Morphological operations

Mathematical morphology and binary operations

- Erode/Dilate
- Open/Close
- Hit-Miss
- Skeleton
- Distance transforms
- Shape features

Mathematical morphology and binary operations

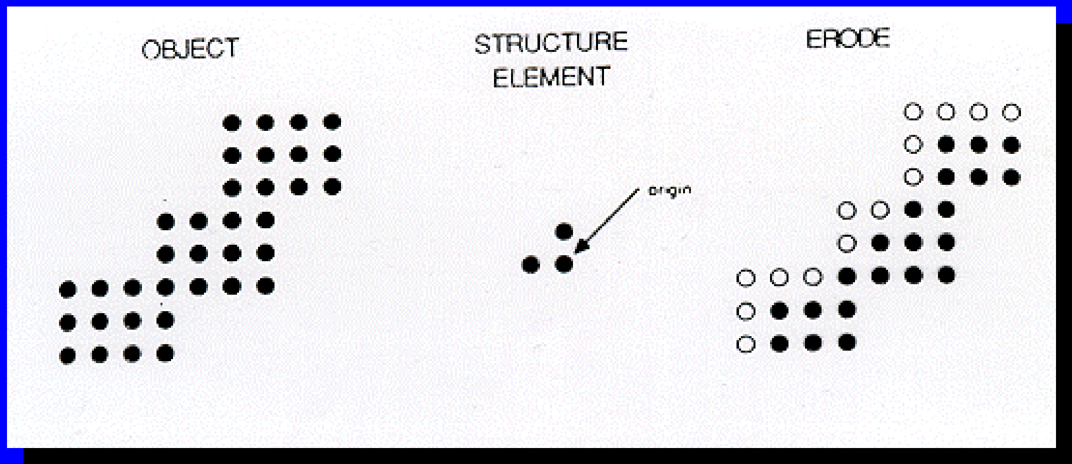
Based on set theory

Loosely: Add or remove pixels from a binary image according to certain rules depending on neighborhood patterns

Erosion

Let B_x denote the translation of B so that its origin is located at x. Then the erosion of X by B is defined as the set of all points x such that B_x is included in X, that is:

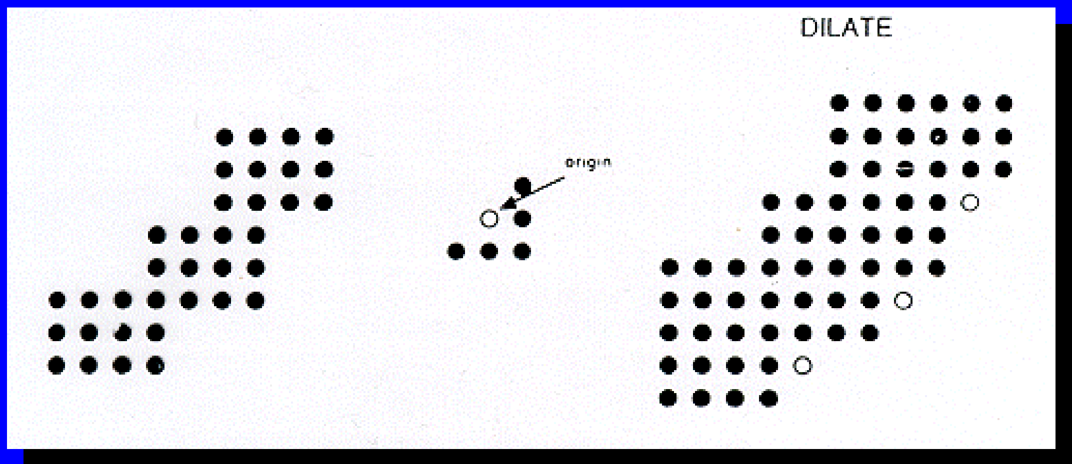
$$X \ominus B \cong \{x | B_x \subset X\}$$



Dilation

Similarly, the dilation of X by B is defined as the set of all points x such that B_x hits X, that is, they have a non-empty intersection:

