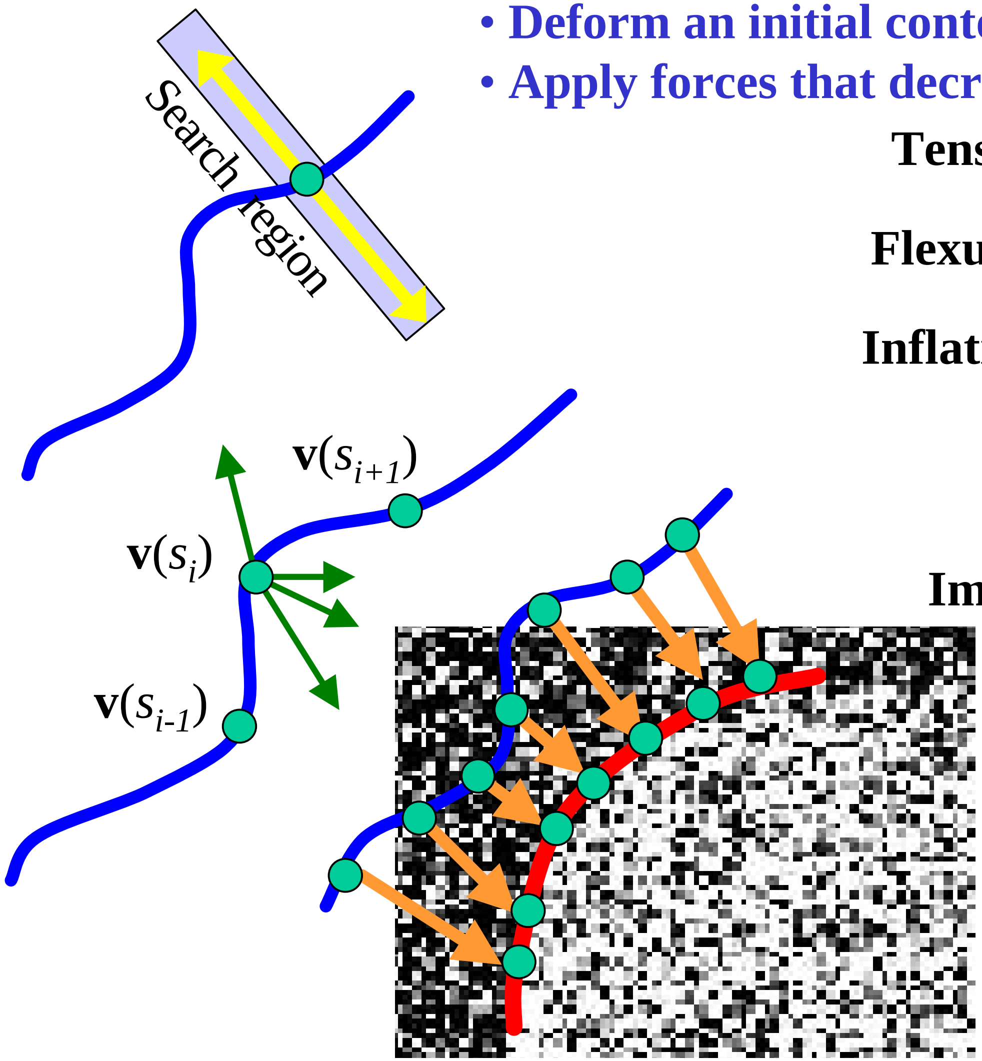
$$\mathbf{v}(s) \rightarrow \mathbf{v}(s_i)\ ;\ i = 1, 2, \ldots, N$$

$$\frac{\partial \mathbf{v}(s)}{\partial s} \rightarrow \mathbf{v}(s_{i+1}) - \mathbf{v}(s_i)$$

$$\int \cdots \, ds \rightarrow \sum$$

Segmentation

- **Deform an initial contour to one having less energy.**
- **Apply forces that decrease the energy.**



$$\textbf{Tensile force}_i = 2\mathbf{v}(s_i) - \mathbf{v}(s_{i-1}) - \mathbf{v}(s_{i+1})$$

$$\textbf{Flexural force}_i = 2a\,(s_i) - a\,(s_{i-1}) - a\,(s_{i+1})$$

$$\textbf{Inflation force}_i = qF(I(x(s_i), y(s_i)))\,\mathbf{n}_i$$

$$F(I(x,y)) = \begin{cases} +1 & \text{if } I(x,y) \geq T \\ -1 & \text{otherwise} \end{cases}$$

$$\textbf{Image force}_i = p\nabla P(x,y)$$

$$P(x,y) = -c|\nabla[G_s * I(x,y)]|$$

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p,q: constants
$\mathbf{n}_i$: unit vector normal to the contour at s_i

