OPTIMAL THRESHOLDING

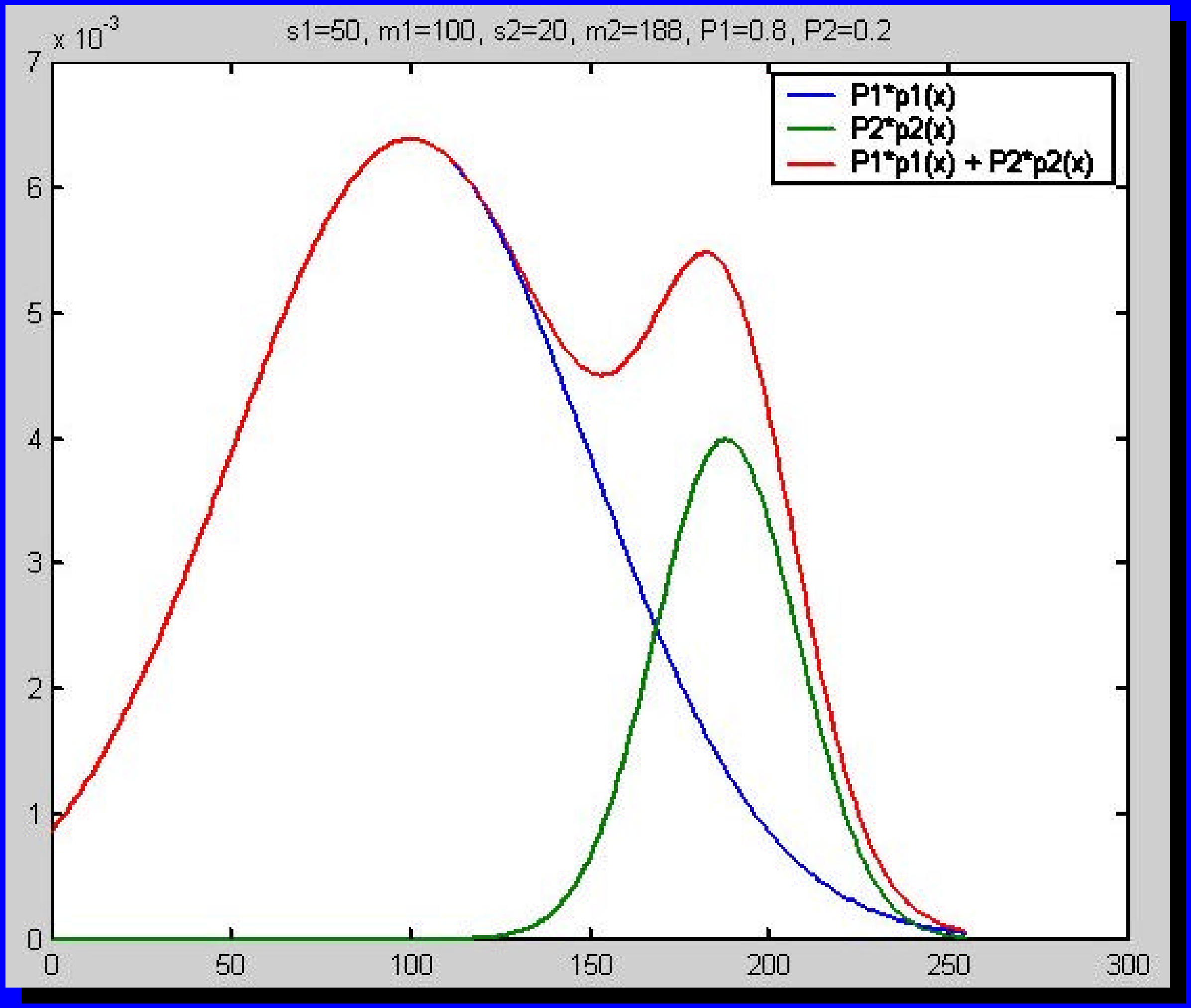
Assume that the original image contains 2 gray-scale components that represent the background and the object, respectively: $p(x) = P_1 p_1(x) + P_2 p_2(x)$

$$p_1(x) = \mathrm{Gauss}(\sigma_1, \mu_1)$$

$$p_2(x) = \mathrm{Gauss}(\sigma_2, \mu_2)$$

P_1 and P_2 represents the a priori probabilities for the background and the object, respectively