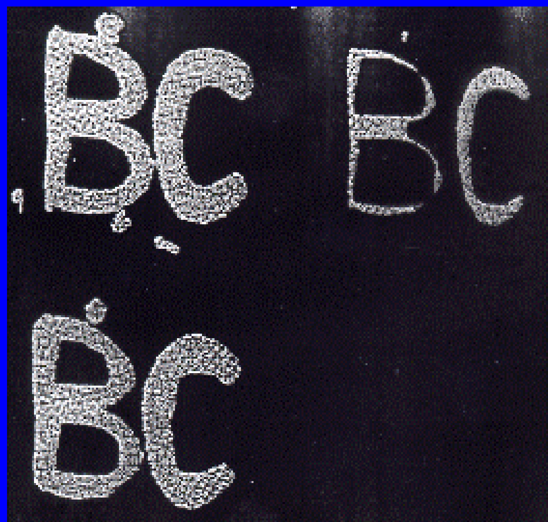
$$X \oplus B \; @ \left\{ x \mid B_x \cap X \neq \varnothing \right\}$$



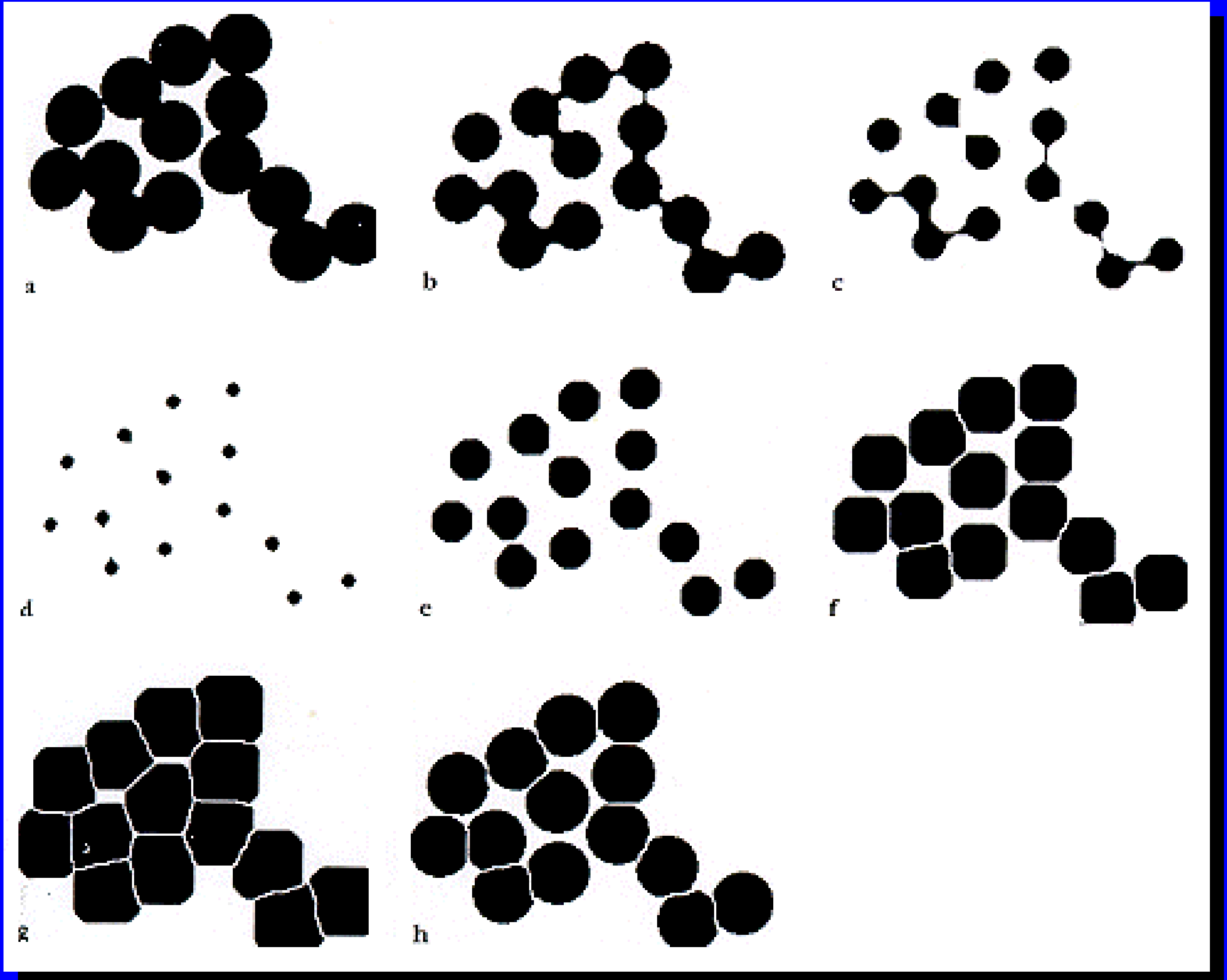
Open

An erosion operation followed by a dilation:

$$X_B = \left(X \ominus B \right) \oplus B$$



Open: example

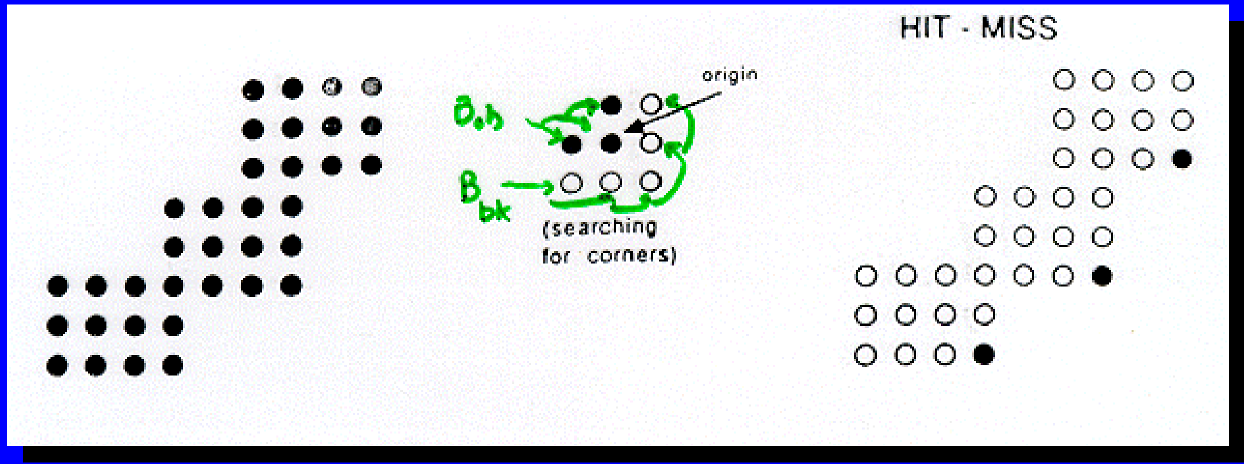


Close

A dilation operation followed by an erosion :

$$X^{B} = \left(X \oplus B \right) \ominus B$$

Hit-Miss



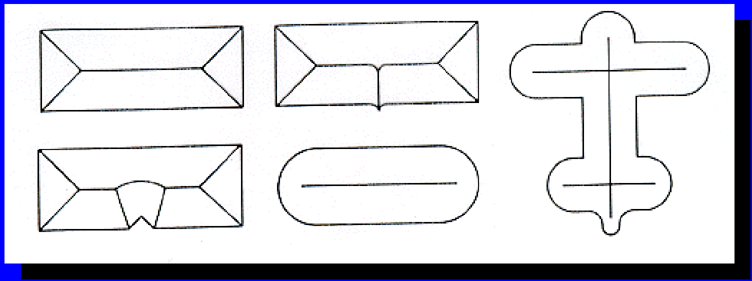
Corresponds to searching for a match or a specific configuration:

B_{ob} and B_{bk} are the sets formed from pixels in B that should belong to the object and the background, respectively:

$$X \otimes B = (X \ominus B_{ob}) / (X \oplus B_{bk})$$

where / means set difference

Skeletonization



Definition: The set of points whose distance from the nearest boundary is locally maximum

Shape recovery: Take the union of circular discs centered on the skeleton and having radii equal to the associated contour distance

Application: Compact shape representation. Works best for long and thin features

Alternative: Medial axis transform. Those points that are equidistant from at least two boundary points (Prairie fire)