Notes

Snakes/Active contour models/deformable contour:
The contour is actively changing and deforming to have less energy (deformable ~ elasticity)

Disadvantages of manual segmentation:
- Difficult to obtain reproducible results
- Operator bias
- Viewing each 2D-slice separately
- Operator fatigue and time consuming

Types of deformable models/active contours:
- snakes
- deformable templates
- dynamic contours

Extensions to basic Snakes:
- Inflation force (makes snake less sensitive to initial conditions)
- ‘dynamic programming’ and ‘simulated annealing’ (search for global minima)
- Region-based image features
- Changing topology

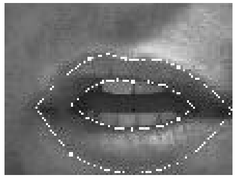
Part II

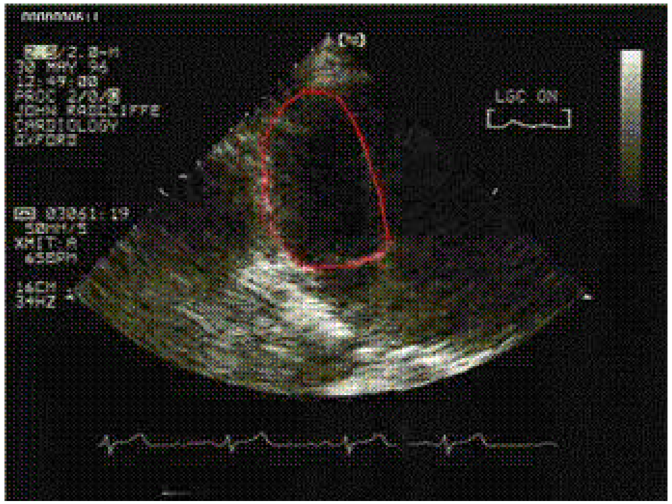
Active Shape Models (Smart Snakes)

Contents for ASM

- Active Shape Models (Smart Snakes)
  - Observation
  - Overview
  - Shape representation
  - Considerations
  - Training ASM
  - Applying ASM
  - Extensions

Observation





Why use the same ‘Snakes’ to locate different types of objects?

Incorporate knowledge about the desired object in the snake model

Train the snake smart snake

Overview

APPLICATION

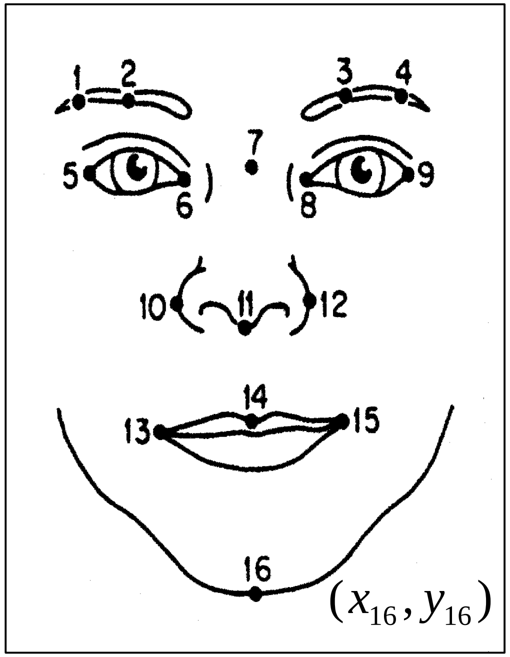
search for the hidden object armed with training results