($P_1 + P_2 = 1$)



The probability to wrongly classify an object pixel as a background pixel is:

$$E_1(T) = \int_{-\infty}^{T} p_2(x)dx$$

The probability to wrongly classify a background pixel as an object pixel is:

$$E_2(T) = \int_{T}^{\infty} p_1(x)dx$$

The total probability of false classification is thus:

$$E(T) = P_2 \cdot E_1(T) + P_1 \cdot E_2(T)$$

Differentiating *E(t)* with respect to *T*, and setting to 0, yields:

$$P_1 \cdot p_1(T) = P_2 \cdot p_2(T)$$

Taking the log of the expression, and after simplification, we obtain a 2nd degree equation:

$$AT^2 + BT + C = 0 \quad \begin{cases} A = \sigma_1^2 - \sigma_2^2 \\ B = 2\left(\mu_1\sigma_2^2 - \mu_2\sigma_1^2\right) \\ C = \sigma_1^2\mu_2^2 - \sigma_2^2\mu_1^2 + \sigma_1^2\sigma_2^2 \ln(\sigma_1 P_1/\sigma_2 P_2) \end{cases}$$

If the variances are equal ($\sigma_1^2 = \sigma_2^2 = \sigma^2$), then we get a single solution for T (otherwise, we get 2 solutions):

$$T = \frac{\mu_1 + \mu_2}{2} + \frac{\sigma^2}{\mu_1 - \mu_2} \ln(P_2/P_1)$$

If $P_1 = P_2$, then T is the mid-value of the mean values: $T = \frac{\mu_1 + \mu_2}{2}$

a=true positive (true object)

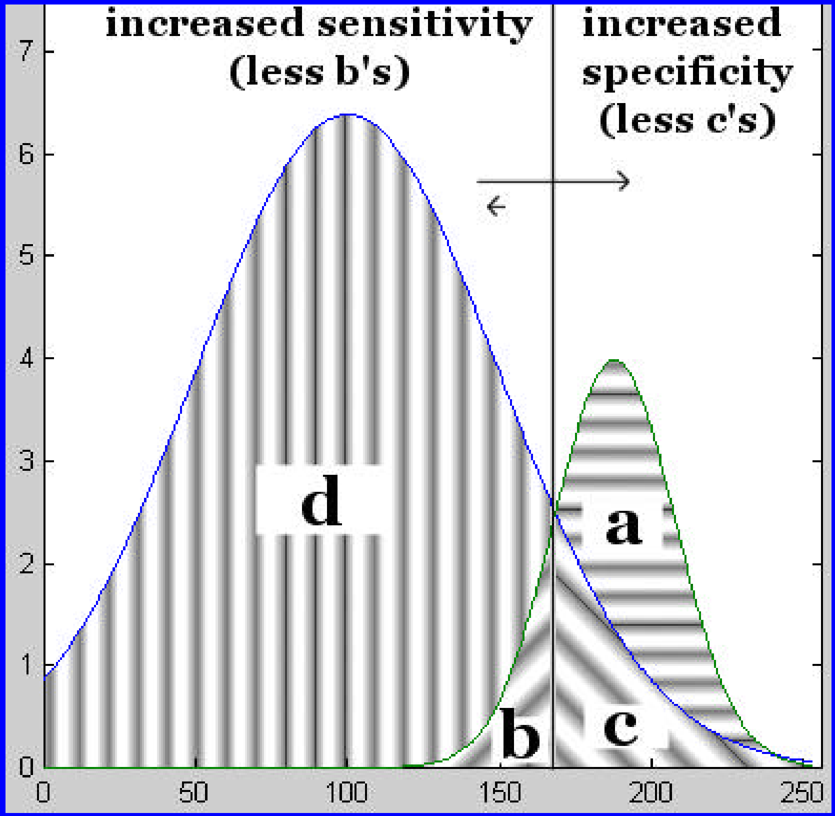
b=false negative (false background)

c=false positive (false object)

d=true negative (true background)

Sensitivity=a/(a+b)

Specificity=d/(c+d)



Increased sensitivity: You will miss less true object pixels but the price you pay is more false object pixels. Good if it is dangerous to miss object pixels

Increased specificity: You will get less false object pixels but may miss true object pixels. Good if it is expensive to respond to false object pixels