Skeletonization

Phase1 : West

	0	
0	1	1
	1	1

	1	1
0	1	1
	0	

		1
0	1	1
		1

	0	
0	1	1
	0	

Phase2 : North

	0	
1	1	0
1	1	

	0	
0	1	1
	1	1

	0	
	1	
1	1	1

	0	
0	1	0
	1	

Phase3 : East

1	1	
1	1	0
	0	

	0	
1	1	0
1	1	

1		
1	1	0
1		

	0	
1	1	0
	0	

Phase4 : South

	1	1
0	1	1
	0	

1	1	
1	1	0
	0	

1	1	1
	1	
	0	

	1	
0	1	0
	0	

Skeletonization

The algorithm runs in four phases: west-north-east-south

Each of the four phases are associated with a set of hit-miss operators.

If the number of operators in the set is 3 (the three leftmost) the object will shrink to a skeleton. If the number is 4, it will shrink to a point.

Skeletonization: algorithm

1. Run phase 1 (west). For each pixel position in the input image, test the west hit-miss operators for a hit.
2. If any of the operators results in a hit, set the corresponding position in the output image to zero. Otherwise, set to one.
3. Proceed in the same manner with phase 2, 3 and 4. If there still remain pixel positions that can be set to zero by a hit, go to 1.
4. The skeleton (or point) is extracted

Distance metrics

a) Euclidian $d_e(x,y)=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

b) City block $d_{cb}(x,y)=|x_1-x_2|+|y_1-y_2|$

c) Chessboard $d_{ch}(x,y)=\max\{|x_1-x_2|,|y_1-y_2|\}$

		3	3	3		
	3	2	2	2	3	
3	2	1	1	1	2	3
3	2	1	**[0]**	1	2	3
3	2	1	1	1	2	3
	3	2	2	2	3	
		3	3	3		

a)

			3			
		3	2	3		
	3	2	1	2	3	
3	2	1	**[0]**	1	2	3
	3	2	1	2	3	
		3	2	3		
			3			

b)

3	3	3	3	3	3	3
3	2	2	2	2	2	3
3	2	1	1	1	2	3
3	2	1	**[0]**	1	2	3
3	2	1	1	1	2	3
3	2	2	2	2	2	3
3	3	3	3	3	3	3

c)

Distance transform

The distance transform uses a binary image as input and produces for each pixel position a value which is the distance from that pixel to the nearest background pixel.

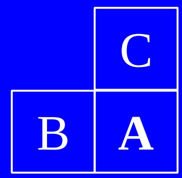
Application areas:

- Fast morphological operations
- Skeletonization
- Computation of shape features (e.g. form factor)

Distance map using chessboard metrics algorithm

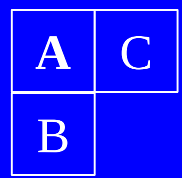
Phase 1: apply this filter from the upper left corner of the image:

If A>0: A=min(B+1, C+1)



Phase 2: apply this filter from the lower right corner of the image:

If A>0: A=min(A,B+1, C+1)



Distance map result

	1	1	1		1	1	1		
	1	2	2		1	2	2		
	1	2	3						
	1	2	3		1	1	1	1	1
	1	2	3		1	2	2	2	2
					1	2	3	3	3
		1	1		1	2	3	4	4
	1	2			1	2	3	4	5

Phase 1

	1	1	1		1	1	1		
	1	2	1		1	1	1		
	1	2	1						
	1	2	1		1	1	1	1	1
	1	1	1		1	2	2	2	1
					1	2	3	2	1
		1	1		1	2	2	2	1
	1	1			1	1	1	1	1