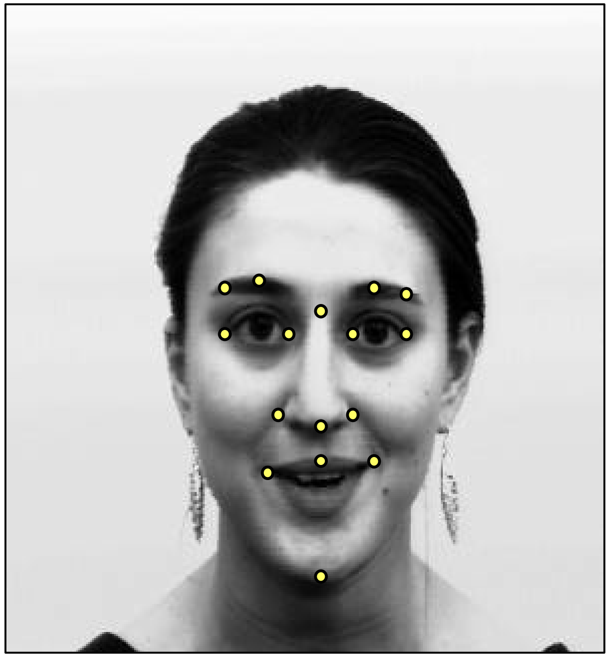
TRAINING &

Shape training
& gray level training

Shape Representation

Shapes are represented by *landmarks*





$$\mathbf{x} = \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ \vdots \\ x_k \\ y_k \\ \vdots \\ x_{16} \\ y_{16} \end{bmatrix}$$

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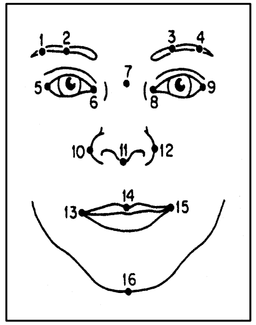
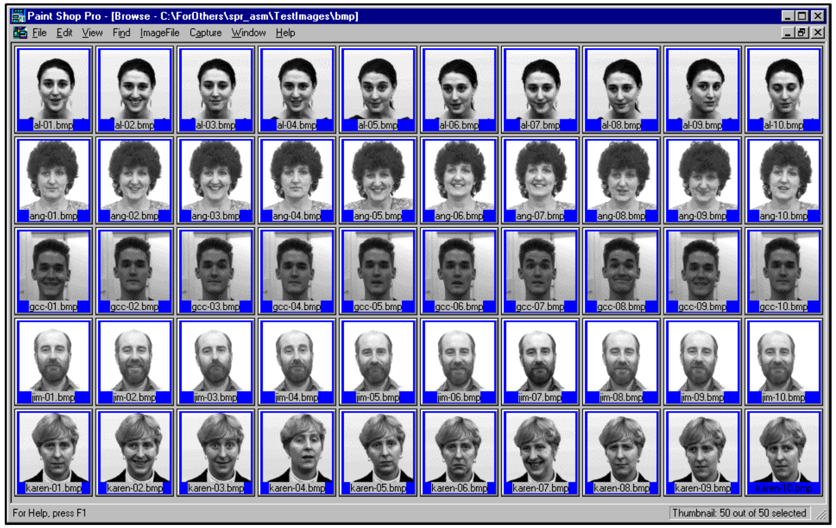
Considerations

Type of object to model

Example: faces

Locations of the Landmarks



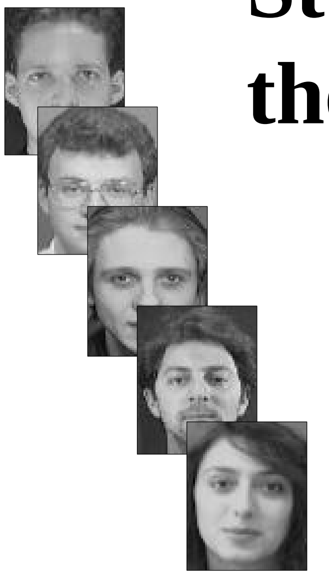


Collect images of the object

Shape Training

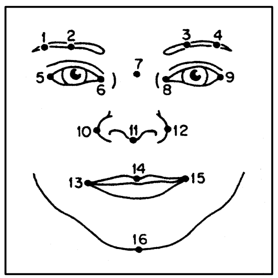
Study the variations of the landmark positions

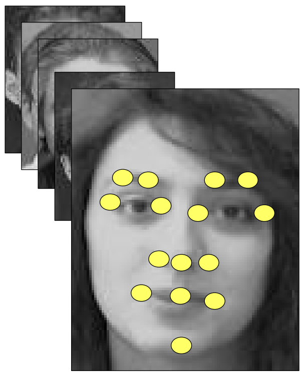
Now, we can...



Training set of images

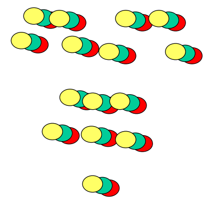
Labeling





Shapes represented by landmarks

Aligning



Aligned shapes

Shape Training Result - PDM

Capture the main modes of variation of the landmark positions

Using Principal Component Analysis (PCA) we obtain the...

Point Distribution Model (PDM)

$$\mathbf{x} = \bar{\mathbf{x}} + \mathbf{P}\mathbf{b}$$

2Lx1 2Lx1 2Lxt tx1

L:number of Landmarks

A Shape = the mean shape + weighted sum of different shape **variation modes**

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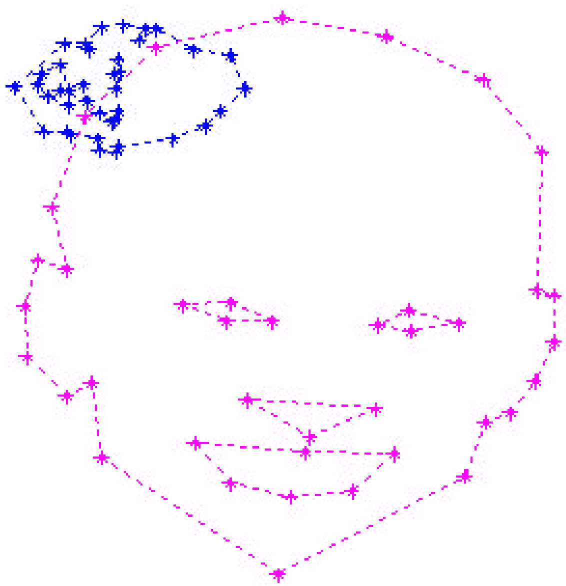
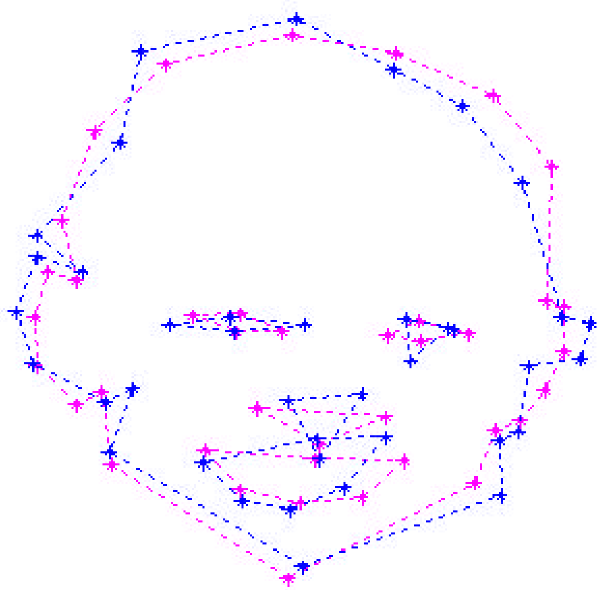
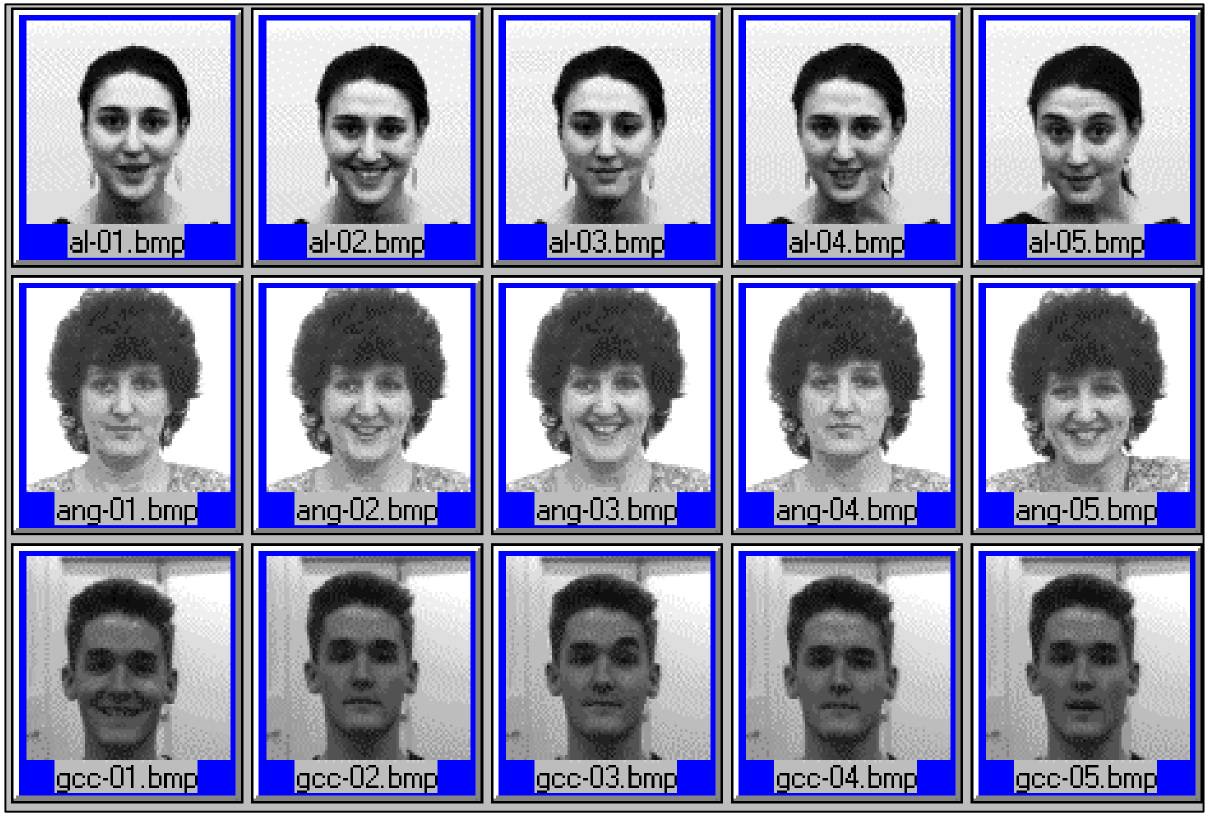
Aligning