Phase 2

Shape features

Shape factor : $S = \frac{p^2}{4p\, A}$

$A = Area$

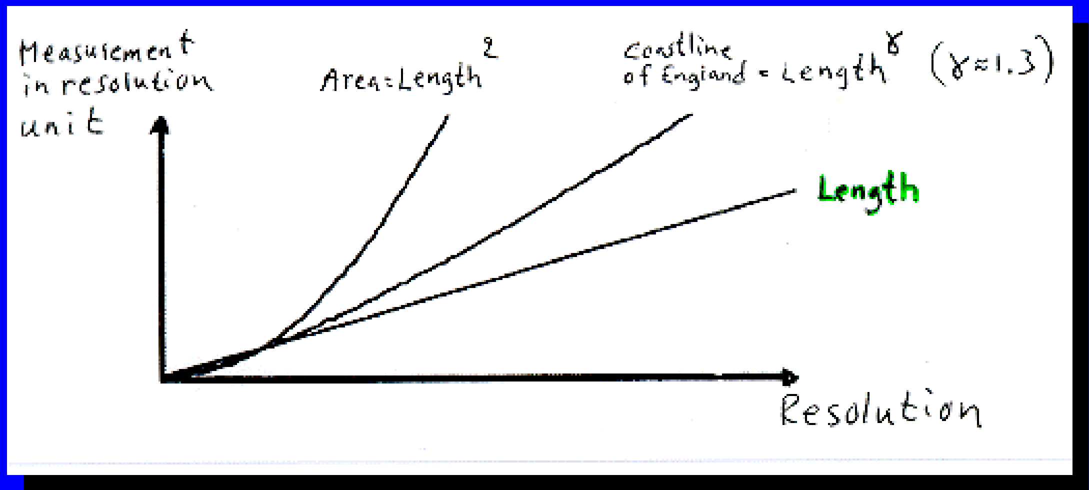
$p = perimeter$

$S = 1$ for a circle

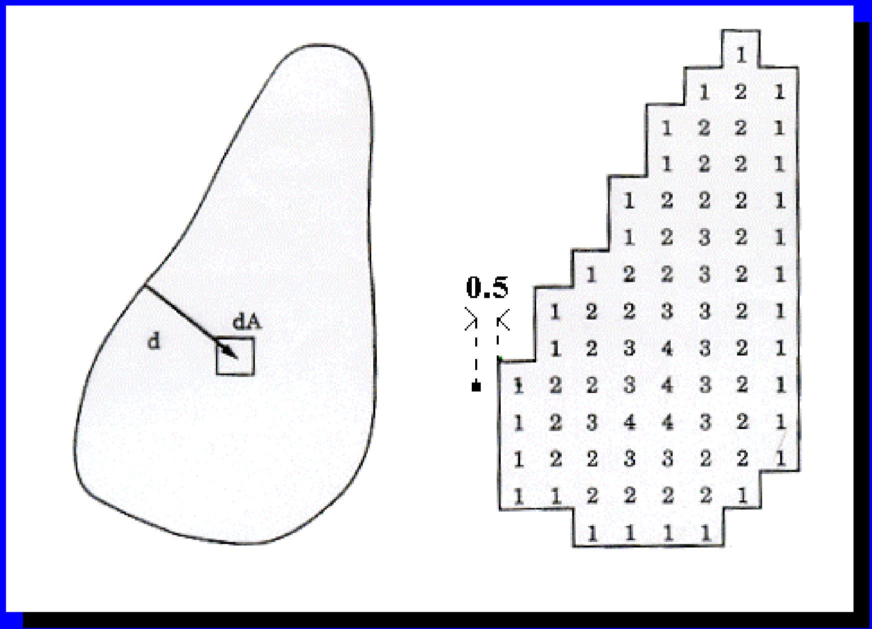
Great Britain

Area can be determined, what about perimeter?

Higher resolution: Perimeter $\rightarrow \infty$



Shape features



$$\bar{d} = \frac{1}{A}\iint_A d(x,y)\,dA \quad \text{(for a circle, } \bar{d} = r/3\text{)}$$

$$FORM = \frac{1}{9p} \cdot \frac{A}{\bar{d}^2} \quad \text{(for a circle, } FORM = 1\text{)}$$

$$\bar{d} = \frac{\sum_A \left[d(x,y) - \frac{1}{2} \right]}{A} = \frac{\sum_A d(x,y)}{A} - \frac{1}{2}$$

where A is the number of pixels in the object

$$\bar{d} = \frac{30 \cdot 1 + 31 \cdot 2 + 12 \cdot 3 + 4 \cdot 4}{77} - 0.5 \approx 1.37$$

$$FORM \approx \frac{1}{9p} \cdot \frac{77}{(1.37)^2} \approx 1.45$$