<u>Rotate</u>, <u>scale</u>, and <u>translate</u> to *minimize* the distance between landmarks



Variation modes



Causes of variations...

- 3D pose
- Expression
- Identity (subject)

Shape Training Equations

$$\mathbf{x} = \bar{\mathbf{x}} + \mathbf{P}\mathbf{b}$$

$$\mathbf{x} = [x_1, y_1, x_2, y_2, \ldots, x_n, y_n]^T$$

$$\bar{\mathbf{x}} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{x}_i$$

$$\mathbf{P} = [\mathbf{p}_0 \quad \mathbf{p}_1 \quad \ldots \quad \mathbf{p}_{t-1}]$$

$$\mathbf{b} = [b_0 \quad b_1 \quad \ldots \quad b_{t-1}]^T$$

Constrained, only allowable variations

$\mathbf{P}$ is obtained from the eigenvectors of the covariance matrix (PCA)

$$\mathbf{C}_\mathbf{x} = \frac{1}{N-1}\sum_{i=1}^{N}(\mathbf{x}_i - \bar{\mathbf{x}})(\mathbf{x}_i - \bar{\mathbf{x}})^T$$

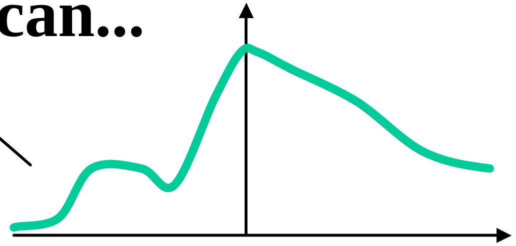
$$\mathbf{C}_\mathbf{x}\mathbf{p}_i = l_i\mathbf{p}_i$$

The corresponding eigenvalues equal the variance explained by each variation mode

Gray Level Training

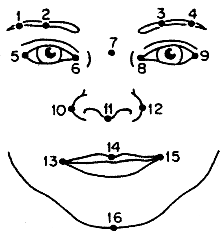
Study the gray levels around the landmarks

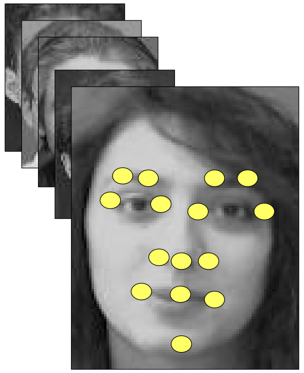
Now, we can...



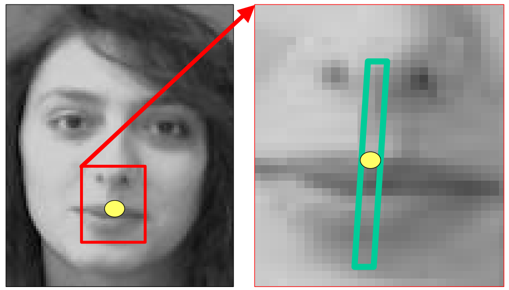
Training set of images

Labeling





Shapes represented by landmarks



Extract gray level profiles around landmarks

Gray Level Training Result

Summarize the information about the Gray levels around landmarks

Each landmark ⟷ **Mean profile**

mean normalized derivative profile

Gray Level Training Equations

$$\mathbf{g}_{ij} = \begin{bmatrix} g_{ij1} & g_{ij2} & \dots & g_{ijn_p} \end{bmatrix}^T$$

Gray level profile for landmark *j in image i*

$$d\mathbf{g}_{ij} = \begin{bmatrix} g_{ij2} - g_{ij1} & g_{ij3} - g_{ij2} & \dots & g_{ijn_p} - g_{ijn_p-1} \end{bmatrix}$$

The derivative profile for landmark *j in image i*

$$\mathbf{y}_{ij} = \frac{d\mathbf{g}_{ij}}{\sum_{k=1}^{n_p-1} \left| d\mathbf{g}_{ijk} \right|}$$

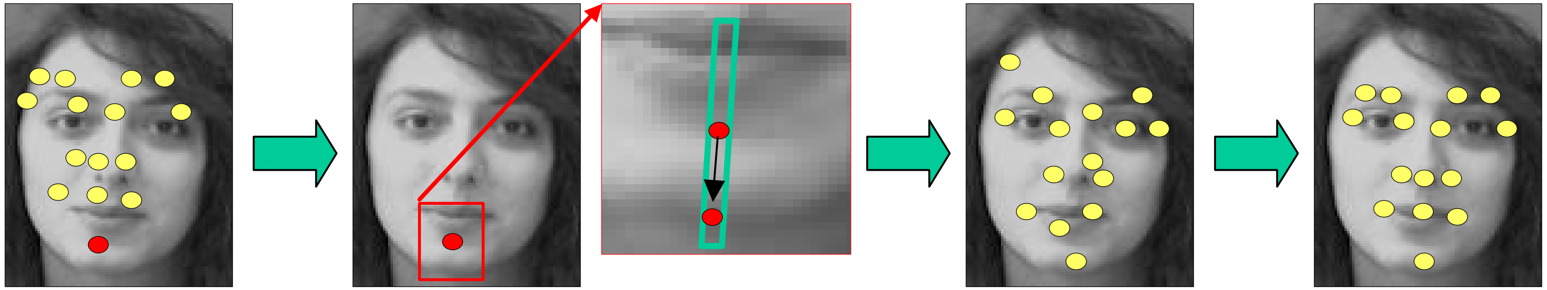
The normalized derivative profile for landmark *j in image i*

$$\overline{\mathbf{y}}_j = \frac{1}{N} \sum_{i=1}^{N} \mathbf{y}_{ij}$$

The mean normalized derivative profile for landmark *j*

Image Search

Using the shape and the gray level information to find instances of shapes in new images



Start with an initial shape

Search around landmarks for expected gray levels and propose a new shape

Generally, an unallowable shape is obtained

Force only main modes of variation

Find the required rotation, scaling, translation, and shape parameters updates needed to deform the current estimate to the proposed shape

Notes

Extensions to ASM

- Automatic landmark generation for PDMs
- Multi resolution image search
- Active Appearance Models (AAMs)
- Genetic algorithms
- Extension to 3D data
- Other shape representations
- Classification using ASM

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References

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- Snakes: Active Contour Models. Kass, Witkin and Terzopoulos. International Journal of Computer Vision. 1(4):321-331, 1987.
- Cootes, Taylor, Cooper and Graham. Active Shape Models - Their Training and Application. Computer Vision and Image Understanding January 1995, volume: 61, No. 1, page(s): 38-59.